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ON THE PROPERTIES OF TWO PULSES PROPAGATING SIMULTANEOUSLY IN DIFFERENT DISPERSION REGIMES IN A NONLINEAR PLANAR WAVEGUIDE

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Abstract

Properties of two pulses propagating simultaneously in different dispersion regimes, anomalous and normal, in a Kerr-type planar waveguide are studied. It is found that the presence of the pulse propagating in normal dispersion regime can cause termination of catastrophic self-focusing of the pulse propagating in anomalous regime. It is also shown that the coupling between pulses can lead to spatio-temporal splitting of the pulse propagating in anomalous dispersion regime, but it does not lead to catastrophic self-focusing of the pulse propagating in normal dispersion regime. For the limiting case when the dispersive term of the pulse propagating in normal dispersion regime can be neglected an indication (based on the variational estimation) to a possibility of a stable self-trapped propagation of both pulses is obtained. This stabilization is similar to the one which was found earlier in media with saturation-type nonlinearity.

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1 Introduction

The propagation of a dispersive light pulse in a planar waveguide with positive, instantaneous Kerr-type nonlinearity can be described by the (2+1)-dimensional nonlinear Schrödinger (NSE) [1]:

$$\iota \frac{\partial}{\partial \zeta}\Psi + \frac{1}{2}\sigma \frac{\partial^2}{\partial \tau^2}\Psi + \frac{1}{2}\frac{\partial^2}{\partial \xi^2}\Psi + |\Psi|^2\Psi = 0. \tag{1}$$

Equation (1) is valid only for pulses in the picosecond range; for shorter pulses additional terms, due to a higher-order dispersion, for example, should be included. The last term in equation (1) describes Kerr-type nonlinearity, second and third terms are associated, respectively, with diffraction, which causes spreading of the pulse in space, and first-order group velocity dispersion, which leads to temporal broadening of the pulse. Parameter σ, which can be either positive (for anomalous dispersion) or negative (for normal dispersion), is the dispersion-to-diffraction ratio [2]. The spatio-temporal dynamics of the pulse depends, to a high degree, on the sign of this parameter.

For anomalous dispersion, equation (1) is similar to NSE [3] describing the propagation of a dispersionless beam in a Kerr-type nonlinear bulk medium if the dispersive term is replaced by a diffraction term. Therefore, in both cases similar effects are expected. One of them is catastrophic self-focusing, which takes place when nonlinearity dominates over transverse diffraction and temporal dispersion. It is highly sensitive to perturbations of the system, so that even small perturbations can dramatically enhance or stop it [4]. An arresting of catastrophic self-focusing can be obtained, for example, by taking into account saturation-type nonlinearity [5], or additional effects like Raman scattering [6], plasma formation [7], multiphoton ionization [8], higher-order group velocity dispersion terms [9], or nonparaxiality of a beam propagating in bulk media [10].

Another way to avoid catastrophic self-focusing is to choose parameters of the system in such a way that the pulse propagates in normal dispersion regime instead of anomalous one. In this case the terms describing dispersion and diffraction have different signs and two different effects, spatial self-focusing and temporal self-defocusing, simultaneously influence on the propagation of the pulse. This causes a breaking of spatio-temporal symmetry and can finally lead to spatio-temporal splitting of the pulse into several sub-pulses. Such splitting was observed in planar waveguides [11, 12] and bulk media [13, 14, 15, 11].

Thus, depending on the sign of dispersion, a dispersive pulse propagating in a Kerr-type planar waveguide reveals different behavior. Catastrophic self-focusing takes place in the case of anomalous dispersion. For normal dispersion the typical process is spatio-temporal splitting. It seems interesting to study an interaction between two pulses co-propagating in such a medium, i.e. a Kerr-type planar waveguide, under the assumption that one of them propagates in normal dispersion regime and the other is in anomalous regime. To the authors knowledge this problem has not been studied in the literature and the main purpose of this paper is to consider it. Note that the importance of the interaction between two pulses in a nonlinear medium has been pointed out already by Agrawal in [16], where an intriguing effect of an induced focusing of two beams co-propagating in a self-defocusing medium has been reported.

The interaction between pulses will be assumed to be limited to cross-phase modulation, a nonlinear effect through which the phase of an optical beam/pulse is affected by another propagating beam/pulse and which can cause a redistribution of energy within each beam/pulse. Another effect, four-wave mixing, will be neglected, so that no energy transfer between both pulses will be taken into consideration. The analysis presented in this paper is based on the variational method [17] and numerical simulations using the split-step spectral method [18].

We proceed as follows. In section 2, two coupled NSEs describing the co-propagation of two dispersive pulses in a nonlinear planar waveguide and basic equations following from the variational method will be introduced. Next, in section 3, the problem of catastrophic self-focusing will be considered. First, the influence of the parameters of the pulse propagating in normal dispersion regime on the threshold of catastrophic self-focusing of the pulse propagating in anomalous dispersion regime will be studied. It will also be examined if catastrophic self-focusing of the pulse propagating in normal dispersion regime can occur as a result of the nonlinear coupling between two pulses. In section 4, which is devoted to the problem of spatio-temporal splitting, it will be investigated if the influence of the pulse propagating in normal dispersion regime can enforce spatio-temporal splitting of the pulse with anomalous dispersion. In the last section, section 5, we will focus on the limiting case when the dispersive term of the normal pulse can be neglected. In this case the problem of two coupled (2+1)-dim NSE will be reduced to the system of (1+1)-dim NSE coupled to (2+1)-dim NSE. The main reason to study this configuration is to investigate a possibility of a stable, self-trapped solution.

Throughout the paper the pulse propagating in anomalous (normal) dispersion regime will be referred to as the *anomalous* (*normal*) pulse.

2 Basic equations

The co-propagation of two optical pulses in a nonlinear planar waveguide can be described by two coupled nonlinear Schrödinger equations:

$$\iota \frac{\partial}{\partial \zeta}\Psi_1 + \frac{1}{2}\sigma_1 \frac{\partial^2}{\partial \tau^2}\Psi_1 + \frac{1}{2}\frac{\partial^2}{\partial \xi^2}\Psi_1 + (\mid \Psi_1 \mid^2 + 2 \mid \Psi_2 \mid^2)\Psi_1 = 0, \tag{2a}$$

$$\iota \frac{\partial}{\partial \zeta}\Psi_2 + \iota\varrho \frac{\partial}{\partial \tau}\Psi_2 + \frac{1}{2}\sigma_2 \frac{\partial^2}{\partial \tau^2}\Psi_2 + \mu\frac{1}{2}\frac{\partial^2}{\partial \xi^2}\Psi_2 + r(\mid \Psi_2 \mid^2 + 2 \mid \Psi_1 \mid^2)\Psi_2 = 0, \tag{2b}$$

where the last terms represent self-phase modulation and the terms before the last ones describe cross-phase modulation, a nonlinear effect which causes a coupling between pulses.

It is assumed that the subscript $j = 1$ ($j = 2$) denotes the anomalous (normal) pulse, hence $\sigma_1 > 0$ and $\sigma_2 < 0$. The notations in equation (2) are explained in Appendix A. In this paper the difference of the group velocities of two pulses, δ, will be neglected. The initial conditions will be taken in the form of the Gaussian pulses

$$\Psi_j(\zeta = 0, \tau, \xi) = \sqrt{\kappa_j} exp\left[-\frac{1}{2}\tau^2 \left(1 + iC_{\tau j}\right)\right] exp\left[-\frac{1}{2}\xi^2 \left(1 + iC_{\xi j}\right)\right]. \tag{3}$$

In equation (3) $\kappa_j := |\Psi_j(\tau = 0, \xi = 0)|^2$ is the strength of nonlinearity, and $C_{\tau j}$ ($C_{\xi j}$) is the temporal (spatial) chirp of the j-th pulse, $j = 1, 2$.

2.1 Variational method

It is known that the set of NSEs (equation (2)) can be obtained from the Lagrangian density given by

$$L = \frac{\iota}{2}\left(\Psi_1^*\frac{\partial\Psi_1}{\partial\zeta} - \Psi_1\frac{\partial\Psi_1^*}{\partial\zeta}\right) + \frac{\iota}{2}\frac{1}{r}\left(\Psi_2^*\frac{\partial\Psi_2}{\partial\zeta} - \Psi_2\frac{\partial\Psi_2^*}{\partial\zeta}\right)$$
$$-\frac{1}{2}\left|\frac{\partial\Psi_1}{\partial\xi}\right|^2 - \frac{1}{2}\sigma_1\left|\frac{\partial\Psi_1}{\partial\tau}\right|^2 - \frac{1}{2}\frac{\mu}{r}\left|\frac{\partial\Psi_2}{\partial\xi}\right|^2 - \frac{1}{2}\frac{\sigma_1}{r}\left|\frac{\partial\Psi_2}{\partial\tau}\right|^2 \tag{4}$$
$$+\frac{\iota}{2}\frac{\varrho}{r}\left(\Psi_2^*\frac{\partial\Psi_2}{\partial\tau} - \Psi_2\frac{\partial\Psi_2^*}{\partial\tau}\right) + \frac{1}{2}\mid\Psi_1\mid^4 + 2\mid\Psi_1\mid^2\mid\Psi_2\mid^2 + \frac{1}{2}\mid\Psi_2\mid^4 .$$

Following the variational method [17] let us choose a proper multi-parametric trial function for the solution of equation (2). Since in this paper we consider the Gaussian initial condition (equation (3)) it is natural to take as the trial function the Gaussian function:

$$\Psi_j = A_j(\zeta)exp\left[-\frac{1}{2}\frac{\tau^2}{w_{\tau j(\zeta)}}\right]exp\left[-\frac{1}{2}\frac{\xi^2}{w_{\xi j}(\zeta)}\right]exp\left[\frac{\iota}{2}\tau^2 C_{\tau j}(\zeta)\right]exp\left[\frac{\iota}{2}\xi^2 C_{\xi j}(\zeta)\right], \tag{5}$$

with twelve parameters: the complex conjugate amplitudes, A_j, A_j^*, the temporal and the spatial widths, $w_{\tau j}, w_{\xi j}$, and the temporal and the spatial chirps, $C_{\tau j}, C_{\xi j}$, where $j = 1, 2$. From the initial condition (equation (3)) it follows that $A_j(\zeta = 0) = \sqrt{\kappa_j}$, $w_{\tau j}(\zeta = 0) = w_{\xi j}(\zeta = 0) = 1$.

