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ELIMINATION OF CHAOS IN MULTIMODE, INTRACAVITY-DOUBLED LASERS IN THE PRESENCE OF SPATIAL HOLE-BURNING

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In this paper possibilities of a stabilization of large amplitude fluctuations in an intracavity-doubled solid-state laser are studied. The modification of the cross-saturation coefficient by the effect of spatial hole-burning is taken into account. The stabilization of the laser radiation by an increase of the number of modes, as proposed in [James *et al.*, 1990b; Magni *et al.*, 1993], is analyzed. It is found that when the cross-saturation coefficient is modulated by the spatial hole-burning the stabilization is not always possible. We propose a new way of obtaining a stable steady-state configuration based on an increase of the strength of nonlinearity, which leads to a strong cancellation of modes, so that during the evolution all modes, but for a single one, are canceled. Such a steady-state solution is found to be stable with respect to small perturbations.

1. Introduction

Solid-state lasers containing frequency-doubling crystals are efficient and compact sources of coherent visible optical radiation. Unfortunately, when they operate in multimode regime, one observes irregular fluctuations of the output intensity. This behavior, referred to as the green problem, has been reported for the first time by Baer [1986]. He found that these instabilities arise from a coupling between longitudinal modes of the laser due to sum-frequency generation. In particular, when such a laser operates in a single longitudinal mode, its output is stable [Kennedy & Barry, 1974]. In the case of two oscillating longitudinal modes, the output intensity is stable only for small values of nonlinearity, otherwise both modes tend to pulse on and off out of phase [Baer, 1986]. When the number of lasing modes is larger than two, the laser can exhibit, depending on the parameters describing it, various types of behavior such as: antiphase dynamics [James *et al.*, 1990b; Wiesenfeld *et al.*, 1990; James *et al.*, 1990a; Roy *et al.*, 1993; Bracikowski & Roy, 1991; Mandel & Wang, 1994; Wang *et al.*, 1995; Otsuka *et al.*, 1997], clustering [Baer, 1986], grouping [Otsuka *et al.*, 1997], and chaotic dynamics [James *et al.*, 1990a; Roy *et al.*, 1993; Bracikowski & Roy, 1991; Liu *et al.*, 1997].

One of the motivations of the large number of papers devoted to intracavity-doubled solid-state lasers, and published during the last decade has been the need to find a configuration and the range of parameters describing the lasers, which could guarantee a stability of their outputs. Several ways to reach this goal have been found, for example, an introduction of a quarter-wave plate into the laser cavity, a proper alignment of the angle between birefringent axes of the active medium and the nonlinear crystal [James *et al.*, 1990a; James *et al.*, 1990b; Roy *et al.*, 1993], a placing of a tilted mirror inside the cavity [Ustyugov *et al.*, 1997], or an utilization of the effect of sum-frequency-mixing [Falter *et al.*, 1997; Danailov & Apai, 1994]. Moreover, it has been predicted analytically [James *et al.*, 1990b] and confirmed experimentally [Magni *et al.*, 1993] that when the number of oscillating longitudinal modes is set to be sufficiently large (for example, by increasing the length of the laser cavity) the output of the laser becomes stable. Besides, the green problem can also be avoided in the case of a single mode operation of the laser, which can be obtained, for example, by use of an intracavity etalon [Baer, 1986] or a birefringent filter [Nagai *et al.*, 1992; Fan, 1991].

The main goal of this paper is to continue the investigations of the green problem. The fact that the cross-saturation coefficient is modulated by the spatial hole-burning effect is taken into account. Firstly, the stabilization of the laser radiation by an increase of the number of longitudinal modes, as proposed in [James *et al.*, 1990b; Magni *et al.*, 1993] is analyzed. The results presented in [James *et al.*, 1990b; Magni *et al.*, 1993], where the cross-saturation coefficient was assummed to be constant for all modes, are compared with the numerical data. It is shown that the theoretically obtained [Wang & Mandel, 1993] linear dependence of the minimal number of modes necessary for stabilization of the laser output on the strength of nonlinearity agrees with the numerical solutions only in the case of sufficiently small nonlinearity. For larger values of nonlinearity, due to the cancellation of modes during the evolution, the minimal number of modes obtained by the numerical simulations is larger than the number which follows from the theoretical predictions. For very large nonlinearity this cancellation is so strong that only a few modes survive (even when there are initially 250 oscillating modes in the laser cavity). Therefore, a large number of simultaneously oscillating modes necessary for stabilization of the laser output cannot be achieved. A similar situation takes place when the cross-saturation coefficient is modulated by the effect of spatial hole-burning, therefore the stabilization of the laser output by an increase of the number of longitudinal modes is not always possible, as a result of the strong competition between modes and a cancellation of some of them during the evolution. However, the problem of the stabilization of the laser output can be solved in another way, namely, by an increase of the strength of nonlinearity, which leads to very strong competition between the modes, so that during the evolution, all, but for a single one, are canceled. As a consequence, a steady-state solution, which is stable against small perturbations, arises. This is the stabilization mechanism proposed in the present paper. It is valid in the case of constant cross-saturation coefficient, as well as in the case when the spatial hole-burning is taken into account.

