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Stochastic resonance in coupled threshold elements on a Barabási–Albert network

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Abstract

Stochastic resonance is investigated in a system of threshold elements located at nodes and coupled along edges of a Barabási–Albert network, driven by a common subthreshold periodic signal and independent noises. Array-enhanced stochastic resonance is observed, i.e., increase of the spectral power amplification evaluated from the mean output of the network due to proper coupling. This enhancement occurs though the response of individual threshold elements to the periodic signal is very diverse due to a power-law distribution of their connectivity. Numerical results are qualitatively explained using simple linear response theory in the mean field approximation.

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Keywords: Stochastic resonance; Array-enhanced stochastic resonance; Scale-free networks

1. Introduction

Stochastic resonance (SR) [1] is a phenomenon where noise plays a constructive role by enhancing response of a nonlinear system to a periodic signal (for review see Refs. [2–4]). This response can be characterized, e.g., by spectral power amplification (SPA) defined as the ratio of the output signal power at the periodic stimulation frequency to the power of the input periodic signal; in systems with SR the SPA shows maximum as a function of the noise intensity [5]. SR was demonstrated in various low-dimensional

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systems, e.g., bistable [5,6] and monostable [7] potential systems, dynamical [8] and non-dynamical [9] threshold-crossing systems, threshold elements [10], etc. An important property of SR is that it can be enhanced due to proper coupling between low-dimensional stochastic resonators, i.e., the maximum SPA can be substantially increased in comparison with that in a single uncoupled resonator. This effect of array-enhanced SR was shown in stochastic systems with mean-field coupling [11], including models of neural networks [12,13], as well as in periodic networks of coupled units, e.g., in chains and lattices of bistable [14–16], monostable [17] and threshold [18] elements with nearest-neighbour coupling, in the Ising model on regular lattices in various dimensions [19,20], etc.

In the last years interest in networks with non-trivial topology, much more complex than simple periodic lattices, has increased rapidly [21,22] (for review see Ref. [23]). It has been motivated by the desire to understand the origin and properties of such weblike structures as social networks, the Internet, world-wide web, spreading of diseases, etc., which are of high importance for the modern society. Apart from the topological properties of complex networks, dynamical phenomena in coupled systems defined on them also received considerable attention. For example, in small-world networks [21], which are created from regular lattices by cutting and rewiring at random a given fraction of connections (edges), phase transitions in the Ising model [24,25], chaos [26], and SR in networks of bistable elements or Ising spins were studied [27–29]. Another important class of networks are evolving networks, first proposed by Barabási and Albert (BA) [22], in which newly added nodes are connected to the existing ones according to a probabilistic rule, preferring connections to nodes which already have many edges. Such networks are scale-free, i.e., the connectivity (number of edges of individual nodes) has a power-law distribution. In this case, much less is known about dynamics on such networks: e.g., synchronization [30], and ferromagnetic phase transition in the Ising model [31,32] were investigated.

In this paper, SR is studied in threshold elements (TE) located on the nodes and coupled along the edges of the BA network, driven by independent noises and a weak periodic signal. The response of individual TE to the periodic stimulation, characterized by their SPA, is highly non-uniform due to the power-law connectivity distribution, which results in a broad distribution of the SPA values. Anyway, numerical simulations and qualitative analysis based on linear response theory in mean-field approximation show that the effect of array-enhanced SR takes place in the BA network, like in networks with more uniform topology. Both the SPA from the average output of all TE and the mean SPA from individual TE can be significantly increased due to proper coupling. Thus, this paper extends the concept of array-enhanced SR to the case of scale-free networks of coupled stochastic resonators.

