# **Refractive Index and Dispersion: Prism Spectrometer**

#### **OBJECTIVES:**

The purpose of this experiment is to study the phenomenon of dispersion i.e. to determine the variation of refractive index of the glass prism as a function of wavelength and to compare the experimental data with the classical normal dispersion function

#### **EQUIPMENT:**

glass prism, spectrometer, sodium and neon lamp

### **INTRODUCTION:**

The dependence of the velocity of propagation of the wave on the properties of the medium gives rise to the phenomena of refraction and reflection, which occur when a wave crosses a surface separating two media, where the wave propagates with different velocities. The reflection and refraction of waves that occur at surfaces of discontinuity can be analyzed geometrically using the ray concept when no other changes happen at the surface. This method is called wave geometry or ray tracing. In particular, for electromagnetic waves in the visible and near visible regions, it constitutes geometrical optics, which is a very important branch of applied physics. In this way we are able to examine optical behavior that does not depend on the nature of light, but only on the straight-line path it travels. Under the approximation of geometrical optics we can say that although the light wave spreads as it moves away from the source, it travels in the straight line.



Figure 1: Light refracting from a medium with an index of refraction  $n_1$  and into a medium with a refractive index  $n_2$ .

Refraction is the bending of light that takes place at a boundary between two materials having different indices of refraction ( $n_1$  and  $n_2$ ). Refraction is due to a change in the speed of light as it passes from one medium to another. No bending of the incident ray occurs if it strikes the boundary along the normal, which is a construction line drawn perpendicular to the boundary at the point of incidence.

The incident ray is the ray approaching the boundary. It strikes the boundary at the point of incidence. The refracted ray is the ray leaving the boundary through the second medium. The reflected ray is the ray undergoing partial (or total) reflection at the boundary. The angle of incidence  $\alpha_1$  is the angle between the incident ray and the normal. The angle of reflection  $\alpha_2$  is the angle between the normal and the reflected ray. The angle of refraction  $\beta$  is the angle between the normal and the reflected ray.

Laws of Reflection and Refraction:

- 1. The directions of incidence, refraction and reflection ray are all in one plane, which is normal to the surface of the separation of two media and therefore contains the normal of the surface.
- 2. The angle of incidence is equal to the angle of reflection. That is

$$\alpha_1 = \alpha_2 \tag{1}$$

3. The ratio between the sine of the angle of incidence and the sine of the angle of refraction is a constant (this ratio is constant for a particular wavelength and a particular set of materials). This is called Snell's Law and it is expressed by

$$\frac{\sin\alpha_1}{\sin\beta} = \frac{n_2}{n_1}$$
(2.a)

Subscript 1 is customarily used to represent the incident medium. Subscript 2 represents the refractive medium. The Equation (1) is valid regardless of the direction in which light is traveling through the two media.

The constant  $n_i$  is called the absolute index of refraction of the medium (i). We define it to be the ratio of the speed of the light in vacuum c to the speed of the light in that medium V<sub>i</sub> (i.e.  $n_i=c/V_i$ ).

Snell's Law can be written also in the form:

$$\frac{\sin \alpha_1}{\sin \beta} = n_{21} = \frac{V_1}{V_2},$$
 (2.a)

where  $n_{12}$  is called the relative index of refraction (index of refraction of the medium (2) relative to the medium (1)), being the ratio of the refractive indices and the ratio of the speeds of light in two media. Its numerical value depends on the nature of the wave and on the properties of two media.

If light is traveling from a less refractive medium to a more refractive medium (i.e.  $n_2 > n_1$ ), the refracted ray will be bent towards the normal (Figure 1). When a light ray

travels from a more dense (with higher refractive index) to a less dense optical material, the ray is bent away from normal.

A dispersive medium is one in which different wavelengths  $\lambda$  of light have slightly different indices of refraction n (i.e. n=n( $\lambda$ )).