2. Basic Equations

The analysis presented in this paper is based on the Baer-type rate equations [Baer, 1986] extended by Roy, Bracikowski and James to the case when the effect of spatial hole-burning is taken into account [Roy *et al.*, 1993; Bracikowski & Roy, 1991].[1] These equations have the following form:

$$\begin{cases} \tau_{c1}\dfrac{\partial I(p,\,t)}{\partial t} = I(p,\,t)\left(-\alpha_p + G(p,\,t) - \varepsilon I(p,\,t) - 2\varepsilon\sum_{q\neq p} I(q,\,t)\right)\,, & \text{(1a)} \\ \tau_f\dfrac{\partial G(p,\,t)}{\partial t} = G_{ap} - G(p,\,t)\left(1 + \beta(p,\,p)I(p,\,t) + \sum_{q\neq p}\beta(p,\,q)I(q,\,t)\right)\,, & \text{(1b)} \end{cases}$$

$$\beta(p,\,q) = \beta_0\frac{\displaystyle\int_{z_0}^{z_0+l}(1-\cos(2k_pz))\,(1-\cos(2k_qz))\,dz}{\displaystyle\int_{z_0}^{z_0+l}(1-\cos(2k_pz))\,(1-\cos(2k_pz))\,dz} \approx \beta_0\left|\frac{z + \dfrac{\sin(2(k_p-k_q)z)}{4(k_p-k_q)}}{\dfrac{3}{2}z}\right|_{z_0}^{z_0+l}\,, \tag{2}$$

[1]Although Roy *et al.* have written down an explicit form for the cross-saturation coefficient, they were using in their analysis the approximation that the cross-saturation coefficient is constant for all modes.

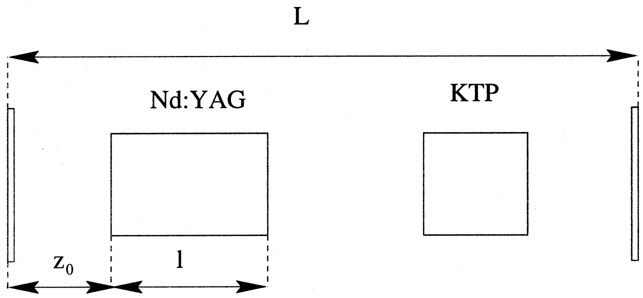


Fig. 1. Schematic diagram of an intracavity-doubled solid-state laser. L denotes the length of the cavity, l is the length of the gain medium, z_0 is the distance of the gain medium from the cavity mirror.

$$k_p = \frac{\pi p}{L}, \qquad p, q = 1, \dots, N,$$

where N is the number of longitudinal modes; τ_c and τ_f are, respectively, the cavity round trip time and the fluorescence lifetime; $I(p, t)$ and $G(p, t)$ are, respectively, the intensity and gain of the pth longitudinal mode; α_p is the cavity loss parameter for the pth mode; γ_p is the small signal gain; $\beta(p, p)$ and $\beta(p, q)$ describe, respectively, self-saturation of the p-mode and cross-saturation between two modes, p and q; $\beta_0 = 0.06$ is the scaling parameter; l is the length of the gain medium; L is the length of the laser cavity, z_0 is the distance of the gain medium from the first cavity mirror (as shown in Fig. 1); $k_p = 2\pi/\lambda_p$ is the wavevector of the longitudinal cavity mode p with the wavelengths $\lambda_p = 2L/p$. Note that even though in Eq. (2) three parameters: L, l, and z_0 are present, the modified cross-saturation coefficient depends only on two rescaling parameters, l/L and z_0/L. The parameter ε is a nonlinear coefficient which describes the conversion efficiency of the fundamental intensity into the doubled intensity. The terms $\varepsilon I(p, t)^2$ and $\varepsilon I(p, t)I(q, t)$ in Eq. (1a) account for the loss in the intensity of the fundamental frequency through second harmonic generation and through sum-frequency generation, respectively.

The set of equations [(1a) and (1b)] has a rather complicated structure and it is not possible to solve it analytically. Therefore, the results presented in this paper has been obtained with the aid of the numerical method of Runge–Kutta.