2. The model and methods of analysis

In order to define the model, first the BA network is created. At the first step, a small number m of fully connected nodes is fixed. Then, step by step, new nodes are added, and each new node is connected to existing nodes with m edges according to the following probabilistic rule: probability of linking to a node i is $p_i = k_i / \sum_i k_i$, where k_i is the actual connectivity of the node i, and $\sum_i k_i$ is the actual number of edges in the whole network. Multiple connections between nodes are allowed. The growth process is continued as long as the total number of nodes N is reached, when the network structure is frozen. For large N, this "preferential attachment" rule results in the network with the mean connectivity $\langle k \rangle = 2m$ and the connectivity distribution $P(k_i) = 2m^2 k_i^{-3}$ [22,23,33–35]. Then, at each node a TE is located, and the network edges are treated as mutual symmetric connections between TE, of a kind typical of artificial neural networks. Finally, the TE are subject to external noise and a subthreshold periodic signal.

The output $y_i(t)$ of each TE i, $i = 1, 2, \ldots, N$, at discrete time steps t is given by

$$y_i(t+1) = \Theta\left[A \sin \omega_0 t + D\eta_i(t) + \frac{w}{\langle k \rangle} \sum_{j=1}^{k_i} y_j(t) - b\right] , \tag{1}$$

where Θ denotes the Heaviside function, A is the amplitude and ω_0 is the frequency of the input periodic signal, D is the noise strength, $\eta_i(t)$ are Gaussian noises with unit variance, uncorrelated in space and time, w is the coupling strength, the summation runs over all nodes connected to the node i, and b is the threshold, with $b > A$. The TE are updated at random, according to the thermal-bath dynamics, and one full-time step consists in updating all TE. It should be noted that uncoupled TE driven by subthreshold periodic signals exhibit SR [10]; moreover, coupled TE were often used to study SR and many related phenomena in coupled systems [18,36,37]. Besides, TE can be used as qualitative models for SR in biological neurons [38]. Finally, the use of TE with discrete time and simple dynamics speeds up numerical simulations and allows the study AB networks with relatively large N.

In order to show numerically SR in the global response to the periodic signal of the model (1), the SPA vs. D is evaluated from the time-dependent mean response of all TE $\langle y(t) \rangle = N^{-1} \sum_{i=1}^{N} y_i(t)$ as

$$\mathrm{SPA} = |P_1|^2/A^2, \quad P_1 = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \langle y(t) \rangle \mathrm{e}^{-\mathrm{i}\omega_0 t} . \tag{2}$$

The numerical results are compared to the predictions of the linear response theory in the mean field approximation. Besides, the SPA_i from the individual signals $y_i(t)$ are also evaluated numerically according to the above definition, and the mean SPA is obtained as their average over N nodes. The mean SPA vs. D and the distribution of SPA_i for fixed D are analysed for various w. In numerical simulations the signal parameters $A = 0.01$, $\omega_0 = 2\pi/128$, $b = 0.6$, and the networks with $m = 5$ and N up to 10^4 were used. The numerical SPA was averaged over several random realizations of the BA network and initial conditions for the individual TE.

3. Spectral power amplification from the linear response theory

Before the numerical results are presented, in this section qualitative theory for SR in the BA network of coupled TE is presented. It is known that in systems with global or nearest-neighbour coupling SR can be analysed theoretically using the linear response theory in the mean-field approximation [11,19,20]. Using similar approximations, here the response of the model (1) to the weak periodic signal is obtained in a form of small oscillations of the mean $\langle y(t)\rangle$ around stable stationary states existing for a given noise intensity in the absence of the periodic modulation.

3.1. Mean-field approximation

In order to obtain stationary states of $\langle y(t)\rangle$ for a given noise intensity D, Eq. (1) is first rewritten as

$$y_i(t+1) = \Theta\left[A\sin\omega_0 t + D\eta_i(t) + \frac{k_i}{\langle k\rangle}\frac{w}{k_i}\sum_{j=1}^{k_i} y_j(t) - b\right] . \tag{3}$$

Introducing the quantity $\lambda_i = k_i/\langle k\rangle = k_i/2m$, assuming that the mean-field approximation $k_i^{-1}\sum_{j=1}^{k_i} y_j(t) \approx \langle y(t)\rangle$ is valid on the right-hand side, and averaging Eq. (3) over N nodes yields

$$\langle y(t+1)\rangle = \frac{1}{N}\sum_{i=1}^{N}\Theta[A\sin\omega_0 t + D\eta_i(t) + w\lambda_i\langle y(t)\rangle - b] . \tag{4}$$

