Two Way Selection Traffic Flow: Mean Field Theory

Bing-Hong Wang[1], P.M. Hui, [2], Chin-Kun Hu [3]

[1] *Department of Modern Physics and Nonlinear Science Center,*

University of Science and Technology of China, Hefei 230026, China

Email: bhwang@ustc.edu.cn

[2] *Department of Physics, The Chinese University of Hong Kong,*

Shatin, New Territories, Hong Kong Email:pmhui@phy.cuhk.edu.hk

[3] *Institute of Physics, Academia Sinica, Nankang, Taipei 11529, Taiwan*

Email: huck@phys.sinica.edu.tw

Using a simple route choice traffic scenario, Ref. [1] studied the influence of dynamical information on the stability of traffic patterns. In order to understand home possible effect of information for traffic controlling, in this paper, we propose and study the mean field theory of route choice decision traffic flow.

Suppose that there are two routes A and B with the same number of cells L. Every time step a road user enters the system and has to chose one route. Two different types of vehicles are considered: "normal" ones and the so-called floating cars (FCs). When leaving the system, FCs can transmit their travel times to a board visible to every vehicle. Hence FCs generate dynamic traffic information which is the basis for the route decisions of new car entering the system. The number of FCs is represented by the variable s_{FC}. The road users are divided as static and dynamic types. Static drivers ignore the dynamic information and select route at random. Dynamic drivers receive the information displayed on the board and always choose the route with the shortest travel time, i.e. they behave "rational". If the travel time on both routes is the same, the route is chosen randomly. The number of dynamic drivers is represented by the variable s_{dyn}. After the route choice both kinds of vehicles behave in the same way. All the cars move ahead along the chosen lane according to Fukui-Ishibashi (FI) traffic flow cellular automaton model [2]. To adopt FI model instead Nagel-Schreckenberg (NS) model is due to the convenience of presenting analytical approach to FI model. The motion of the vehicles can be described by the following rules (parallel dynamics):

R1: rapid acceleration: $v_i \to \min(v_{\max}, C_i)$

R2: random delay for the cars with the maximal velocity: If $v_i = v_{\max}; v_i \to v_{\max} - 1$ with probability p.

It is important to note that because the travel time information does not represent the current real situation of crowding along the route, the decisions of dynamic drivers are not always correct. The information delay introduces the oscillation of the system. We consider the mean field theory of the decision dynamics in first order approximation for two way choice traffic flow model.

It can be proved that for FI CA model with maximal velocity M and delay probability p, if at every time step, a car at the entry enters a route with probability w, the average traffic flux along the road is

$$\Phi(M, w, p) = (1-p)^M (w - \frac{w^{M+1}}{M+1}) + [1-(1-p)^M](w - \frac{w^M}{M})$$

If all the dynamic cars choose the route A according to the travel time information on the board, the probabilities of entering the route A and B for a car at the entry in any time step will be respectively $w_1 = S_{\text{dyn}} + (1 - S_{\text{dyn}})/2$ and $w_2 = (1 - S_{\text{dyn}})/2$

It can also be proved that the oscillation period of decision dynamics for two ways choice traffic is $T = 2\tau$ where τ is the average travel time of a car along a route when the entering probability of a car at the entry keeps as w_1.

For FI traffic flow CA model, we have obtained the exact analytical expression of the fundamental diagram, i.e. the average speed v as the function of vehicle density ρ, the maximal velocity of a car M, and the delay probability p as follows: [3]

$$v(M, p, \rho) = \frac{M - 1 + \frac{1}{\rho} - \sqrt{(\frac{1}{\rho} - 1 - M + 2p)^2 + 4p(1-p)}}{2}$$

When the probability of entering route A for a car at the entry keeps as $w_1 = S_{\text{dyn}} + (1 - S_{\text{dyn}})/2$, the average traffic speed v_1 along the route A and the average vehicle density

ρ_1 can be determined from $\Phi(M, p, w_1) = \rho_1 v_1(M, p, \rho_1)$. Similarly, when the probability of entering route B keeps as $w_2 = (1 - S_{\text{dyn}})/2$, the average traffic speed v_2 and average vehicle density ρ_2 along the route B can be determined from $\Phi(M, p, w_2) = \rho_2 v_2(M, p, \rho_2)$. Since the travel time can be solved as $\tau = L/v$, the theoretical value of oscillation period for two-way choice traffic model can be determined as

$$T_{\text{theory}} = 2L\rho_1/\Phi(M, p, w_1) = 2L\rho_2/\Phi(M, p, w_2)$$

The minimal (maximal) number of cars distributed on one of routes $N_{\min}$ ($N_{\max}$) for the two-way traffic model can be determined as $N_{\min} = L\rho_2$ and $N_{\max} = L\rho_1$ Hence, our mean field theory can present the oscillation phenomena of decision dynamics for two-way choice traffic. After comparing the theory with simulation result, we can see good agreement not only qualitatively but also quantitatively between our mean field theory in first order approximation and simulations for decision dynamics of two-way choice traffic flow.

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