3. Simplified Model — No Spatial-Hole Burning Effect

In this section the approximation that the cross-saturation coefficient is constant for all modes: $\beta(p, q) = \beta(\bar{p}, \bar{q}) = 2/3$, is used. It is assumed that losses and small signal gains are the same for all modes, i.e. $\alpha_p = \alpha_q = \alpha$, $G_{ap} = G_{aq} = G_a$, where $p, q = 1, \dots, N$. Other parameters describing the system have been chosen as follows: $\tau_{c1} = 10$[ns], $\tau_f = 0.24$[ms], $\alpha = 0.015$, $\gamma = 0.12$, $\beta_0 = 0.06$. The number of longitudinal modes, $N = 1, \dots, 250$, and the strength of nonlinearity, $\varepsilon = 10^{-7} \div 10^{-3}$, are not fixed and they are varied in the analysis.

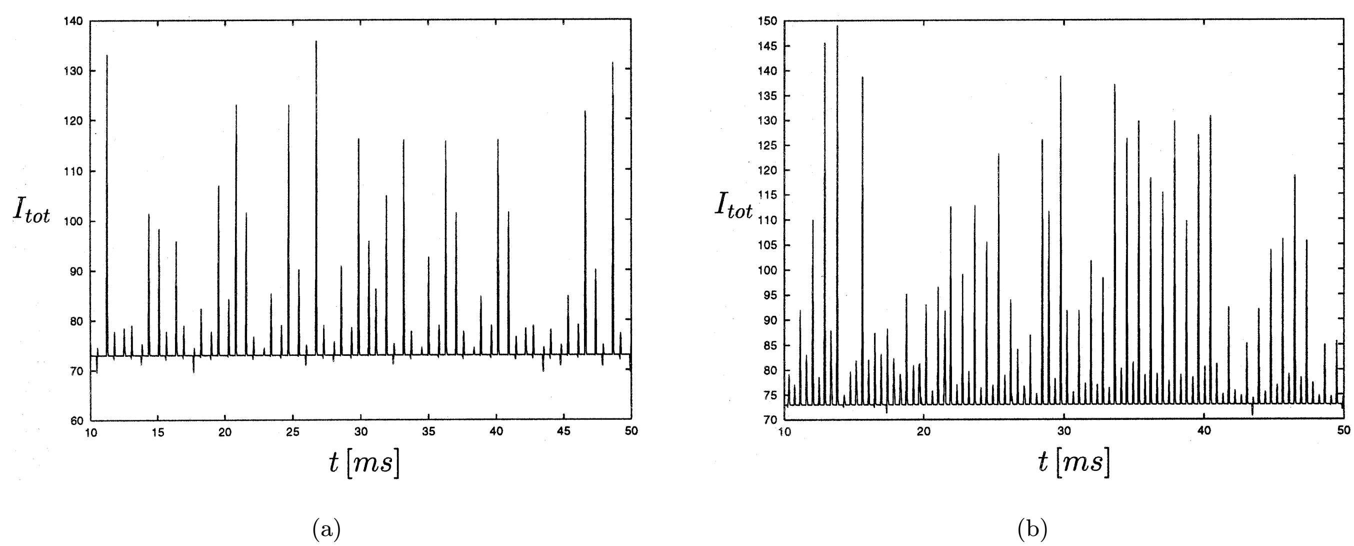
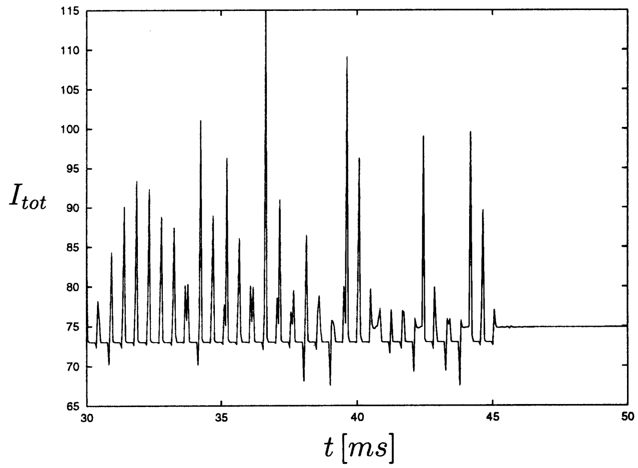
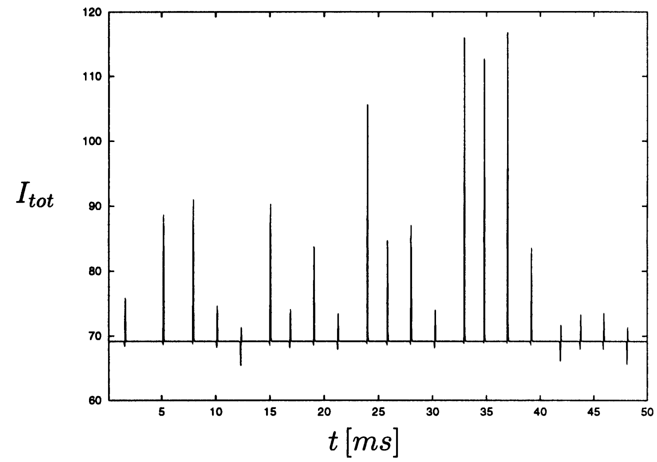