For the scale-free network with the power-law connectivity distribution, the above mean-field approximation is a strong simplification, since the network consists of highly non-equivalent nodes whose ability to perceive the mean behaviour of the network much depends on the number of connections they possess (for more elaborate mean-field approach applied to the Ising model on the BA network see Ref. [32]). Replacing the average over N nodes by the average over the distribution of λ_i, which is $P(\lambda_i) = \lambda_i^{-3}/2$, yields

$$\begin{aligned}\langle y(t+1)\rangle &= \sum_{\lambda_i} P(\lambda_i) P(y_i(t+1) = 1 \mid \lambda_i) \\ &= \frac{1}{2}\sum_{\lambda_i} P(\lambda_i)\left[1 - \mathrm{erf}\left(\frac{b - A\sin\omega_0 t - w\lambda_i\langle y(t)\rangle}{\sqrt{2}D}\right)\right] ,\end{aligned} \tag{5}$$

where $P(y_i(t)=1 \mid \lambda_i)$ is the probability that $y_i(t)=1$ if the connectivity of the node i is $k_i = \lambda_i\langle k\rangle$, and the sum runs over a discrete set of λ_i, $\frac{1}{2} \leqslant \lambda_i \leqslant N/2m$. When obtaining Eq. (5) again a strong approximation was made that the probability to have $y_i(t)=1$ is equal for all nodes with the same λ_i, though in fact it depends also on the connectivity k_j of the nodes connected to the node i. The next step is to change to continuous distribution of λ, denoted as $\rho(\lambda) = \lambda^{-3}/2$, and to continuous time, and consider the limit $N \to \infty$, which yields the following equation for the mean $\langle y(t)\rangle$:

$$\frac{\mathrm{d}\langle y\rangle}{\mathrm{d}t} = -\langle y\rangle + \frac{1}{2}\int_{1/2}^{\infty}\mathrm{d}\lambda\rho(\lambda)\left[1 - \mathrm{erf}\left(\frac{b - A\sin\omega_0 t - w\lambda\langle y(t)\rangle}{\sqrt{2}D}\right)\right] . \tag{6}$$