The evolution equations for the parameters of the trial function are obtained by varying the reduced Lagrangian

$$\langle L\rangle := \int_{-\infty}^{\infty} L d\xi d\tau,$$

into which the trial function (equation (5)) is inserted, with respect to the parameters of the trial function, A_j, A_j^*, $w_{\tau j}$, $w_{\xi j}$, $C_{\tau j}$, $C_{\xi j}$. We obtain the following 12 coupled ordinary differential equations:

$$\frac{\delta}{\delta z}\mathcal{I}_1 = 0 \tag{6a}$$

$$\frac{\delta}{\delta z}\mathcal{I}_2 = 0 \tag{6b}$$

$$\frac{\delta^2 w_{\tau 1}}{\delta\zeta^2} = \frac{\sigma_1^2}{w_{\tau 1}^3} - \sigma_1\frac{1}{2}\frac{\mathcal{I}_1}{w_{\tau 1}^2 w_{\xi 1}} - \frac{4\mathcal{I}_2 w_{\tau 1}\sigma_1}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{3}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{1}{2}}} \tag{7a}$$

$$\frac{\delta^2 w_{\xi 1}}{\delta\zeta^2} = \frac{1}{w_{\xi 1}^3} - \frac{1}{2}\frac{\mathcal{I}_1}{w_{\tau 1} w_{\xi 1}^2} - \frac{4\mathcal{I}_2 w_{\xi 1}}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{1}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{3}{2}}} \tag{7b}$$

$$\frac{\delta^2 w_{\tau 2}}{\delta\zeta^2} = \frac{\sigma_2^2}{w_{\tau 2}^3} - \sigma_2\frac{1}{2}\frac{\mathcal{I}_2 r}{w_{\tau 2}^2 w_{\xi 2}} - \frac{4\mathcal{I}_1 w_{\tau 2}\sigma_2 r}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{3}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{1}{2}}} \tag{7c}$$

$$\frac{\delta^2 w_{\xi 2}}{\delta\zeta^2} = \frac{\mu^2}{w_{\xi 2}^3} - \mu\frac{1}{2}\frac{\mathcal{I}_2 r}{w_{\tau 2} w_{\xi 2}^2} - \frac{4\mathcal{I}_1 w_{\xi 2}\mu r}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{1}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{3}{2}}} \tag{7d}$$

$$C_{\tau 1} = -\frac{1}{\sigma 1}\frac{\delta\, ln(w_{\tau 1})}{\delta z} \tag{8a}$$

$$C_{\xi 1} = -\frac{\delta\, ln(w_{\xi 1})}{\delta z} \tag{8b}$$

$$C_{\tau 2} = -\frac{1}{\sigma 2}\frac{\delta\, ln(w_{\tau 2})}{\delta z} \qquad (8c)$$

$$C_{\xi 2} = -\frac{1}{\mu}\frac{\delta\, ln(w_{\xi 2})}{\delta z} \qquad (8d)$$

$$\frac{\delta \phi_1}{\delta z} = \frac{3}{4}|A_1|^2 - \frac{1}{2}\frac{\sigma 1}{w_{\tau 1}^2} + \frac{1}{w_{\xi 1}^2} + \mathcal{I}_1 \frac{2 + w_{\tau 1}^2/(w_{\tau 1}^2 + w_{\tau 2}^2) + w_{\xi 1}^2/(w_{\xi 1}^2 + w_{\xi 2}^2)}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{1}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{1}{2}}} \qquad (9a)$$

$$\frac{\delta \phi_2}{\delta z} = \frac{3}{4}r|A_2|^2 - \frac{1}{2}\frac{\sigma 2}{w_{\tau 1}^2} + \frac{\mu}{w_{\xi 2}^2} + \mathcal{I}_2 r \frac{2 + w_{\tau 2}^2/(w_{\tau 1}^2 + w_{\tau 2}^2) + w_{\xi 2}^2/(w_{\xi 1}^2 + w_{\xi 2}^2)}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{\frac{1}{2}}(w_{\xi 1}^2 + w_{\xi 2}^2)^{\frac{1}{2}}} \qquad (9b)$$

where $\mathcal{I}_j := w_{\tau j}(0) w_{\xi j}(0)|A(0)|^2 = \kappa_j$. From equations (6 a) and (6 b), which are actually the energy conservation laws for two pulses, $N_j := \int\int |\Psi_j|^2 d\tau d\xi = \pi \mathcal{I}_j$, it follows that there is no energy transfer between the pulses.

The set of equations (6 a) - (9 b) is rather complicated and only in the special case when $\sigma_1 = \sigma_2 = 1$ the analytical solution

$$w_{\xi j}(\zeta) = w_{\tau j}(\zeta) = \left[1 + \zeta^2\left(1 - \frac{\kappa_j}{2}\right)\right]^{\frac{1}{2}}, \quad j = 1, 2$$

is available [19]. More general situations should be treated numerically, e.g. using the Runge-Kutta method [20].

3 Catastrophic self-focusing

This section is devoted to the problem of catastrophic self-focusing, which can occur in the solution of the set of equation (2). Our analysis will be based on the variational method and numerical simulations and the comparison of the results of both.

From the point of view of analytical estimations, which can be done using the method of moments [21, 22] or the variational method [23], catastrophic self-focusing is identified with a development of a singularity in the solution at a finite distance of propagation. For NSE (equation (1)) and the Gaussian initial condition (equation (3)) the threshold of catastrophic self-focusing given by those estimations is

$$\kappa_{cat} = \sigma + 1.$$

In our numerical simulations catastrophic self-focusing is identified with a discontinuity of the phase $\phi(\tau, \xi, \zeta)$ of the amplitude $\Psi := |\Psi| e^{i\phi}$ in the central point of the coordinate system, $\tau = 0, \xi = 0$, and with non-monotonic behavior of the intensity $|\Psi|^2$ in the central point after catastrophic self-focusing has been reached [24]. The threshold of catastrophic self-focusing given by the numerical analysis [25, 13, 24]

$$\kappa_{cat} \approx \sigma + 0.85$$

is lower than the one given by analytical estimations.

In order to study the influence of the parameters of the normal pulse on the threshold of catastrophic self-focusing of the anomalous pulse, the parameters of the anomalous pulse have been chosen in such a way that the relations $\kappa_1 > 1 + \sigma_1$ (in the variational method), and

$\kappa_1 > 0.85 + \sigma_1$ (in the numerical simulations) are satisfied, which means that catastrophic self-focusing will take place when there is no coupling between pulses. Then the parameters of the normal pulse, i.e. the strength of nonlinearity, κ_2, and the dispersion-to-diffraction ratio, σ_2, are varied. We found that catastrophic self-focusing of the pulse propagating in anomalous dispersion regime can be arrested by the influence of the pulse propagating in normal dispersion regime.

The results, following from the variational method, are shown in figure 1. The shaded area denotes the range of the parameters of the normal pulse, κ_2 and σ_2, for which catastrophic self-focusing of the anomalous pulse does not occur. It is evident that for small nonlinearity of the normal pulse, κ_2, the term describing cross-phase modulation of the anomalous pulse is negligible as compared with self-phase modulation. Therefore, the process of catastrophic self-focusing cannot be stopped and it takes place for all values of σ_2. When the strength of nonlinearity κ_2 increases, the influence of the normal pulse on the anomalous pulse through the cross-phase modulation term increases and then it is possible, for some values of the dispersion-to-diffraction ratio, $|\sigma_l(\kappa_2)| < |\sigma_2| < |\sigma_u(\kappa_2)|$, to stop catastrophic self-focusing. The lower threshold, $|\sigma_l(\kappa_2)|$, in the beginning decreases with an increase of the strength of nonlinearity of the normal pulse, κ_2. For a sufficiently large nonlinearity, $\kappa_2 > \kappa_{cat}$, the lower threshold becomes zero. The upper threshold, $|\sigma_u(\kappa_2)|$, increases with an increase of nonlinearity. The existence of the lower threshold can be explained as follows: when $|\sigma_2| < |\sigma_l|$ is small, the dispersive term of the normal pulse is negligible as compared with diffraction. Therefore, the most important role in the propagation of the normal pulse is played by self-focusing,which not only does not lead to an arresting of catastrophic self-focusing of the anomalous pulse, but even additionally enhances it. A similar situation is known, for example, in a configuration of two beams, which co-propagate in a bulk medium and have the same amplitudes [18]. Namely, the critical value of nonlinearity necessary for catastrophic self-focusing is three times smaller than in the case when they propagate as single pulses. On the other hand, while for large σ_2, we have a broadening of the normal pulse with a significant spreading of the energy out from the center of the coordinate system, $\xi = 0, \tau = 0$, for the anomalous pulse there is a tendency of the energy to concentrate in the center. Then the overlap of two pulses becomes negligible, so that the coupling between them through cross-phase modulation is very small and catastrophic self-focusing of the anomalous pulse cannot be stopped by the influence of the normal pulse.

The results obtained with the aid of the numerical calculations are shown in figure 2. They confirm predictions of the variational method. Namely, catastrophic self-focusing of the anomalous pulse can be arrested by the pulses propagating in normal dispersion regime when the strength of nonlinearity is sufficiently large, $\kappa_2 > \kappa_{cat}$, and of dispersion-to-diffraction ratio satisfies the relation $|\sigma_l(\kappa_2)| < |\sigma_2| < |\sigma_l(\kappa_2)|$.

Another question is whether the nonlinear coupling between pulses can cause catastrophic self-focusing of the pulse propagating in normal dispersion regime. The variational method and the numerical calculations demonstrate that catastrophic self-focusing of a normal pulse in planar waveguides does not take place both when it propagates as a single pulse or when it is accompanied by an anomalous pulse.

a)

b)