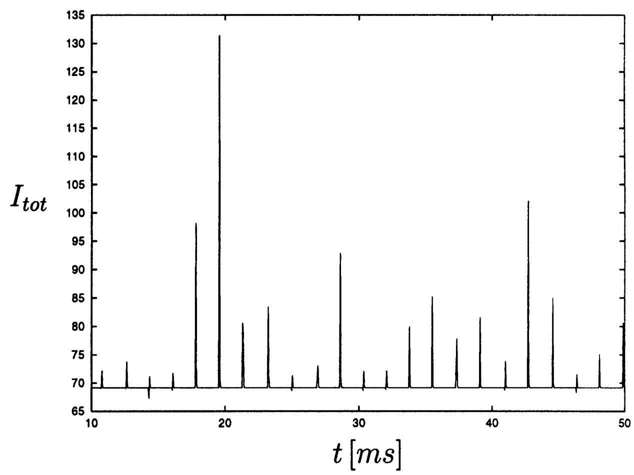
Fig. 2. The evolution of the total intensity, $I_{\rm tot} = \sum_{n=0}^{n=N} I(n, t)$, of the laser for different values of the initial number of longitudinal modes: (a) $N = 3$, (b) $N = 70$, (c) $N = 80$, (d) $N = 5$, (e) $N = 100$, (f) $N = 250$ and different values of nonlinearity: (a–c) $\varepsilon = 0.0001$, (d–f) $\varepsilon = 0.00012$; the case of the cross-saturation coefficient constant for all modes.



(c)



(d)



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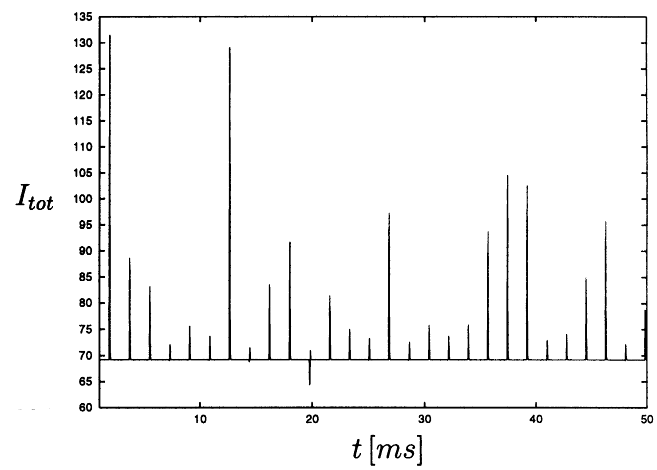


(f)

Fig. 2. (*Continued*)

As the first step let us consider the dependence of the laser output on the number of longitudinal modes initially excited in the laser cavity. From the numerical simulations it follows that for very small nonlinearity, $\varepsilon \approx 0$, the laser output is stable, even for a large number of longitudinal modes. In the case of larger nonlinearity, $\varepsilon = 1.0 \times 10^{-4}$, as it can be seen from Fig. 2(a), the behavior of the laser output is complicated and already for three simultaneously oscillating longitudinal modes the total intensity exhibits irregular oscillations. When the number of modes increases, amplitudes of those oscillations slowly increase (see Fig. 2(b) representing the results for 70 modes). Finally, when the number of modes is larger than the critical value, the total stabilization of the laser output occurs, as shown in Fig. 2(c).

However, for slightly higher than previously considered value of nonlinearity, $\varepsilon = 1.2 \times 10^{-4}$, we have not observed the stabilization of the laser output, even for very large number of longitudinal modes, $N = 200$ (compare the total intensity of the laser output for $N = 25$, $N = 100$, and $N = 200$ modes, shown, respectively, in Figs. 2(d)–2(f)). This happens because of cancelation of modes, as a result of a competition between them, which takes place during the evolution.