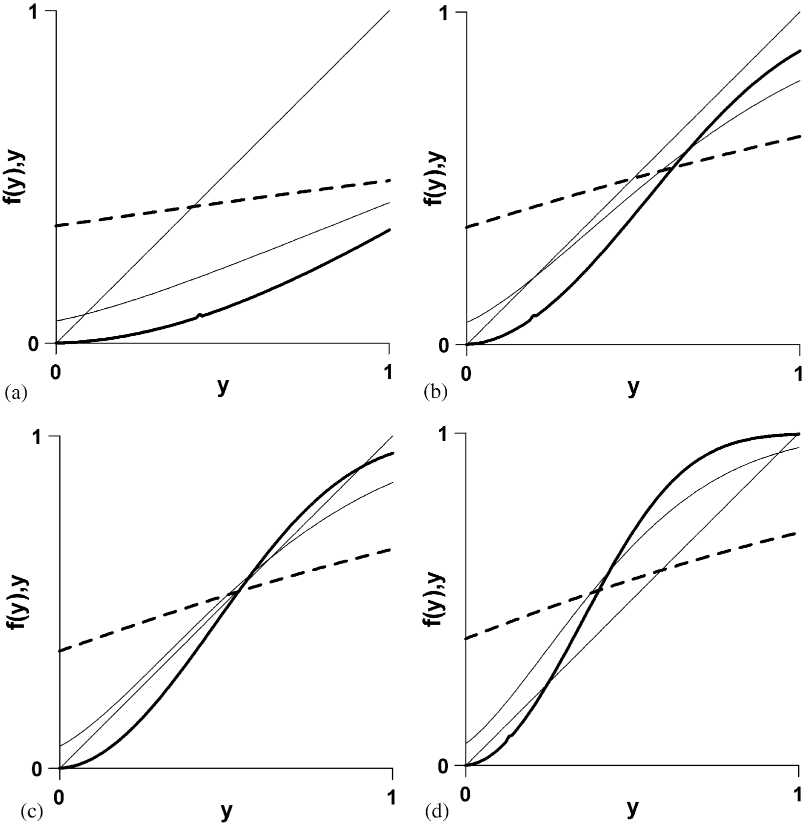

Fig. 1. Graphical solution of Eq. (7) for (a) w=0.6, (b) w=1.3, (c) w=1.5, (d) w=2.0; thick line—right-hand side of Eq. (7) for $D = 0.2$, thin line—for $D = 0.4$, dashed line—for $D = 1.6$; intersection points with the diagonal yield the stationary states $\langle y \rangle_0$ of the mean-field theory.

In the absence of the periodic forcing, stationary (possibly stable) states $\langle y \rangle_0$ of Eq. (6) for given D can be obtained as solutions of the equation

$$\langle y \rangle_0 = \frac{1}{2} \int_{1/2}^{\infty} \mathrm{d}\lambda \rho(\lambda) \left[1 - \mathrm{erf}\left(\frac{b - w\lambda \langle y \rangle_0}{\sqrt{2}D} \right) \right] . \tag{7}$$

For the parameters from Section 2, the following cases are possible (Fig. 1). For small and moderate w there is only one stable solution: $\langle y \rangle_0 \approx 0$ for small D, corresponding to approximately ordered phase, rising to $\langle y \rangle_0 > 0$ for moderate and large D, which corresponds to increasing disorder (Fig. 1(a), (b)). For large w there are three or one solutions. For small D there are two asymmetric stable solutions $\langle y \rangle_0 \approx 0$ and $\langle y \rangle_0 \approx 1$, corresponding to approximately ordered phases, separated by one unstable solution $0 < \langle y \rangle_0 < 1$ (Fig. 1(c), (d)). As D increases, the stable solutions approach each other, but usually do not merge: in contrast, one of the solutions disappears, and only one stable solution $\langle y \rangle_0 > 0$ remains, corresponding to the disordered phase.

3.2. Linear response theory

For given noise intensity it can be assumed that under the influence of the periodic signal with $A \to 0$ the mean value $\langle y(t)\rangle$ oscillates around the stable stationary state, i.e., $\langle y(t)\rangle = \langle y\rangle_0 + \xi(t)$, where $\xi(t) \to 0$. Inserting this into Eq. (6), and expanding the error function on the right-hand side in the Taylor series with respect to a small quantity $w\lambda\xi(t) + A\sin\omega_0 t$ yields

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = -\frac{\xi}{\tau} + \frac{A}{\sqrt{2\pi}D}\int_{1/2}^{\infty} \mathrm{d}\lambda\rho(\lambda)\exp\left[-\frac{(b-w\lambda\langle y\rangle_0)^2}{2D^2}\right]\sin\omega_0 t\ , \tag{8}$$

where τ is the relaxation time,

$$\tau = \left\{1 - \frac{w}{\sqrt{2\pi}D}\int_{1/2}^{\infty} \mathrm{d}\lambda\rho(\lambda)\lambda\exp\left[-\frac{(b-w\lambda\langle y\rangle_0)^2}{2D^2}\right]\right\}^{-1}\ . \tag{9}$$