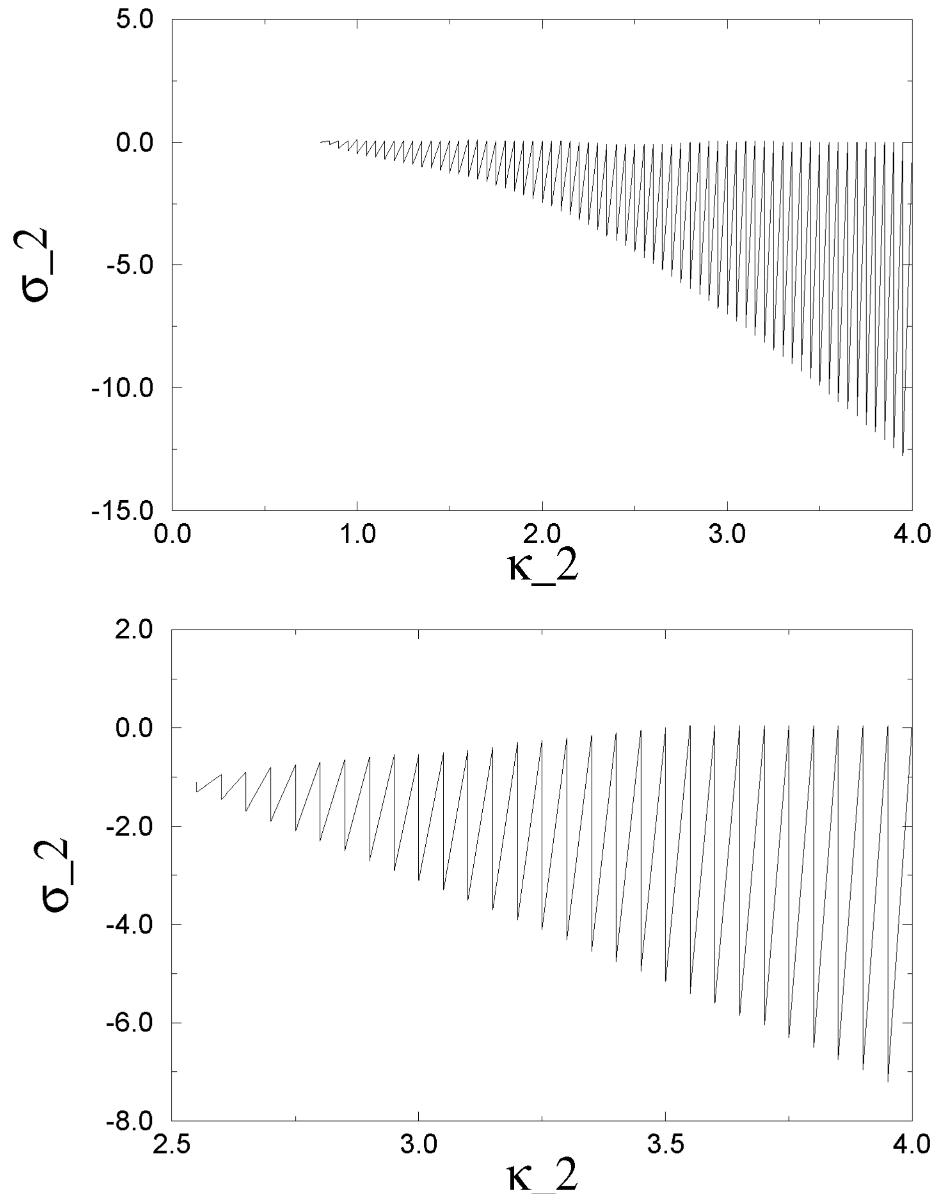


Figure 1: The results of the variational method displaying the dependence of the threshold of catastrophic self-focusing of the pulse propagating in anomalous dispersion regime, Ψ_1, on the parameters of the pulse propagating in normal dispersion regime, Ψ_2. The shaded area denotes the range of the parameters, the strength of nonlinearity, κ_2, and the dispersion-to-diffraction ratio, σ_2, for which catastrophic self-focusing occurs. The parameters of the anomalous pulse have been chosen in such a way that they are above the threshold of catastrophic self-focusing in a single propagation regime, i.e., for (a) $\kappa_1 = 2.2, \sigma_1 = 1.0$, for (b) $\kappa_1 = 2.3, \sigma_1 = 1.0$.

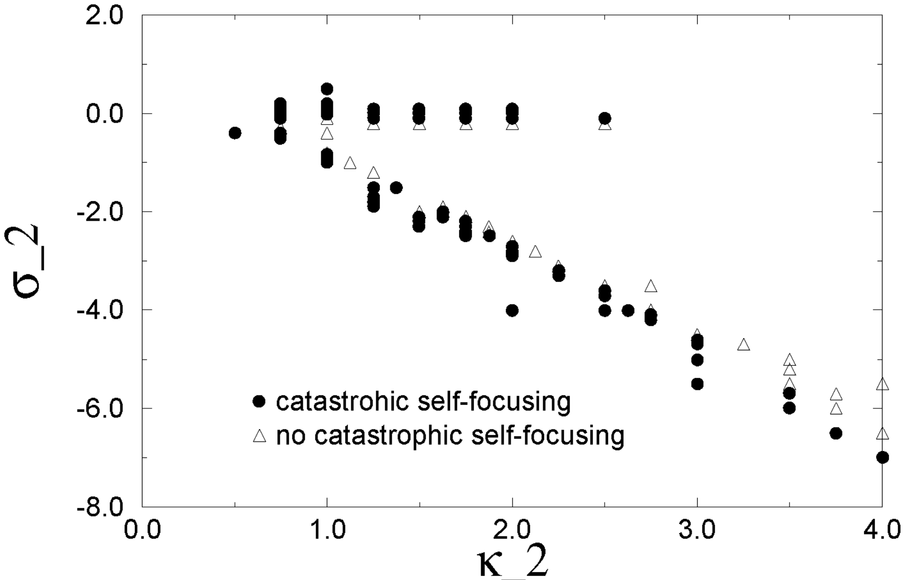


Figure 2: The results of the numerical simulations displaying the dependence of the threshold of catastrophic self-focusing of the pulse propagating in anomalous dispersion regime, Ψ_1, on the parameters of the pulse propagating in normal dispersion regime, Ψ_2, i.e. the strength of nonlinearity, κ_2, and the dispersion-to-diffraction ratio σ_2. Full circle (empty triangle) points denote occurrence (lack) of the catastrophic self-focusing. The parameters of the anomalous pulse have been chosen in such a way that they are above the threshold of catastrophic self-focusing in a single propagation regime, i.e., $\kappa_1 = 2.0, \sigma_1 = 1.0$.

4 Spatio-temporal splitting

In this section the problem of spatio-temporal splitting is discussed in more detail. The origin of spatio-temporal splitting of a pulse propagating in normal dispersion regime in Kerr-type planar waveguides [11, 12] or bulk media [13, 14, 15, 11] is the fact that spatial self-focusing and temporal self-defocusing act simultaneously during the propagation. Therefore, in space, there is a tendency of the energy to concentrate in the center of the coordinate system, $\tau = 0, \xi = 0$, whereas in time the spreading of the energy away from the center takes place. When both effects are combined, local focusing areas develop away from the center and, as a result, spatio-temporal splitting of the pulse into several sub-pulses takes place. The number of sub-pulses emerging in this way depends on the propagation distance and the parameters of the system.

In all cases when spatio-temporal splitting of pulses has been observed the numerical simulations have been used [11, 12, 13, 14, 15]. The variational method is not appropriate to predict splitting of pulses, since it requires the solution to have a shape which does not change in propagation. When a Gaussian function is chosen as the initial condition, as we have done in this paper, it is difficult (if not impossible) to guess a trial function which would satisfy the initial condition and also could describe spatio-temporal splitting of the pulse. In this connection it is worthy to recall that the variational method cannot be applied to predict, for example, the formation of higher-order solitons in planar waveguides or optical fibers [17].

Since the variational method is not applicable to the study of spatio-temporal splitting of two pulses propagating simultaneously in a nonlinear planar waveguide, the results of this section are due to numerical simulations. Figures 3 and 4 show the spatio-temporal dependences of intensities of both pulses, anomalous one (figure 3(a) and 4(a)), and normal one (figure 3(b) and 4(b)), for different longitudinal variables, ζ. Parameters of the pulses were chosen in such a way that when they propagate as single pulses the following effects take place: symmetric, spatio-temporal broadening of the anomalous pulse (see figures 3(c) and 4(c)) and large asymmetrical, spatio-temporal broadening of the normal pulse without splitting into sub-pulses (see figures 3(d) and 4(d)). Therefore, for the anomalous pulse the conditions $\sigma_1 = 1$ and $\kappa_1 < 1 + \sigma_1$ are satisfied.

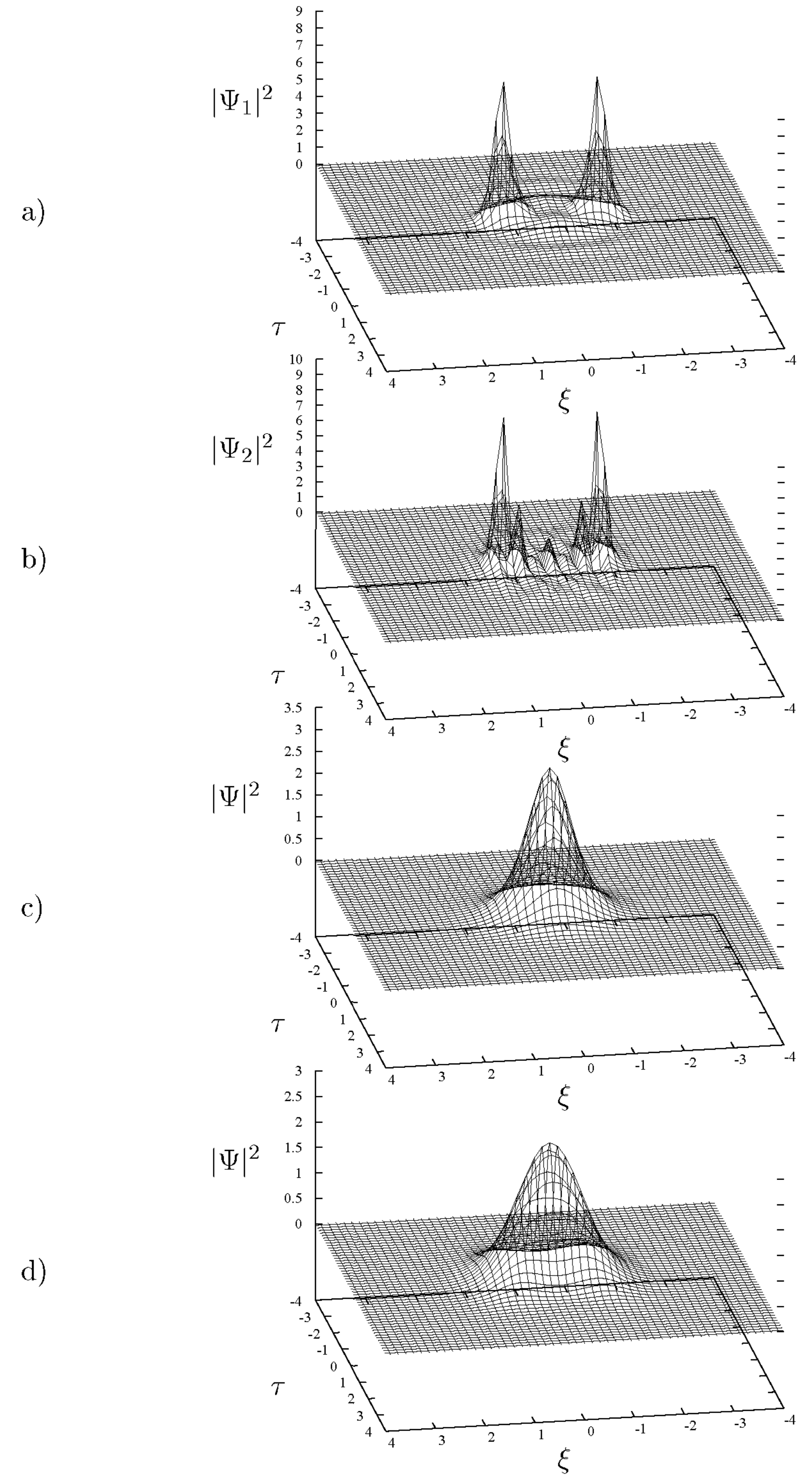
When the pulses propagate simultaneously, i.e. there is a nonlinear coupling between them, the situation becomes qualitatively different. Namely, spatio-temporal splitting of both pulses can develop, so that for $\zeta = 2$ the anomalous (normal) pulse becomes divided into $n = 3$ ($n > 10$) sub-pulses. The effect of splitting of the anomalous pulse, which does not occur when it propagates as a single pulse, can be explained as follows. When the nonlinear coupling between pulses through cross-phase modulation is present one pulse can induce a redistribution of energy of the other pulse. Therefore, if there are some local focusing areas in the distribution of energy of one pulse, the energy of the other pulse tends to concentrate there. Such a tendency has already been pointed out by Agrawal, who has observed the occurrence of local focusing areas in the distribution of energy of two beams which co-propagate in a defocusing nonlinear medium [16].