The dependence of the minimal number of modes necessary for stabilization of the laser output on the strength of nonlinearity is presented in

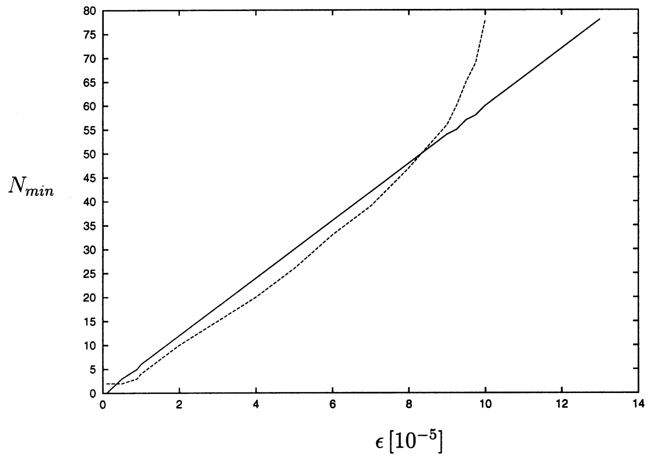
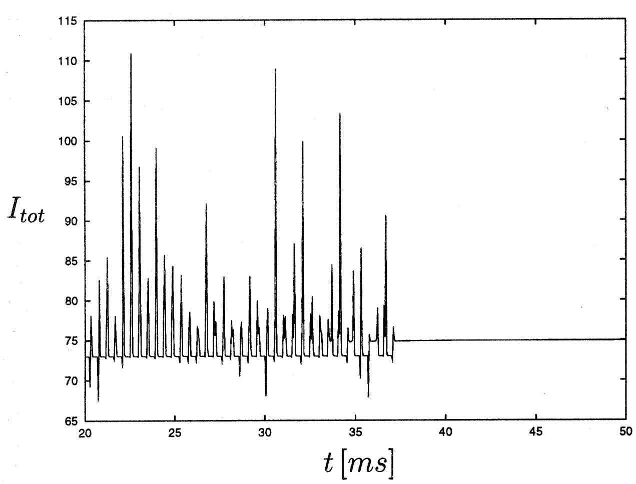


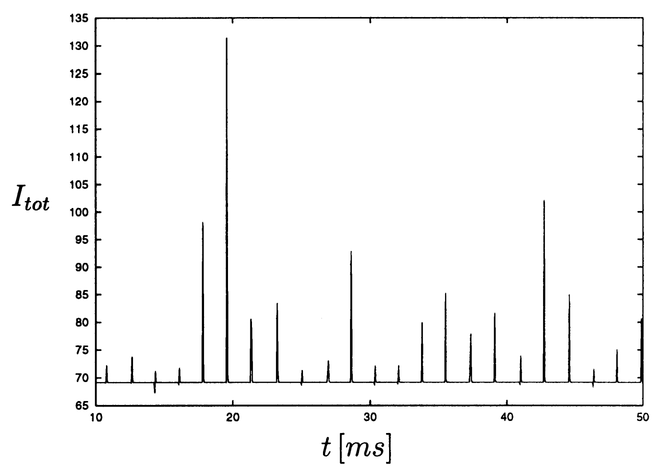
Fig. 3. The dependence of the minimal number of modes, $N_{\rm min}$ necessary to obtain a stabilization of the laser output as a function of the nonlinear coefficient, ε; the comparison between the theoretical predictions (the straight line) and the results of the numerical simulations; the case of the cross-saturation coefficient constant for all modes.

Fig. 3. As it can be seen, for small nonlinearity, $\varepsilon < 0.8 \times 10^{-4}$, there is a rather good agreement between theoretically [Wang & Mandel, 1993] and numerically obtained results. However, when nonlinearity increases, $0.8 \times 10^{-4} < \varepsilon < 1.0 \times 10^{-4}$, the minimal number of modes obtained in numerical simulations is larger than the number predicted theoretically. This discrepancy is the result of the competition between modes, which leads to cancellation of some during the evolution. When $\varepsilon > 1.2 \times 10^{-4}$ the cancellation is so large that after some time only few modes survive, thus a sufficiently large number of modes necessary for stabilization of the laser cannot be realized.