The well-known solution of Eq. (8) is [20]

$$\begin{aligned}
&\xi(t) = \xi_0\sin(\omega_0 t - \phi_0)\ ,\\
&\xi_0 = \frac{A}{\sqrt{2\pi}D}\frac{\tau}{\sqrt{1+\omega_0^2\tau^2}}\int_{1/2}^{\infty}\mathrm{d}\lambda\rho(\lambda)\exp\left[-\frac{(b-w\lambda\langle y\rangle_0)^2}{2D^2}\right]\ ,\\
&\phi_0 = \arctan(\omega_0\tau)\ ,
\end{aligned} \tag{10}$$

thus the SPA evaluated from $\langle y(t)\rangle$ in the linear response approximation is

$$\mathrm{SPA} = \xi_0^2/(4A^2)\ . \tag{11}$$

For $\rho(\lambda) = \lambda^{-3}/2$ the integrals in Eqs. (7), (9), and (10) can be evaluated numerically. Eq. (11) is valid only in the limit of vanishing A. Moreover, the above theory neglects the possibility that if two stable stationary states of Eq. (7) coexist, as in Fig. 1(c), (d), the periodic signal with finite amplitude can remove this bistability, and the system can jump between the two states which leads to dramatic increase of the SPA. Another problem in the case of asymmetric bistable stationary states is the choice of $\langle y\rangle_0$ for the evaluation of the SPA from Eqs. (10), (11). Since, as mentioned in Section 3.1, only one stable stationary solution usually exists in the whole range of D, and the other one vanishes as the noise intensity increases, the value of $\langle y\rangle_0$ corresponding to the former solution is always taken to obtain SPA for any D. However, even for small A the SPA averaged over many realizations of the network can result from oscillations around both stationary states.

4. Results and discussion

Typical numerical results for the SPA vs. D from $\langle y(t)\rangle$ in system (1) with $N = 10\,000$ are shown in Fig. 2. For a wide range of the coupling strength w these curves show maxima at $D > 0$, thus SR is found. For small and moderate $w > 0$ the SPA

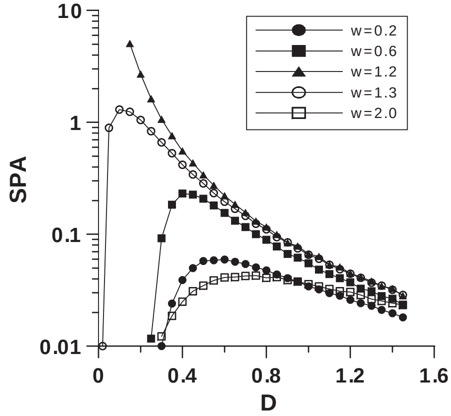

Fig. 2. Results of numerical simulations: SPA vs. D from the time-dependent mean response $\langle y(t)\rangle$ for the BA network with $N = 10\,000$ and various connection strength w (see legend for explanation of symbols, solid lines are guides to the eye).

is enhanced due to coupling, while too large coupling decreases the SPA in comparison with that in an uncoupled TE. For a particular choice of w the SPA becomes a monotonically decreasing function of D and SR disappears. Thus, qualitative dependence of SR on the coupling strength resembles that in a regular chain of TE with nearest-neighbour coupling [18]. The array-enhanced SR effect can be also explained similarly as in systems on regular lattices [14]. Since all TE are driven by the same periodic signal, for all i maximum probability that $y_i(t) = 1$ occurs when the periodic signal is also maximum. Hence, small or moderate coupling is equivalent to providing additional signal with a noticeable periodic component at the input of each TE. This increases the probability that the output of all TE is synchronized with the periodic signal, and enhances SR. In contrast, when w is too large, random fluctuations at the output of each TE influence strongly the activity of the coupled TE, correlation between the output of all TE and the periodic signal is diminished, and SR is deteriorated. Besides, the simulations show that for a particular w the system becomes very sensitive to small periodic signals even in the limit $D = 0$, which leads to the monotonic decrease of the SPA with D.

Taking into account the simplifications used to derive Eq. (11), the qualitative agreement between the numerical and theoretical curves SPA vs. D is good (Fig. 3). For small and large w both kinds of curves agree even quantitatively. The most striking discrepancy occurs for moderate w; nevertheless, the theory predicts correctly first the enhancement and then the decrease of the SPA with the rise of the coupling strength. However, the disappearance of SR for a certain moderate w is not predicted. Instead, optimum w is expected to exist for which the maximum SPA at $D > 0$ reaches supremum value, comparable with the coupling strength for which SR disappears in the numerical simulations. For the optimum w, the amplitude ξ_0 (10) of the linearized response to the periodic signal is very high, hence the SPA is strongly increased.