5 The limiting case of vanishing dispersion of the normal pulse

In this section we consider the limiting case when the dispersive term of the normal pulse can be neglected. We will apply the variational method and numerical simulations and compare their results. We will assume that the initial condition has a shape of the Gaussian function given by equation (3) and concentrate basically on the question as to whether there exists a stable self-trapped solution of the above mentioned system of equations.

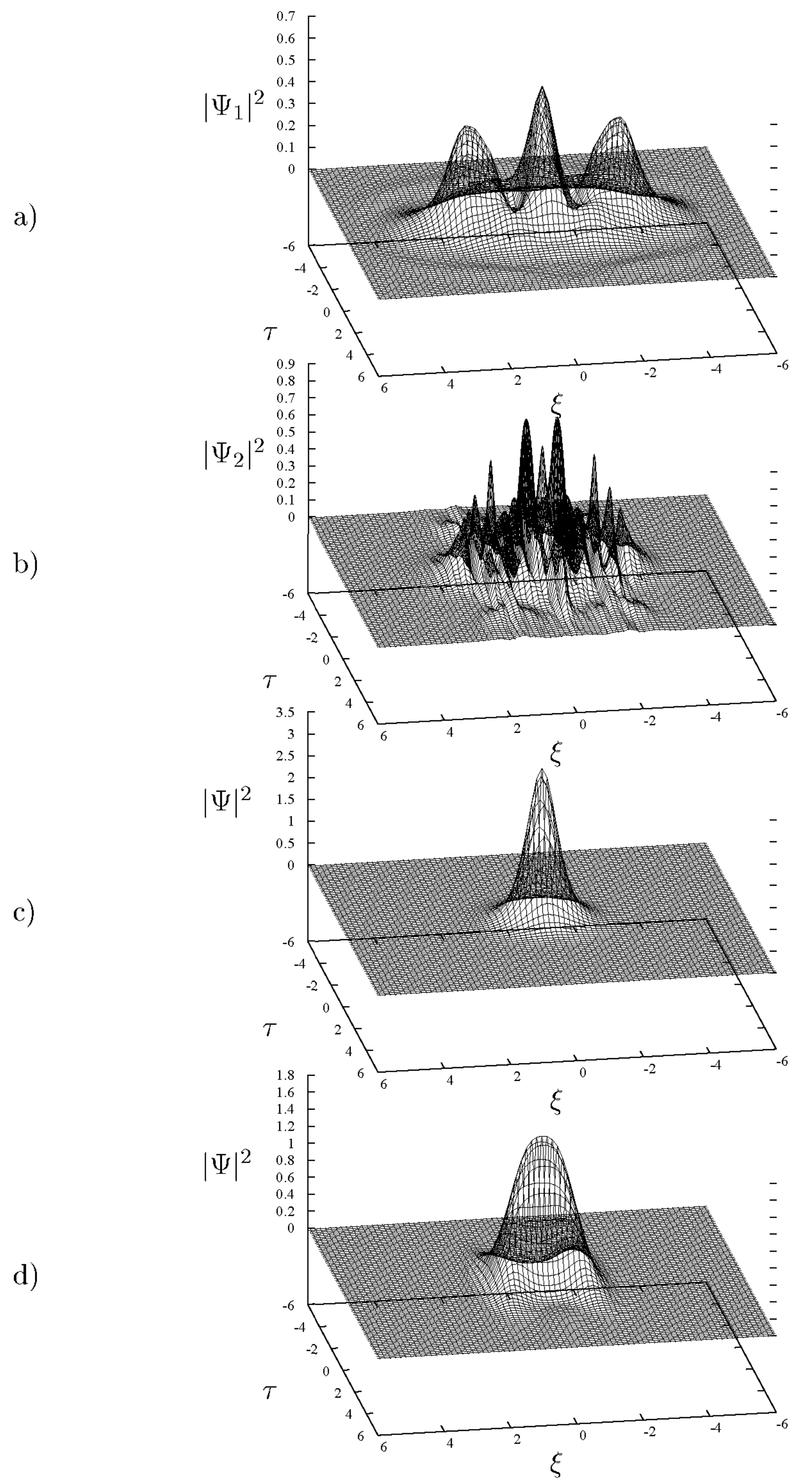
First we briefly discuss the case when the pulses propagate in a planar waveguide separately, i.e. when there is no coupling between them. Specifically, we consider (i) the propagation of a pulse with anomalous dispersion and (ii) the propagation of a dispersionless beam. Case (i) can be described by the (2+1)-dim NSE, which does not have stable, self-trapped solutions. Thus, depending on parameters of the system, either spatio-temporal spreading of the pulse or catastrophic self-focusing take place. Case (ii) is described by the (1+1)-dim NSE which depends only on one transverse variable ξ and being an integrable system possesses the familiar soliton solution given by *sech* function [3]. Taking the Gaussian function (equation (3)) which depends on two transverse variables τ and ξ from the variational method we obtain that the temporal width of the pulse is constant while the spatial width oscillates. These oscillations are due to the fact that the shape of the Gaussian trial function differs from the exact soliton solution given by *sech* function [19]. However, numerical simulations lead to a different behavior. Namely, the temporal width of the pulse appears to oscillate synchronically with the spatial width. Amplitudes of both oscillations decrease with the longitudinal variable ζ and vanish at finite ζ when the spatial soliton is formed [26, 27].

Now, let us take into account the nonlinear coupling between pulses. From the variational method it follows that the evolution of the normal pulse coupled to the anomalous one is essen-



$\zeta = 1.0$

Figure 3: The spatio-temporal dependence of the intensity of the anomalous pulse (a) co-propagating with the normal pulse (b) and the spatio-temporal dependence of the intensities of both pulses, anomalous one (c) and normal one (d) when they propagate separately. For (a) and (b) $\sigma_1 = 1, \kappa_1 = 1.88, \sigma_2 = -0.1, \kappa_2 = 2$, for (c) and (d) $\sigma = 1, \kappa = 1.88$ and $\sigma = -0.1, \kappa = 2$, respectively; the propagation distance $\zeta = 1.0$.



$\zeta = 1.0$

Figure 4: The spatio-temporal dependence of the intensity of the anomalous pulse (a) co-propagating with the normal pulse (b) and the spatio-temporal dependence of the intensities of both pulses, anomalous one (c) and normal one (d) when they propagate separately. For (a) and (b) $\sigma_1 = 1, \kappa_1 = 1.88, \sigma_2 = -0.1, \kappa_2 = 2$, for (c) and (d) $\sigma = 1, \kappa = 1.88$ and $\sigma = -0.1, \kappa = 2$, respectively; the propagation distance $\zeta = 2.0$.

tially similar to that of the single normal pulse. Namely, the temporal width of the pulse does not depend on the longitudinal variable, ζ, as it is seen from equation (8 c) with the neglected dispersion of the normal pulse $\sigma_2 = 0$, while the spatial width of the pulse undergoes periodic oscillations (See figure 5(b)). The propagation of the anomalous pulse coupled to the normal one is, however, qualitatively different as compared with the behavior of a single anomalous pulse. Namely, both temporal and spatial widths of the pulse undergo periodic oscillations (See figure 5(a)). Therefore, neither spatio-temporal spreading nor catastrophic self-focusing of the anomalous pulse can develop and a self-trapped solution arises. Note that a similar self-trapped solution was found in the case of (2+1)-dim NSE with the saturation of nonlinearity [5].

We also performed numerical simulations for the case of simultaneously propagating pulses. The results are displayed in figure 6 from which it is evident that the temporal and spatial widths of both pulses oscillate synchronically, with the amplitude of the temporal oscillations smaller than the amplitude of the spatial ones. Unfortunately, the numerical calculations are rather labourous and we were not yet able to calculate evolution for longer longitudinal variables, ζ, so that we do not know if the amplitude of oscillations decreases with ζ and if no spreading and catastrophic self-focusing of the anomalous pulse develop. Nevertheless, the currently available numerical results suggest that a self-trapped solution can exist in the configuration under discussion. Further calculations should clarify this question.

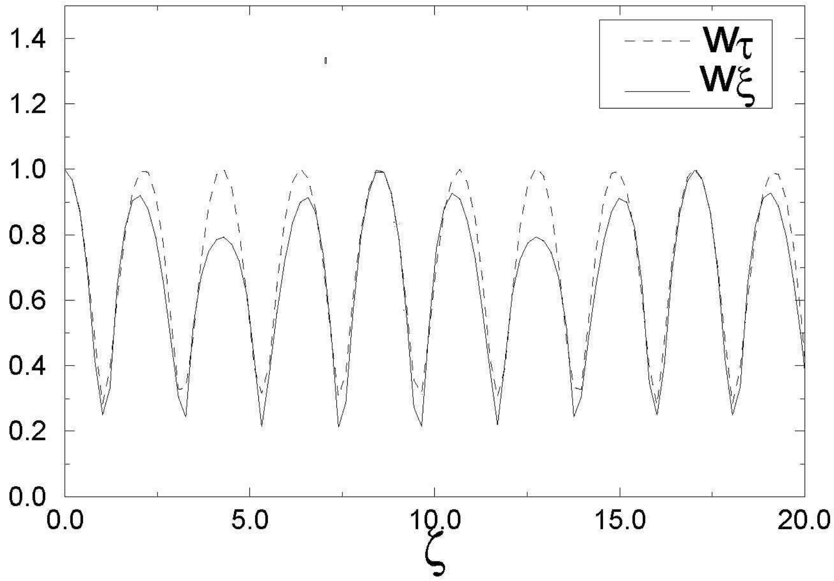
Note that, a configuration of two simultaneously propagating pulses could also be used in optical compression techniques for, as it is seen from figure 6(a), for some particular values of the longitudinal distance ζ the temporal width of the anomalous pulse decreases about 5 times with respect to the initial width.

6 Conclusions

In this paper properties of two pulses propagating simultaneously in different dispersion regimes, i.e. anomalous and normal, in a Kerr-type planar waveguide are considered. The propagation is described by two coupled NSE. The interaction between pulses is assumed to be limited to cross-phase modulation. Four wave mixing is neglected, i.e. no energy transfer between pulses is taken into account. The accuracy of another assumption used in the analysis, the omitting of the difference of group velocities of the pulses, is discussed in Appendix B. Our analysis is based on the variational method and numerical simulations.