In conclusion, depending on the relation between nonlinearity and the number of longitudinal modes, the dynamics of a multimode, intracavity-doubled, solid-state laser can be divided into four regions as illustrated in Fig. 5. In particular, when nonlinearity is small and the number of modes is larger than a critical value the output of the laser is stable. Examples of such a behavior, which is labeled as Region I, are shown in Fig. 4(a). With increasing nonlinearity the complexity of the behavior of the laser increases and irregular fluctuations of the output intensity appear. This behavior, depicted in Fig. 4(b), corresponds to Region II. For even larger values of the strength of nonlinearity the cancellation of modes starts to take place. When nonlinearity exceeds the critical value $\varepsilon_{\rm crit1}$ the evolution of the output intensity leads to the cancellation of a large number of modes, so that only a few of them survive. As a result, a stabilization of the laser output cannot be obtained. This behavior, illustrated in Fig. 4(c), is classified as Region III. When nonlinearity is larger than the critical value $\varepsilon_{\rm crit2}$ there is such a large mutual cancellation of modes that after some time only one of them survives. Therefore, the steady-state

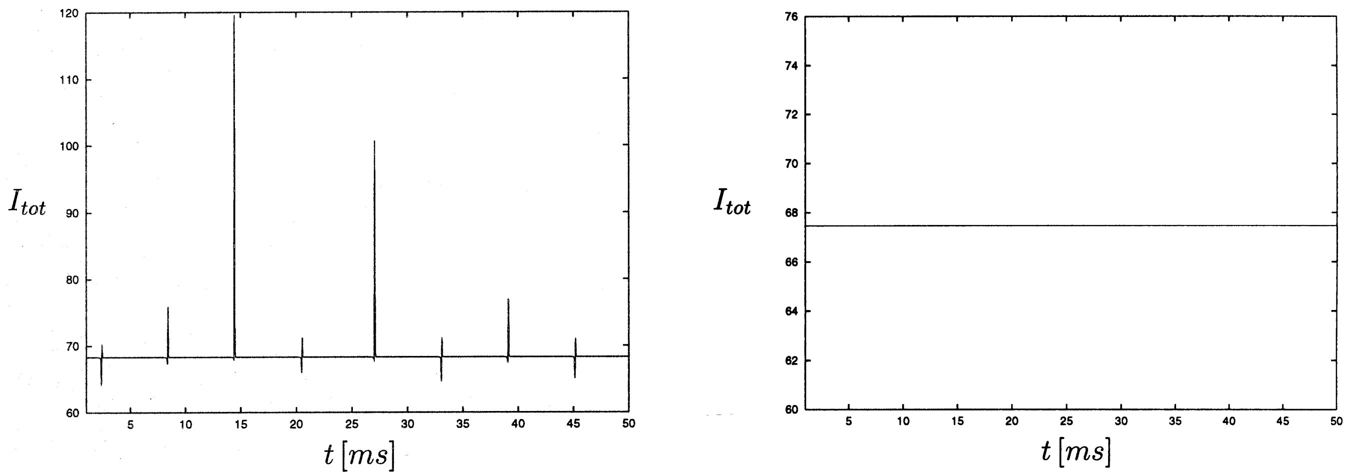


(a)



(b)

Fig. 4. The evolution of the total intensity, $I_{\rm tot} = \sum_{n=0}^{n=N} I(n,t)$, of the laser, for different values of nonlinearity: (a) $\varepsilon = 0.0001$, (b) $\varepsilon = 0.00012$, (c) $\varepsilon = 0.000125$, (c) $\varepsilon = 0.00013$ and a constant value of the number of longitudinal modes, $N = 100$; the case of the cross-saturation constant for all modes.



(c) (d)

Fig. 4. (*Continued*)

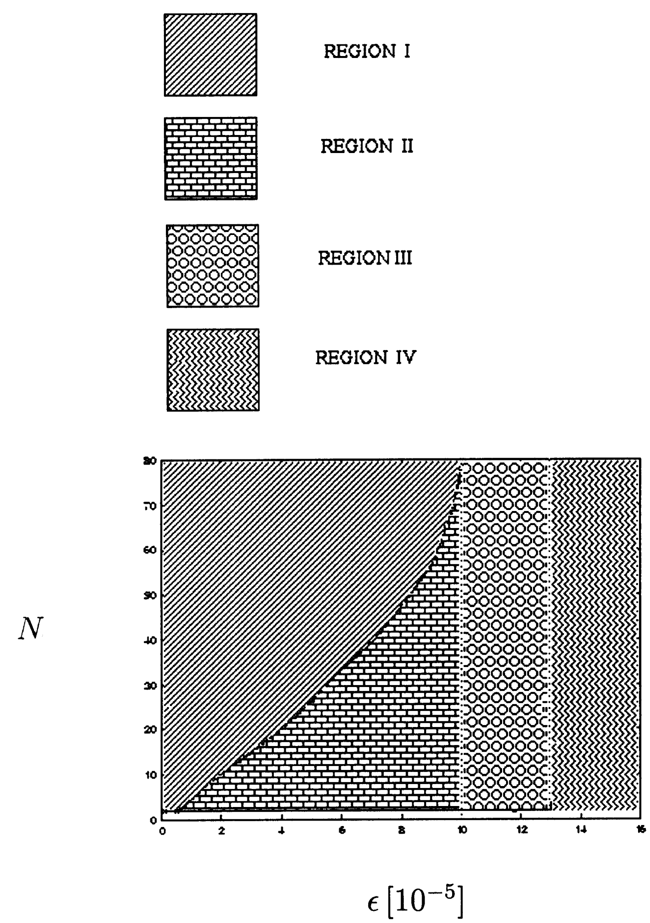


Fig. 5. Different regions of the laser behavior, depending on the number of longitudinal modes, N, and the nonlinear coefficient, ε.

solution, shown in Fig. 4(d), which is stable against small perturbations, arises.