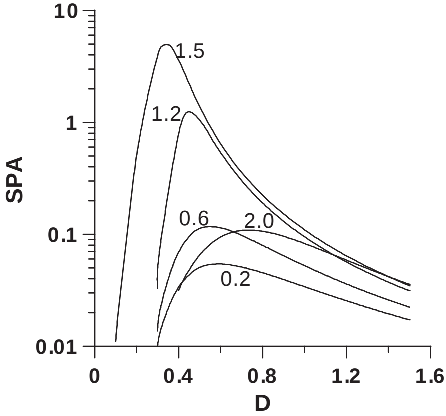

Fig. 3. Results of the theory from Section 3: SPA vs. D from the time-dependent mean response $\langle y(t)\rangle$ for the BA network with various connection strength w (given by numbers close to the respective curves).

It should be noted that numerical simulations revealed that for $A = 0$ the system (1) in fact reached a stationary state, with the value $\langle y(t)\rangle = \text{const}$ dependent on D and w, although possibly different from $\langle y\rangle_0$ from Eq. (7). For $A > 0$ the mean $\langle y(t)\rangle$ showed oscillations around the stationary state; the increase of the SPA was related to the increase of the amplitude of these oscillations. Hence, the theory of Section 3 provides good qualitative explanation of the origin of the array-enhanced SR effect in system (1). Numerically, it was also found that the disappearance of SR for particular w was caused by a very strong response of the system to the periodic signal at $D = 0$. The latter effect is beyond the limits of applicability of the linear response theory, and can be probably attributed to the jumps of $\langle y(t)\rangle$ between different stationary states under the influence of the periodic signal with finite amplitude.

Recent investigation of the ferromagnetic transition in the Ising model on the BA network has shown strong dependence of the critical temperature on the logarithm of the network size [31,32], which suggests that dynamical phenomena in systems on the BA network can be sensitive to the number of nodes. However, numerical simulations in system (1) do not show strong dependence of the SPA on N. In contrast, the curves SPA vs. D for a given w seem to saturate with rising N (Fig. 4). Such a result is typical in coupled systems, where it can be usually ascribed to the suppression of fluctuations in large systems. This shows that the theory of Section 3 can be used to describe qualitatively SR in system (1) independently of its size. Hence, further studies are necessary to find the possible influence of the network size on SR in other systems defined on the BA network.

The mean SPA vs. D, evaluated as the average of the SPA_i, $i = 1, 2, \ldots, N$, has similar properties as the SPA from the mean $\langle y(t)\rangle$ (Fig. 5(a)). The distribution of the individual SPA_i is rather broad (Fig. 5(b)); this is in contrast with systems on networks with more uniform topology, where all coupled stochastic resonators can be treated as equivalent. Fig. 5(b) shows that in this paper array-enhanced SR was observed in a

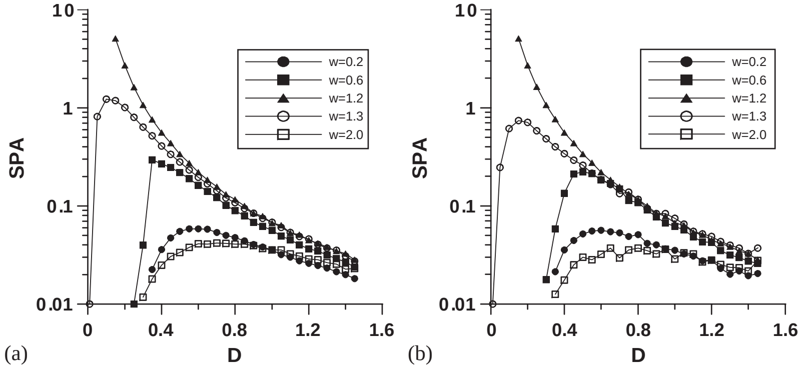

Fig. 4. Results of numerical simulations: SPA vs. D from the time-dependent mean response $\langle y(t)\rangle$ for the BA network with (a) $N = 1000$ and (b) $N = 100$ and various connection strength w (see legend for explanation of symbols, solid lines are guides to the eye).