First we have studied the influence of the parameters of the pulse propagating in normal dispersion regime on the threshold of the catastrophic self-focusing of the pulse with anomalous dispersion. We observed that catastrophic self-focusing of the pulse propagating in anomalous dispersion regime can be arrested by the pulse propagating in normal dispersion regime when the strength of nonlinearity is sufficiently large, $\kappa_2 > \kappa_{cat}$ and the dispersion-to-diffraction ratio satisfies the relation: $|\sigma_l(\kappa_2)| < |\sigma_2| < |\sigma_l(\kappa_2)|$. We also investigated whether the nonlinear coupling between pulses can cause catastrophic self-focusing of the pulse propagating in normal dispersion regime. Both the variational method and numerical calculations, show that the answer to this question is negative. Namely, catastrophic self-focusing of a normal pulse in planar waveguides does not take place both when it propagates as a single pulse or when it is accompanied by an anomalous pulse to which it is nonlinearly coupled.

a)



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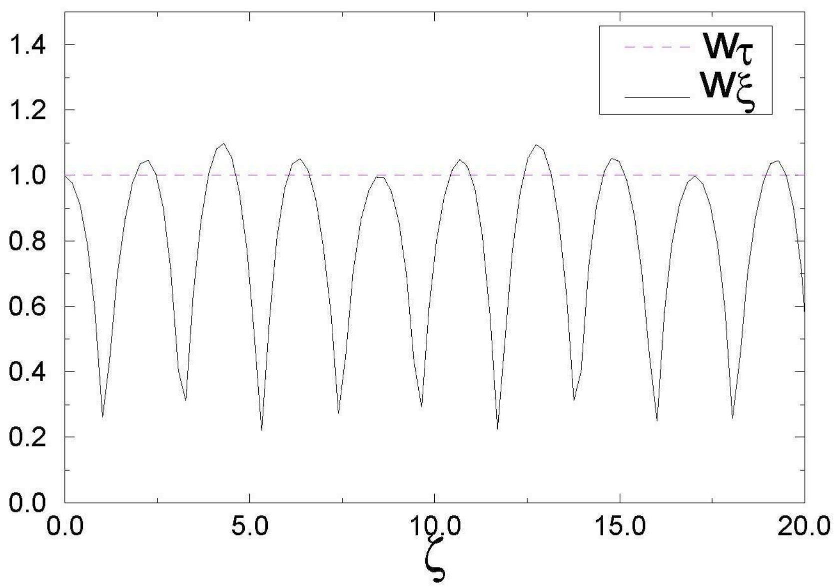
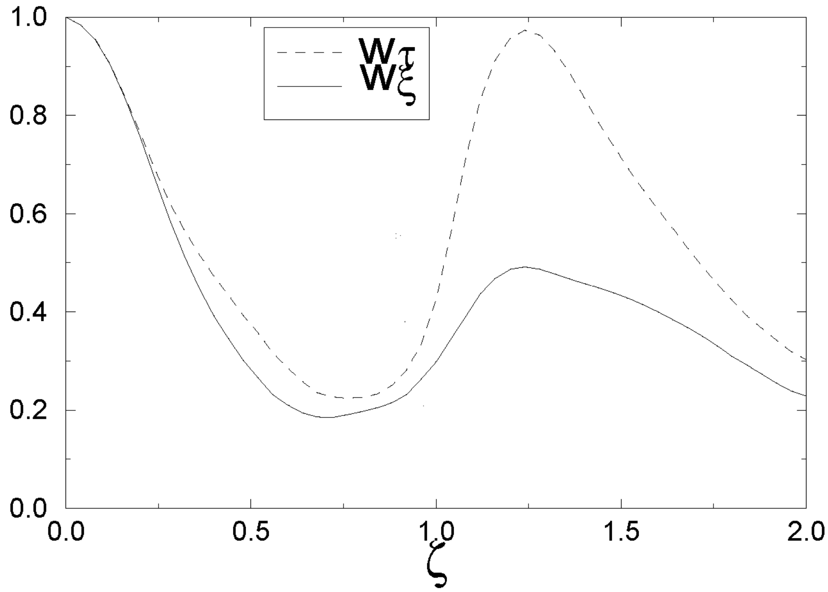


Figure 5: The results obtained using the variational method displaying the dependence of the temporal, w_τ, and spatial w_ξ, widths of the anomalous pulse (a) co-propagating with the normal pulse (b).

a)



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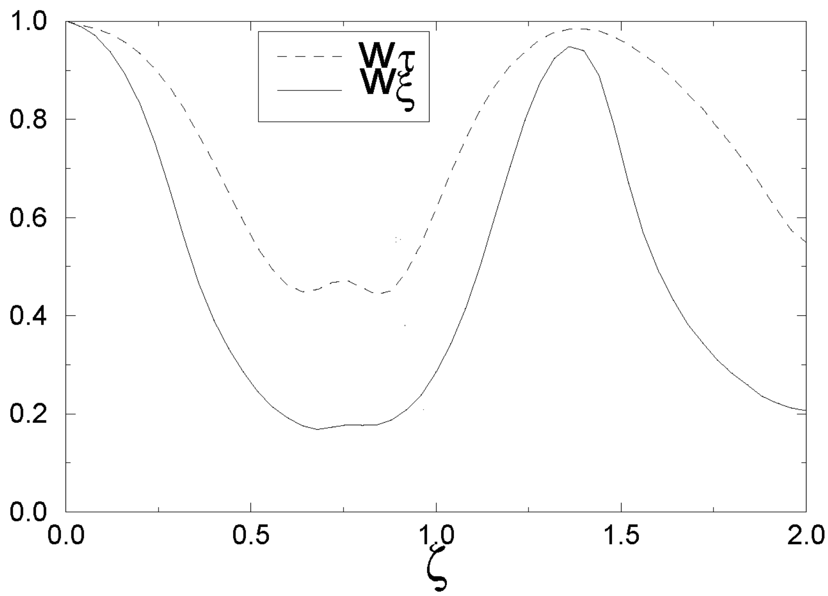

Figure 6: The results obtained using the numerical simulations displaying the dependence of the temporal, w_τ, and spatial w_ξ, widths of the anomalous pulse (a) co-propagating with the normal pulse (b).

We have also found, using the numerical simulations, that the presence of the pulse propagating in normal dispersion regime can lead to spatio-temporal splitting of the pulse propagating in anomalous dispersion regime. Recall that splitting of an anomalous pulse into several pulses does not occur when it propagates as a single pulse.

We also considered the limiting case of vanishing dispersion of the pulse propagating in normal dispersion regime. The main motivation was to study whether such configuration can lead to a stable self-trapped propagation of a pulse with anomalous dispersion. The positive answer has been obtained within the variational method which confirms that neither spatio-temporal spreading nor catastrophic self-focusing of the anomalous pulse can develop thus giving rise to a self-trapped solution. Note, that this kind of stabilization is similar to the one which was earlier found in media with the saturation-type nonlinearity [5]. Though the existing data supports the existence of a self-trapped solution, conclusive results require labourous simulations at high values of the longitudinal variable ζ and are not yet available (work in progress).

In conclusion, note that the existence of a stable self-trapped solution could be useful for example in the optical switching devices. Besides, the configuration of two simultaneously propagating pulses in a planar waveguide could be of use in optical compression techniques.

Acknowledgments

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Appendix A

The notation in equation (2) is as follows [2]: $\zeta = z/z_{DF1}$ is the longitudinal coordinate normalized to the Fresnel diffraction length of the anomalous pulse, $\xi = x/w_1$ is the spatial transverse coordinate normalized to the initial spatial width of the anomalous pulse, $\tau = (t - \beta_1^{(1)} z)/t_1$ is the local time normalized to the initial temporal width of the anomalous pulse. The parameters $\sigma_j = z_{DF1}/z_{DSj}$, $\mu = z_{DF1}/z_{DF2}$, $\varrho = (1/v_{g1} - 1/v_{g2})(z_{DF1}/t_2)$, $r = \lambda_1/\lambda_2 = \omega_2/\omega_1$ denotes respectively the dispersion-to-diffraction ratio, the ratio of the Fresnel diffraction length of the anomalous pulse to the Fresnel diffraction length of the normal pulse, the difference of the group velocities of two pulses, and finally the ratio of the carrier frequency of the anomalous pulse to the carrier frequency of the normal pulse. $\Psi_j := \beta_j^{(0)} w_j \sqrt{n_0(\omega_j) n_2(\omega_j)}\bar{\Psi}_j$ denotes the normalized amplitude of the j-s pulse, where $\bar{\Psi}_j$ is the amplitude of the slowly varying envelope of the electric field. The dispersive terms are defined as follows: $\beta_j^{(0)} := \beta^{(0)}(\omega_j) = \omega_j/c$ is the wavenumber, $\beta_j^{(1)} := \delta\beta/\delta\omega|_{\omega=\omega_j} = 1/v_{gj}$ is the reverse group velocity, and $\beta_j^{(2)} := \delta^2\beta/\delta\omega^2|_{\omega=\omega_j}$ is the group velocity dispersion. The parameters $z_{DFj} := \beta_j^{(0)} n_0(\omega_j) w_i^2$, $z_{DSj} := t_j^2/\beta_j^{(2)}$, w_j, t_j denotes, respectively, the Fresnel diffraction length, the dispersive length, the initial spatial width and the initial temporal width of the j-s pulse. In the above notation $j = 1, 2$, where the subscript $j = 1$ ($j = 2$) refers to the anomalous (normal) pulse.

Appendix B

Since we have assumed that pulses have different wavelengths and different group velocity dispersions, it is physically evident that they should also have different group velocities. Therefore, the assumption that $\delta = 0$ is a simplification accepted in this paper and should be viewed as a first step of the analysis. When $\delta \neq 0$ the pulses propagate with different velocities and the overlap between them decreases with the longitudinal variable. Therefore, the nonlinear coupling between them also decreases. In the limiting case of $\delta \to \infty$ the coupling between pulses becomes zero and the problem of simultaneous propagation of two pulses reduces to the case when they propagate separately. Note that in the variational method the inclusion of the parameter δ becomes problematic since for the trial function given by a Gaussian function the term $\iota\frac{1}{2}\varrho/r\,[\Psi_2^*(\partial\Psi_2/\partial\tau) - \Psi_2(\partial\Psi_2^*/\partial\tau)]$ in the Lagrangian (equation (4)) becomes zero and one should consider another candidate function for the trial function.

However, we believe that the inclusion of the parameter δ will not cause qualitative changes in the results of this paper, such as the possibility of an arresting of catastrophic self-focusing of the pulse propagating in anomalous dispersion regime by the influence of the pulse propagating in normal dispersion regime. The only difference we expect is a change of the values of the parameters, $\sigma_l, \sigma_u, \kappa_{cat}$, which describe the threshold of catastrophic self-focusing. This quantitative changes would be proportional to the value of the parameter δ.