This mechanism of stabilization, achieved by forcing the laser to operate in the one-mode regime, is similar to other approaches presented in the literature, where the stabilization is obtained by inserting into the laser cavity an additional element like an etalon [Baer, 1986] or a birefringent crystal [Nagai *et al.*, 1992; Fan, 1991]. However, the method proposed here seems to be a better solution, since no additional element is involved.

4. Inclusion of the Effect of Spatial Hole-Burning

In this section the analysis presented in the previous section is extended to take into account the effect of spatial hole-burning. The present analysis is based on the Baer-type rate equations, Eq. (1), modified by Bracikowski, Roy and James [Roy *et al.*, 1993; Bracikowski & Roy, 1991], who have written down an explicit form for the cross-saturation coefficient, Eq. (2), but were not investigating the influence of the spatial hole-burning on the laser dynamics. As far as the authors know, the analysis presented in this section is the first study of this issue.

Same as in the previous section, two ways of the laser stabilization are analyzed: by increasing the number of longitudinal modes and by increasing the

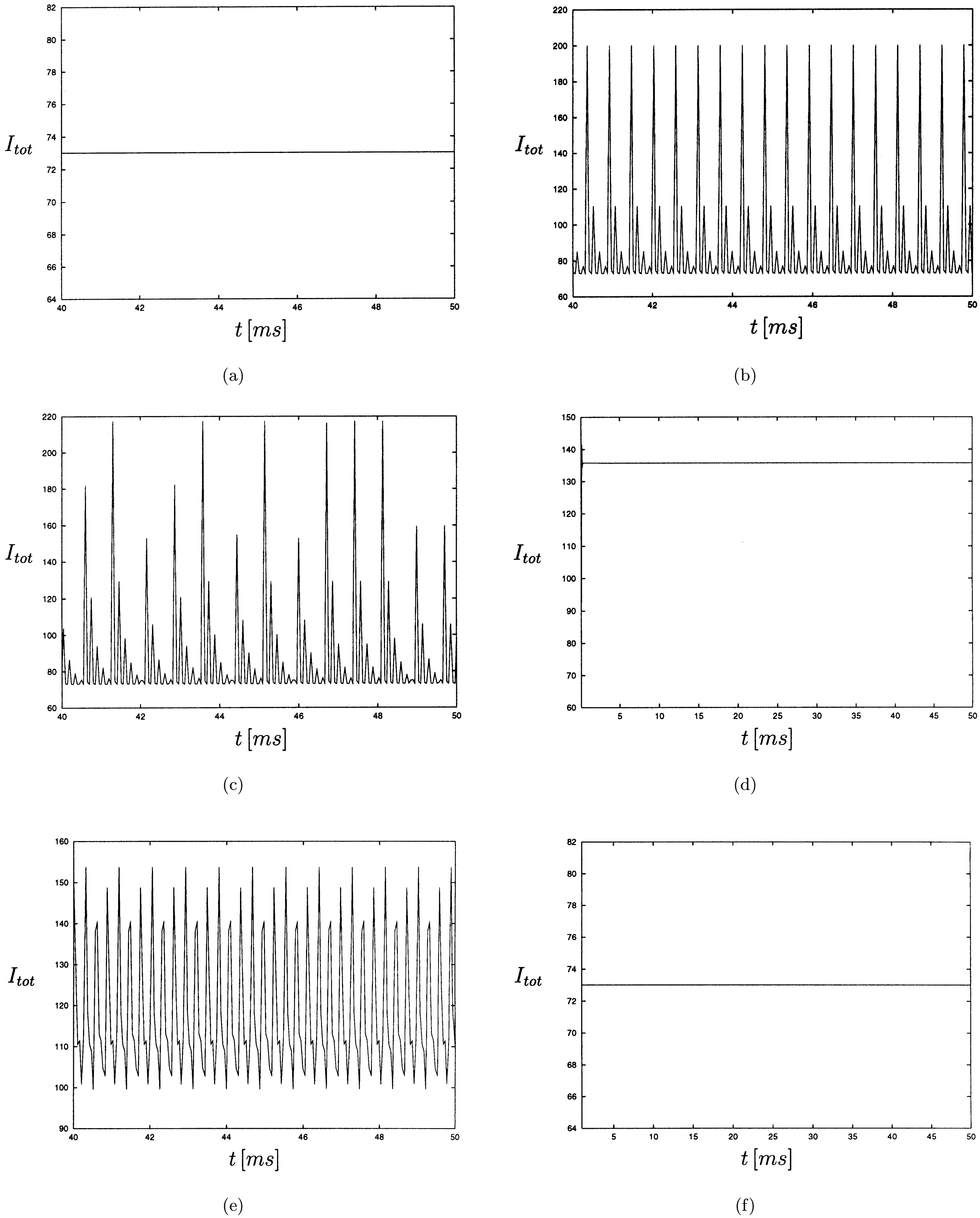


Fig. 6. The evolution of the total intensity, $I_{\text{tot}} = \sum_{n=0}^{n=N} I(n, t)$, of the laser for different values of the initial number of longitudinal modes: (a) $N = 25$, (b) $N = 100$, (c) $N = 200$ and a constant value of nonlinearity, $\varepsilon = 0.0001$; and also for different values of nonlinearity: (a) $\varepsilon = 1.010^{-6}$, (b) $\varepsilon = 1.010^{-5}$, (c) $\varepsilon = 1.010^{-4}$ and a constant number of longitudinal modes, $N = 20$; the case of the cross-saturation coefficient modulated by the effect of spatial hole-burning.

strength of nonlinearity. Parameters describing the laser cavity are chosen to be as follows: $L = 1.2$[m], $l = 6$[mm], $z_0 = 12$[mm].

Our results show that when the spatial hole-burning effect is taken into account (i) the competition between the modes is much stronger than in the case of the cross-saturation coefficient constant for all modes (ii). Therefore, the minimal number of modes which are necessary to obtain the stabilization of the laser output is in the case (i) larger than in the case (ii). For example, for the strength of nonlinearity $\varepsilon = 0.0001$ only 80 modes are necessary in the case (ii), as shown in Fig. 2(c), while in the case (i) even 200 initially oscillating modes are not sufficient for the stabilization of the laser output, as shown in Fig. 6(c).

Therefore, an alternative approach of the stabilization of the laser output based on an increase of the strength of nonlinearity, which has been proposed in the previous section, can be examined. Indeed, from the results of the numerical simulations, displayed in Fig. 6(f) it follows that the stabilization of the laser output can be obtained when the nonlinearity is larger than a critical value. This occurs because of an extensive cancellation of modes, due to which the laser operates in a single longitudinal mode.