Figure 2: The index of refraction as a function of wavelength for fused quartz.

Water, glass, transparent plastics, and quartz are all dispersive materials. Generally, in the case of normal dispersion, the shorter wavelengths travel with slightly smaller wave velocities than do longer waves (i.e. refractive index decreasing with is а wavelength).

This phenomenon is characteristic for a transparent media. As an example, the index of refraction for quartz that varies with the wavelength

in the visible and near-visible region is shown in Figure 2.

In the case of light absorption the anomaly dispersion is observed, what means that refractive index is increasing as a function of the wavelength.

When a wave is refracted into a dispersive medium which index of refraction depends on the wavelength, the angle of refraction also depends on the wavelength. If the incident wave, instead of being monochromatic is composed of several wavelengths, each component wavelength will be refracted trough a different angle.

The variation of the refractive index of material as a function of the wavelengths is most easily seen in the action of the prism. In his experiments on optics Isaac



Newton performed the first detailed investigations on dispersion. He demonstrated how a prism disperses white light into its spectral components (see Figure 3).

Figure 3: A triangular prism separating white light into its component colors. Chromatic dispersion occurs at the first surface and is increased at the second surface. When the narrow beam of white light strikes the face of the glass prism surface at angle, some is reflected and some passes into

the glass. It is refracted according to the Snell's law, but since the indices of refraction are slightly different for the various wavelengths, each color refracts trough a different angle. Red light moves more guickly in glass than violet light and it bends (refracts) less sharply. A triangular prism is shaped to bend the light twice, and disperse it as much as possible. The observed result is separation of white light into its color components (spectrum). For this reason prisms are wildly used for analyzing light in instruments called spectroscopes.

#### Prism:

A prism is a medium bounded by the two plane surfaces making the angle  $\varphi$  (called as a prism or apex angle). We assume that the medium has an index of refraction n surrounded by a medium having unit index, such as air (see Figure 4). An incident ray such as PQ suffers two refractions and emerges deviated at angle  $\varepsilon$  relative to the incident direction. From the Figure 4 it is easily seen that the following relations hold:

$$\sin\alpha_1 = n\sin\beta_1, \tag{3.a}$$

$$\sin \alpha_2 = n \sin \beta_2, \tag{3.b}$$

$$\varphi = \beta_1 + \beta_2 \tag{3.c}$$

$$\varepsilon = (\alpha_1 - \beta_1) + (\alpha_2 - \beta_2) \Longrightarrow \varepsilon = \alpha_1 + \alpha_2 - \varphi$$
(3.d)



Figure 4: Path of the ray trough a prism.

The first and second Equations are simply Snell's law applied to the refractions at Q and R. The third follows when we use triangle QTR, and forth when we use triangle QRU. The first three equations serve to trace the path of the ray and the last allows us to find a ray deviation. The deflection (deviation) of the ray is investigated as a function of the incidence angle  $\alpha_1$ , the refraction

(being dependent index n on wavelength) and the refraction angle of the prism  $\phi$ .

Minimum deviation is obtained by making  $d\epsilon/d\alpha_1 = 0$ . From Equation (3.d), we have that  $d\epsilon/d\alpha_1 = 1 + d\alpha_2/d\alpha_1$ , and for  $d\epsilon/d\alpha_1 = 0$  we must have

$$\frac{\mathrm{d}\alpha_2}{\mathrm{d}\alpha_1} = -1. \tag{4.a}$$

Equations  $\cos \alpha_1 d\alpha_1 = n \cos \beta_1 d\beta_1$ , From (3.a)-(3.c) we have that  $\cos \alpha_2 d\alpha_2 = n \cos \beta_2 d\beta_2$  and  $d\beta_1 = -d\beta_2$ . Therefore

$\frac{d\alpha_2}{d\alpha_2} = -$	$\cos \alpha_1 \cos \beta_2$	(4)	h١
$d\alpha_1$	$\cos \alpha_2 \cos \beta_1$	(+.)	5)

Since four angles  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$  and  $\beta_2$  are smaller than  $\pi/2$  and satisfy the symmetric conditions (3.a) and (3.b), Equations (4.a) and (4.b) can be satisfied simultaneously only if  $\alpha_1 = \alpha_2$  and  $\beta_1 = \beta_2$ .