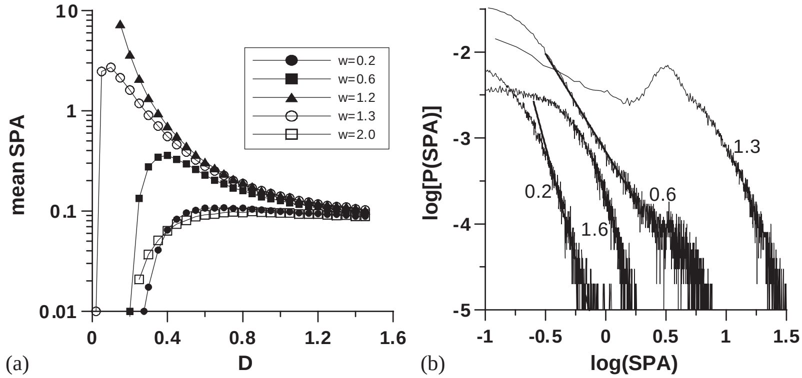

Fig. 5. Results of numerical simulations: (a) mean SPA vs. D, averaged over all network nodes (see legend for explanation of symbols, solid lines are guides to the eye); (b) probability distributions $P(\text{SPA})$ of the values of SPA_i from individual nodes for the BA network with various connection strength w (given by numbers close to the respective curves); for each w, the distributions were obtained for D corresponding to the maxima of the respective mean SPA in (a); these distributions in some cases show power-law behaviour with a non-universal exponent, shown with straight lines (the slope for $w = 0.2$ is -5.2 ± 0.8, for $w = 0.6$ it is -2.3 ± 0.5). The results were obtained for the BA network with $N = 10\,000$.

system of coupled TE with strongly non-uniform response to the periodic stimulation. No particular form of the probability distribution $P(\text{SPA})$ of the SPA_i could be found. It seems that over small intervals this distribution can be approximated by a power law, with a non-universal scaling exponent dependent on w. However, in other cases the tails of this distribution decay faster (Fig. 5(b)). Since the SPA_i is obviously not directly proportional to k_i, it is not surprising that the power-law connectivity distribution $P(k)$ is not simply transferred to $P(\text{SPA})$.

5. Conclusions

SR was investigated in a BA network of coupled TE driven by a common periodic signal and independent noises. The enhancement of SR due to coupling was observed, as in systems of coupled stochastic resonators on networks with more uniform topology. Both the SPA from the mean response of the system to the periodic signal, and the mean SPA averaged over all TE were significantly increased for the proper coupling strength. This result shows that SR can be enhanced due to coupling between highly non-equivalent stochastic units, with a broad distribution of the individual values of the SPA. No clear dependence of SR on the network size was observed, and the SPA saturated with the increasing number of TE. Simplified theory based on the mean-field approximation and the linear response assumption provided good qualitative, and in some cases quantitative, explanation of the numerical results.

This paper extends the investigation of the effect of coupling on SR to a rapidly developing area of scale-free networks. Possible applications of the obtained results include, e.g., the study of SR in social sciences, where the structure of the collaboration networks, and thus of the professional information-spreading channels between individuals (actors, scientists, etc.) is believed to be scale-free [23], investigation of SR in the Ising model on the BA network (with potentially strong size dependence), etc. From a more general point of view, it can be interesting to compare various networks to find optimum coupling topologies for processing weak periodic signals immersed in the noisy background via SR. In this context, the BA networks seem to have an important property that the distribution of the output SPA from individual units is broad, and the performance of a certain fraction of units can be highly improved due to proper coupling.

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