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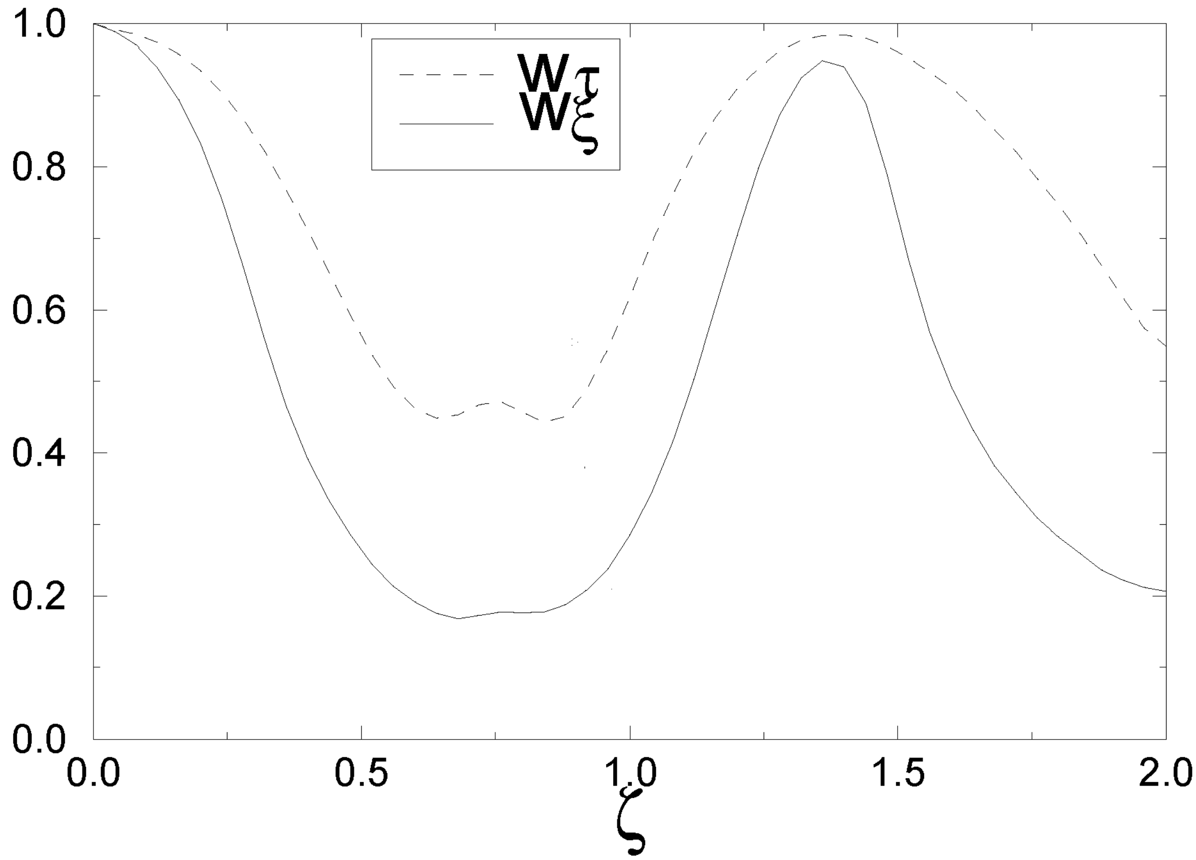

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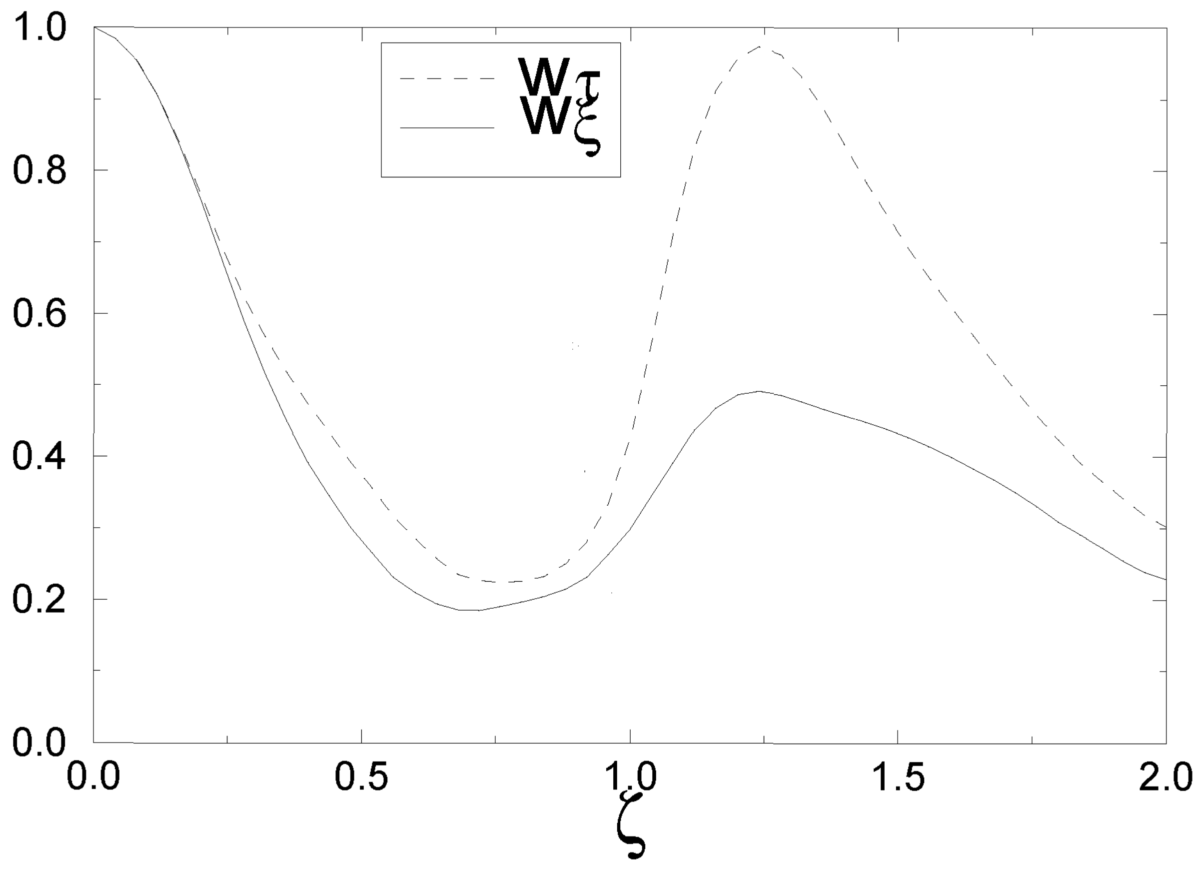

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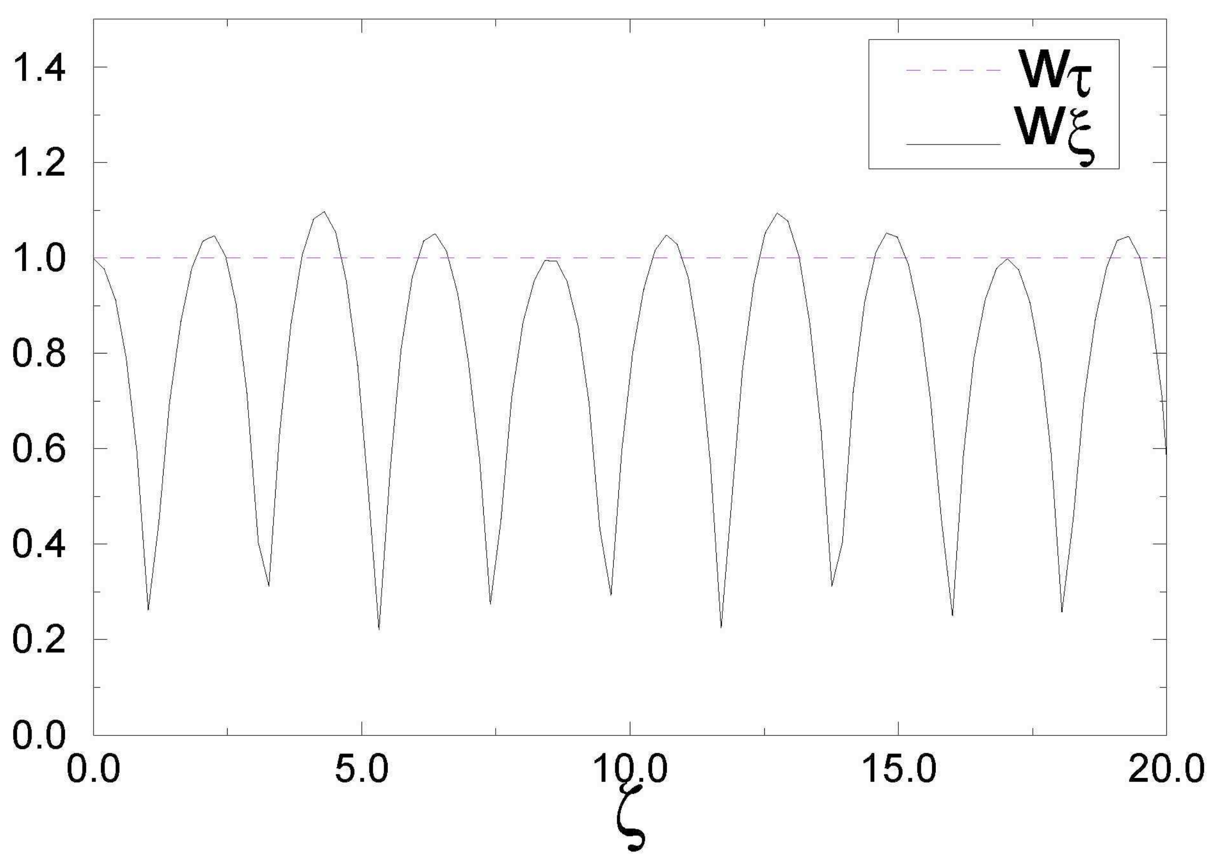
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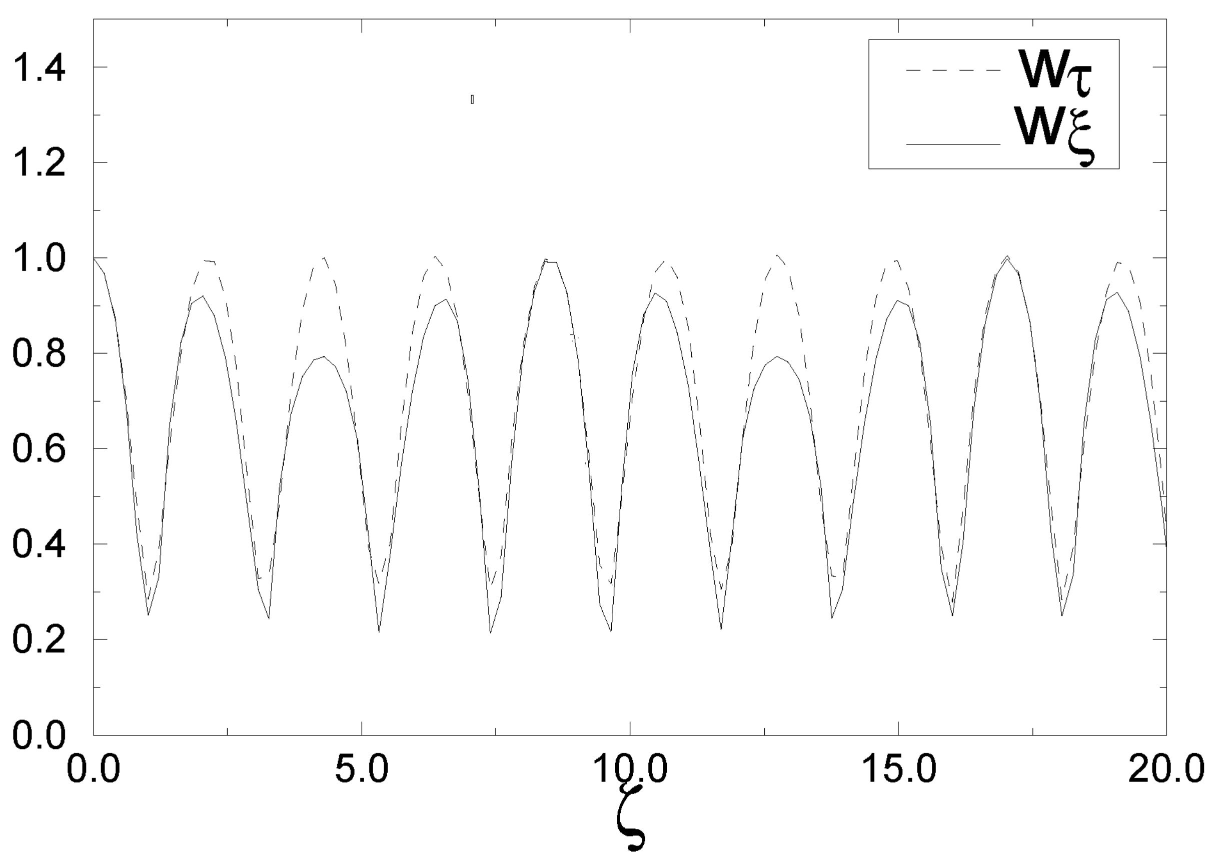