It means that the deviation has a minimum value  $\varepsilon_{min}$  when  $\alpha_1 = \alpha_2$  and therefore  $\beta_1 = \beta_2$ . According to Equations (3.c) and (3.d) this requires that

$$\alpha_1 = \frac{1}{2} (\varepsilon_{\min} + \varphi), \quad \beta_1 = \frac{1}{2} \varphi.$$
(4.c)



Note that in this case the path of the ray is symmetric with respect to the two faces of the prism. To obtain the angle of minimum deviation  $\varepsilon_{min}$  the angle of incidence  $\alpha$  is chosen so that the emerging ray also makes the same angle  $\alpha$  with a normal to the other face (see Figure 5).

Figure 5: Symmetrical path of the ray trough a prism in the case of minimum deviation.

Introducing Equations (4.c) in Equation (3.a) we obtain that:

$$n = \frac{\sin\left(\frac{\epsilon_{\min} + \phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)},$$
(5)

which is a convenient formula for measuring the index of refraction n of a material by finding  $\varepsilon_{min}$  experimentally in a prism of known angle  $\varphi$ .

#### Spectroscope:

A simple type of prism spectroscope is shown in Figure 6. It is an optical instrument designed to observe the effect of optical dispersion and to permit analysis of spectra. Light emitted by the source, and limited by the slit, is transformed into parallel rays by the collimating lens (A collimating lens is a converging lens that focuses a diverging light beam into a parallel beam). After being dispersed by the prism, initially parallel beams emerge, each at a different angle according to its wavelength. A telescope focuses the rays and allows the observer to see the image of the slit. Most spectroscopes have a calibrated circular scale, which enables the observer to measure the angle of the emerging light for each image. Each image of the slit

(corresponding to the different values of the wavelength) is called a spectral line of the light coming from the source. This is an origin of the term "line spectra". If the slit is narrow enough only thin strip of the source is observed trough a prism, the band of the images has very little overlap and bright spectral colors across the band are visible.

Most modern spectrometers are made with diffraction grating in place of prism because the dispersion of the grating can be made much greater than that of the prism.



Figure 6: A prism spectroscope.

# **EXPERIMENTAL PROCEDURES:**

# **Spectrometer Setup**

- 1. Remove prism from the turntable.
- 2. Position the instrument so that the telescope can be pointed at some distant object
- 3. Adjust the eyepiece of the telescope until the crosswires are in focus.
- 4. Focus the telescope on the distant object (infinity).
- 5. Position the instrument on the laboratory table.
- 6. Position a sodium lamp close to the slit at the end of the collimator.

7. Rotate the telescope so that it faces the collimator and you can observe the slit image.

8. Adjust the collimator only until the image of the wide slit is in focus (it allows to place a slit in the focal point of collimator lens and generate parallel rays coming out from collimator).

9. Adjust the slit width until its image is just wider than the crosswires (i.e. make a slit as narrow as possible to make measurements more precise).

# Turntable Setup

The rotation axis of the turntable should be perpendicular to the plane containing the principal axes of the telescope and collimator.

1. Place a prism on the centre of the turntable.

2. Adjust the height of the turntable using screws  $S_1$ ,  $S_2$ ,  $S_3$  (Figure 6) until the collimator is centered on the vertical dimension of the prism.

## Measurement of Angle $\varphi$ of Prism (Apex Angle)



For this part of the experiment any type of may be used light (sodium or neon light). It can be simply shown and proved using Snell's Law that if a parallel beam is incident at a corner of a prism. the angle between two reflected rays is twice the prism (apex) angle. See 7, Figure where angular positions of the

Figure 7: Measurement of Apex Angle  $\varphi$ .

telescope corresponding to the rays reflected from the faces of the prism are:  $a=360^{\circ}-2\alpha$  and  $b=2\beta$ .