5. Conclusions

In this paper the properties of the dynamics of multimode intracavity-doubled solid-state lasers have been studied. The influence of spatial hole-burning on the cross-saturation coefficient was taken into account. The system was described by the rate equations of Baer-type, which were solved numerically with the aid of the Runge–Kutta method.

Firstly, the case of the cross-saturation coefficient constant for all modes was considered. The minimal number of modes necessary for the stabilization of the laser output was defined and its dependence on the strength of nonlinearity was studied. It was shown that for small nonlinearity this dependence is linear and agrees with the theoretical predictions. However, for larger values of nonlinearity the minimal number of modes was found to be larger than the number obtained in theoretical considerations. As a result of this discrepancy the effect of the competition between the modes and cancellation of some during the evolution was given. It was also observed that with increasing nonlinearity the competition between modes increases, so that for some values of nonlinearity the initially large number of oscillating modes is reduced to only a few. For much higher values of nonlinearity this competition is so strong that only a few modes survive. Therefore, a large number of simultaneously oscillating modes cannot be realized and the stabilization cannot be obtained in this way. However, we propose another method based on the increase of the strength of nonlinearity. This stabilization occurs for such a high value of nonlinearity, for which all modes, besides a single one, are canceled during the evolution. In this case a steady-state solution, stable against small perturbations, arises.

A similar analysis was accomplished for the cross-saturation coefficient modulated by the effect of spatial hole-burning. We have shown that in this case the stabilization of the laser output by increase of the number of longitudinal modes is hardly possible. Therefore, an alternative approach of the stabilization based on an increase of the strength of nonlinearity have been examined and shown to be an appropriate solution.

The method of stabilization of a laser output, as proposed in our paper, is similar to other approaches presented in the literature where the stable output of the laser is achieved by forcing the laser to operate in a single-mode regime, for example, by inserting into the cavity an etalon or a birefringent crystal. However, the method proposed here offers a better solution, since no insertion of an additional element into the laser cavity is needed.

Acknowledgments

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