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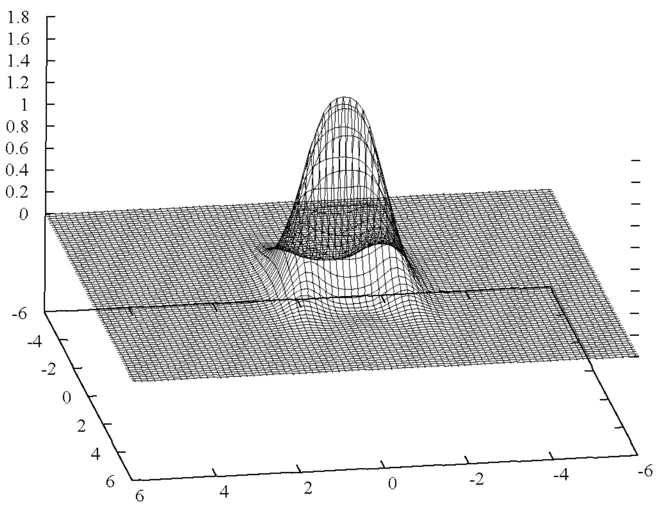
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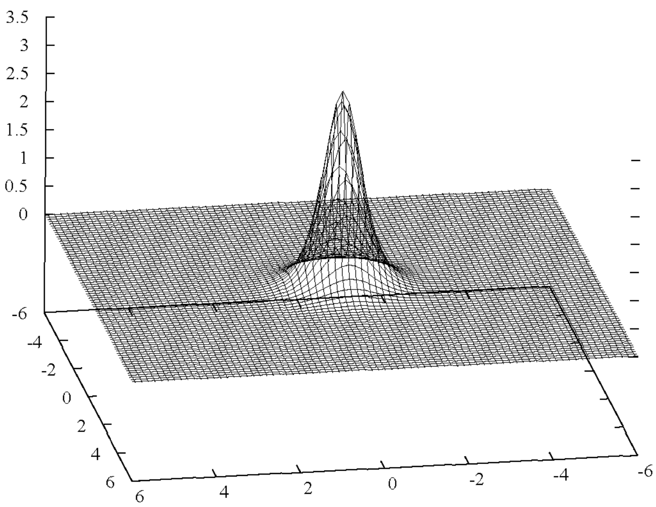

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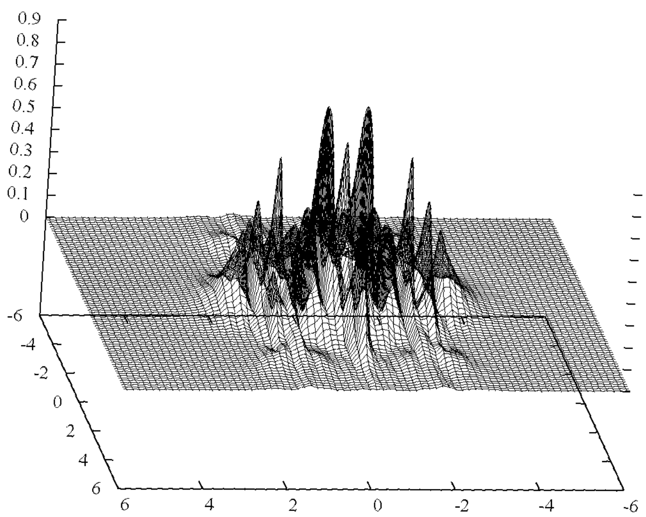
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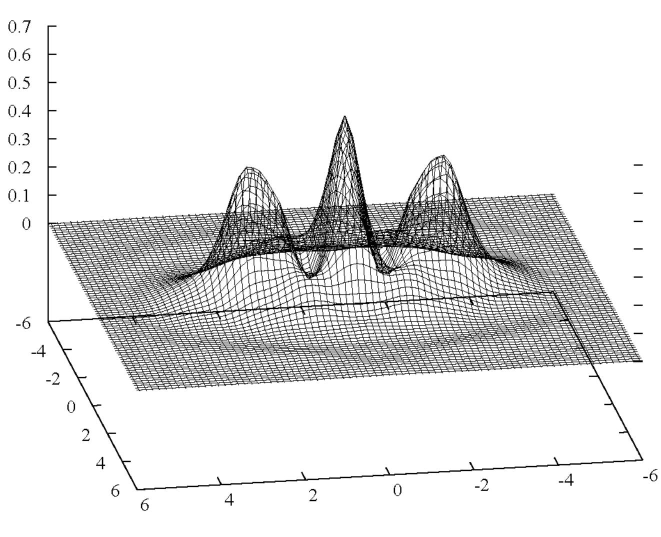

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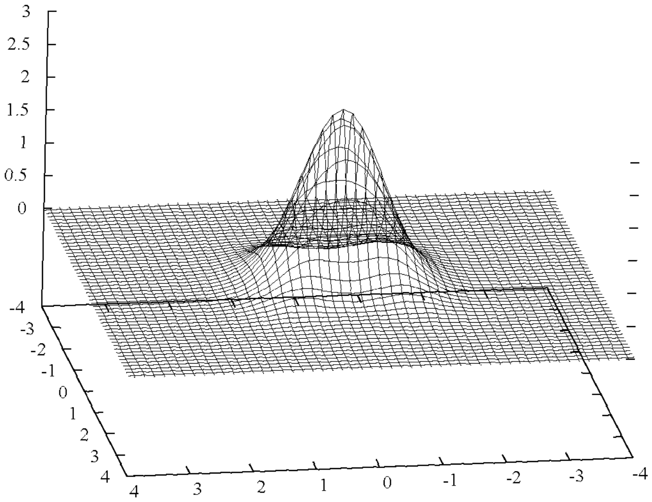

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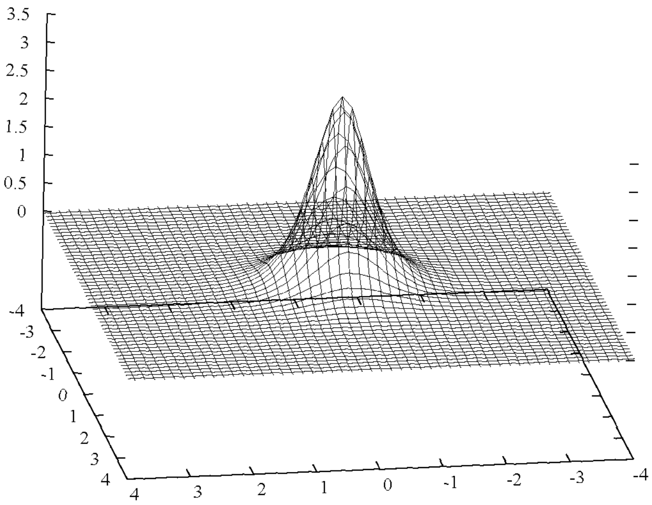
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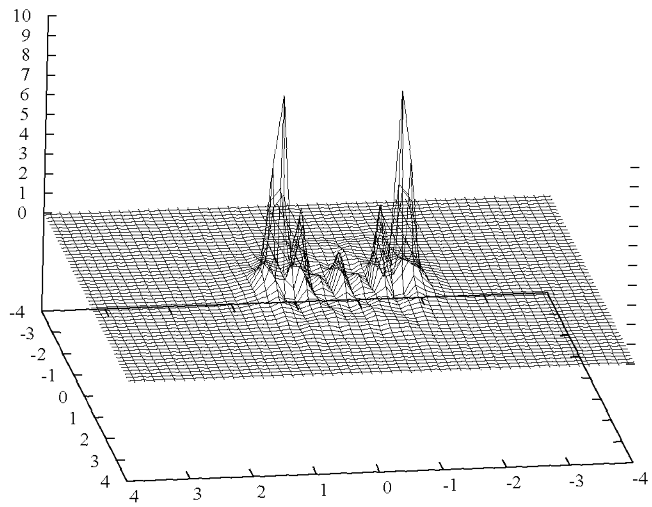
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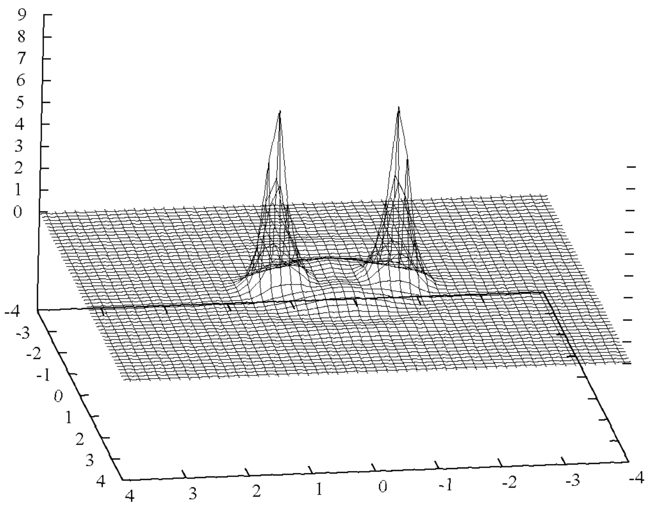