Therefore  $a-b=360^{\circ}-2\alpha-2\beta$ 

and while  $\alpha = 90^{\circ} - \phi_1$  and  $\beta = 90^{\circ} - \phi_2$  we get that  $a-b=2(90^{\circ} - \phi_1)-2(90^{\circ} - \phi_2)=2\phi_1-2\phi_2=2\phi$  what means that:

$$\varphi = \frac{a-b}{2}, \tag{6}$$

where a and b are angular positions of the telescope for two reflected rays.

### **Experimental Procedure:**

1. Rotate the turntable until the refracting edge of the prism is approximately pointing towards the collimator. Light from the collimator will be reflected from both sides of the prism.

2. Rotate the telescope until the first of the reflected slit images is centered on the crosswires. Record the angle of the telescope.

3. Rotate the telescope until the second of the reflected slit images is centered on the crosswires. Record the angle of the telescope.

4. The angle between the two telescope positions is twice the prism (apex) angle  $\varphi$ .

The error of apex angle  $\Delta \phi$  is a sum of two elements: one being the smallest-marked-scale-division (referring to precision of angle measurement) and the second is given by the half of the angular width of the slit.

## Measurement of the Minimum Angle of Deviation ε<sub>min</sub>

For this part of the experiment the light of the wavelength at which n is to be measured is required. The task is to find the refractive index n of the glass for different wavelengths being the spectral components of neon lamp.

The prism and the spectrometer can be used to measure the refractive index at a given wavelength using the formula:

$$n = \frac{\sin\left(\frac{\epsilon_{\min} + \phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)},$$
(7)

where  $\phi$  is the prism angle and  $\epsilon_{\text{min}}$  is angle of minimum deviation for that wavelength.

### **Experimental Procedure:**

1. Rotate the turntable and telescope until light will pass approximately symmetrically through the prism (see Figure 5)

2. Rotate the telescope until the succession of views 1-5 shown in Figure 8 is observed as the turntable is rotated consistently in the same direction. Record the angle of the telescope. Repeat this procedure for different wavelengths.

3. Remove a prism and measure the angle for the direction of incident ray. Record the angle of the telescope.

4. The angle between the two telescope positions from step 2 and 3 is the minimum deviation angle  $\epsilon_{min}$ .

The error of the measured angles of minimum deviation  $\Delta \epsilon_{min}$  is a sum of three elements: two of them are identical as in the case of  $\Delta \phi$  and third is given by the half of the angular dead interval. The death interval is an angle of the turntable rotation (constantly in the same direction) from the moment when the slit image is stopped in the field of view until the instant when it starts to move back.



Figure 8: Successive views as the turntable is rotated through the angle of minimum deviation.

## CALCULATIONS AND DATA ANALYSES:

Basing on Equation (6) calculate the apex angle  $\phi$  of the prism and give the error  $\Delta \phi$  of obtain value.

Calculate the angle of minimum deviation  $\epsilon_{min}$  for different observed lines of neon spectrum and give the errors  $\Delta \epsilon_{min}$  of calculated values.

From Equation (7) calculate the refractive indices  $n_{\lambda}$  for following lines of neon spectrum. Calculate the errors  $\Delta n_{\lambda}$ , which depend on the errors of measurements of  $\phi$  and  $\varepsilon_{min}$  angles, basing on the total differential method. In your calculations remember to use the values of  $\Delta \phi$  and  $\Delta \varepsilon_{min}$  which should be expressed in radians.

Plot the curve of dispersion i.e. the dependence of the calculated refractive index as a function of the wavelength (Figure 9). Do your experimental results show the same trend of the theoretical curve of normal dispersion?