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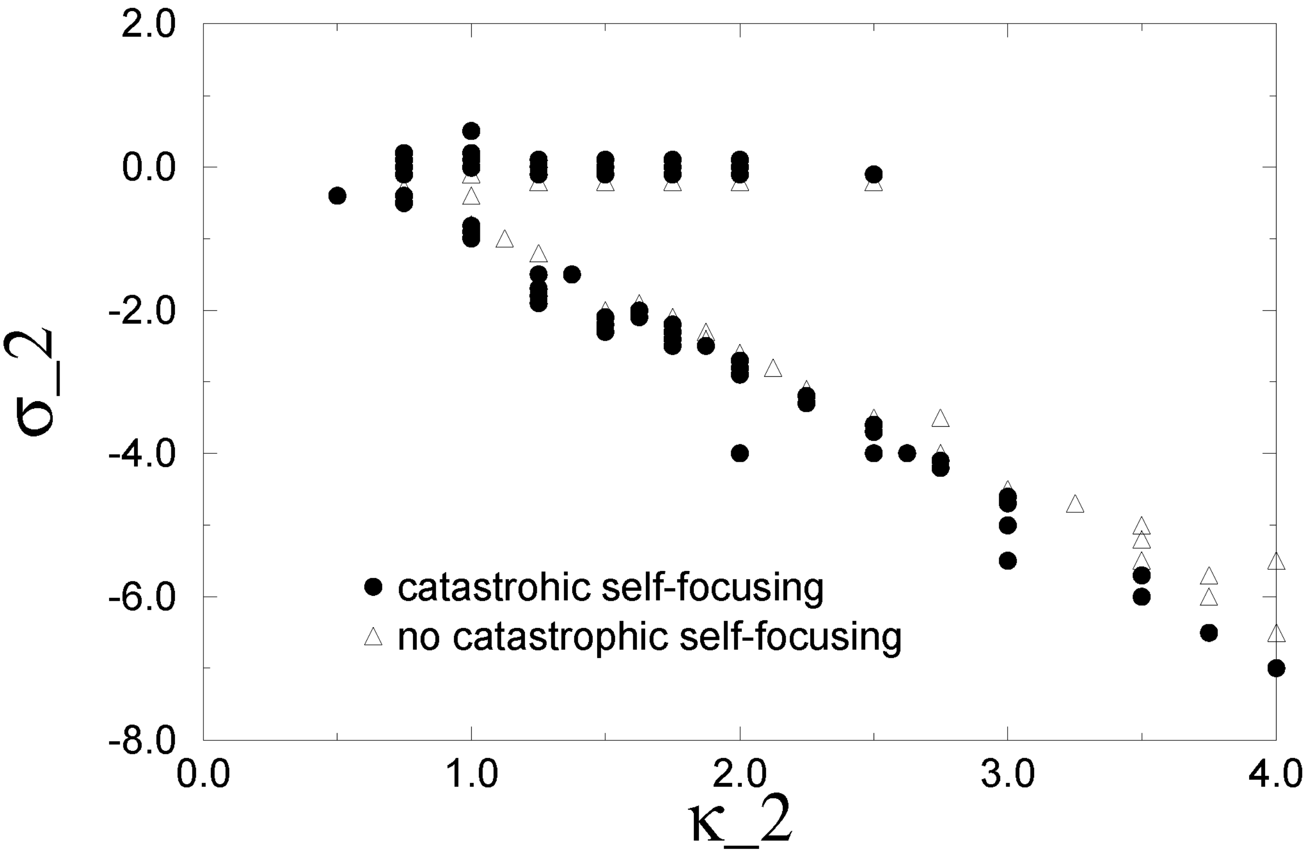

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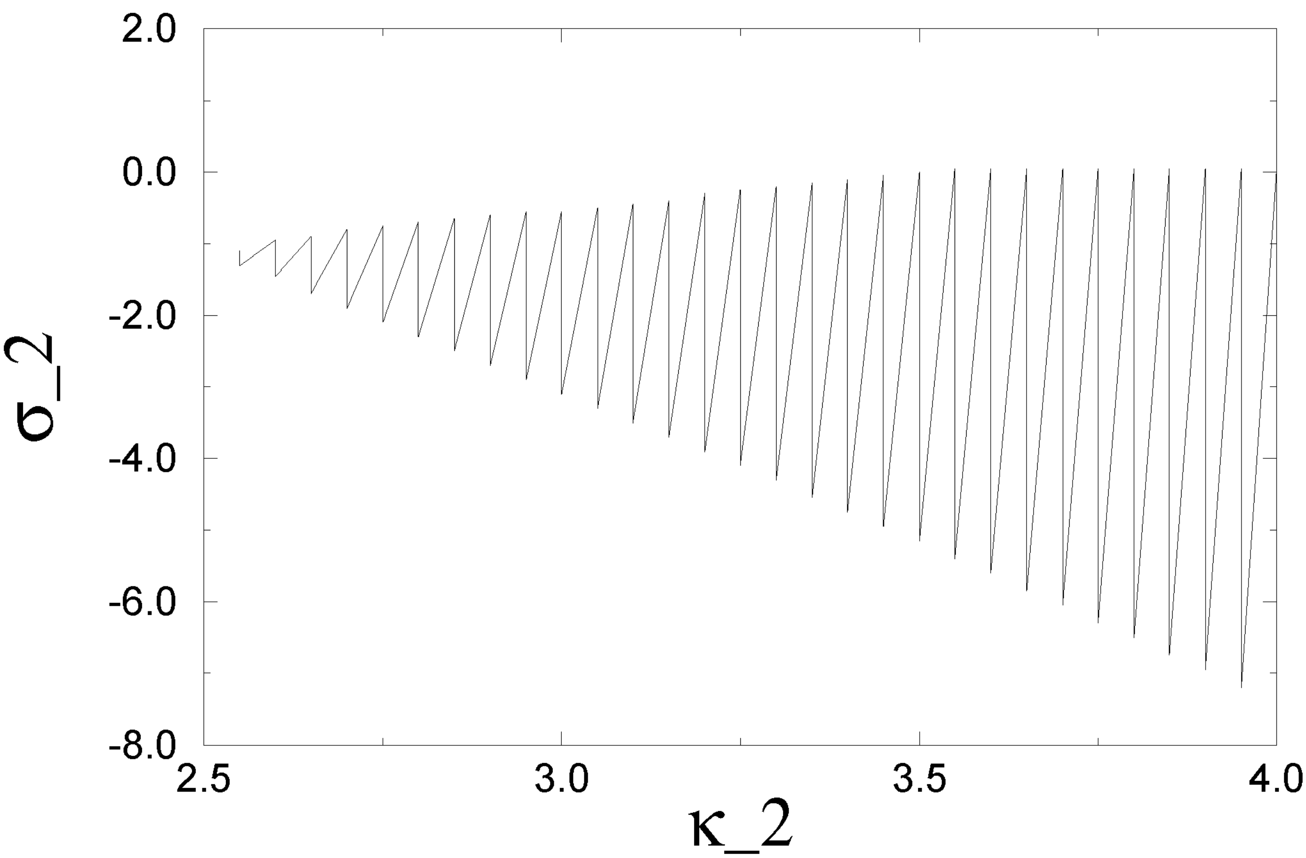

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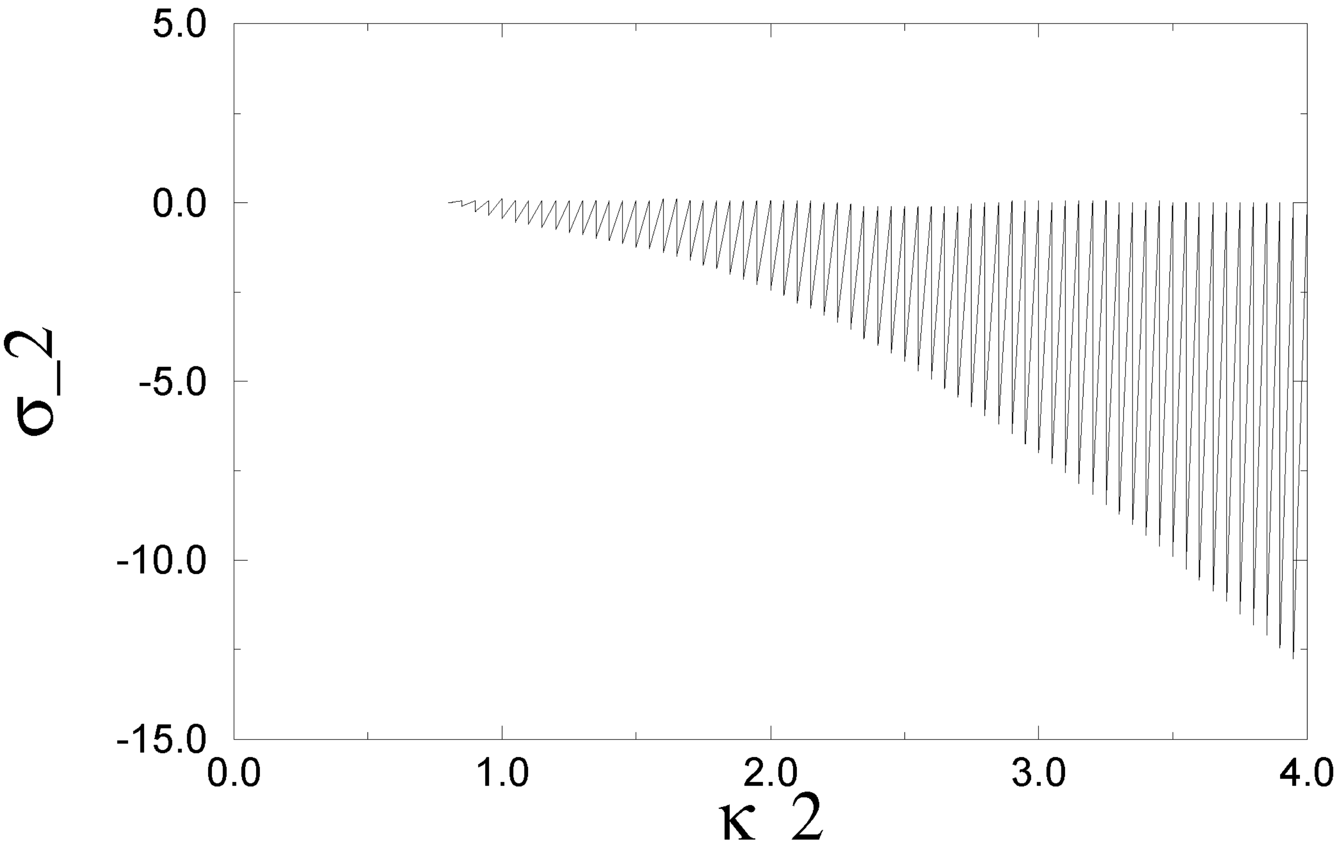
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