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Influence of nonlinear coupling of pulses on spatio-temporal compression

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Abstract. Properties of a novel configuration of an optical (spatio-temporal) pulse compressor, that is based on a Kerr-type planar waveguide into which two pulses are simultaneously launched, are studied. It is assumed that the pulse which is the subject of the compression propagates in the anomalous dispersion regime, while the auxiliary pulse is in normal dispersion. The best parameters of the proposed compressor are obtained when duration of the auxiliary pulse is so large that this dispersion can be neglected, while energy of the second pulse is above the threshold of first-order soliton generation. It is observed that in such a configuration the compression occurs simultaneously with the generation of a soliton-like solution. It is argued that the proposed configuration with two simultaneously propagating pulses has advantages over the configuration with a single pulse, namely the maximal compression factor and the optimal length of the compressor is, respectively, more than 3 times larger and, at least, 10 times greater than the corresponding values of the compressor with a single pulse. It is also demonstrated that such a compressor can be considered as a universal device, since its operation depends only slightly on the initial parameters of the pulse subject to the compression.

1. Introduction

In the last decade intensive research in the field of pulse compression has been stimulated by the increasing demand for short optical pulses, which are useful for a variety of applications, e.g. measurement of extremely fast physical processes [1], optoelectronics sampling [2, 3], generation of ultrafast X-ray radiation [4] and ultrahigh-order harmonics [5]. They can also be employed to design spectroscopic [6] and imaging [7, 8] devices for investigations of atomic or molecular systems, including diagnostic and therapeutic tools for microbiology and medicine [9]. Moreover, generation of short optical pulses is of great interest for laser satellite communication [10], ultrahigh bit-rate and long distance optical communication [11], ultrafast optical storage [12] and data processing [13, 14]. Furthermore, in the future, short pulses may play a crucial role in a development of all-optical computers [15].

Depending on the application, the desired output of the compressor parameters of the pulse and the form of the input pulse, various pulse compression techniques

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have been developed. In most cases these techniques make use of Kerr-type nonlinear effects, which can occur in optical fibres, planar waveguides or bulk media. In the case of an optical fibre three different compression techniques can, in general, be distinguished: namely (i) a higher-order soliton compression [16, 17]; (ii) an adiabatic pulse compression, which can be realized in dispersion-decreasing fibres [18], step-like dispersion profiled fibres [19], as well as in uniform fibres with distributed amplification [20]; and finally (iii) a compression based on self-phase modulation in a single-mode fibre followed by a dispersive delay consisting of prism pairs [21], a diffracting grating [22], or a coupled waveguide structure [23]. Moreover, in some applications, e.g. for nanostructuring [24, 25], it is essential that pulses are confined not only in time, but also in space. An operation of a compressor perfectly meeting this requirement can be based on, e.g. a Kerr-type bulk medium [26] or a planar waveguide [27, 28], in which a dispersive pulse propagates in the anomalous or normal dispersion regime.

Currently, elaborating new methods for pulse compression or upgrading existing ones is of current interest; the aim is to construct such a compressor, which on the output would give sufficiently short, transform-limited (i.e. with the time-bandwidth product equal to the time-bandwidth of a chirp-free sech pulse [17]) pulses, would not require difficult and time consuming trial-and-error optimization of the length [22], and whose operation would not be restricted to a peculiar class of pulses, e.g. with high intensities. One way to achieve this goal is to launch into the compressor an auxiliary pulse that modifies the refractive index of the medium in which both pulses propagate and, as a result, the evolution of effects on the pulse which is the subject of the compression is altered. In this way large compression, even for small energy of the compressed pulse, can be obtained. An advantage of the two-pulses configuration, which has also a potential application for all-optical control of pulses, already has been verified for various compression techniques. Many of these techniques operate on the basis of optical fibres, for example, a compression similar to higher-order soliton compression employing an interplay between cross-phase modulation and group velocity dispersion (GVD) [29, 30]; a compression in a single mode fibre followed by a grating-pair dispersive delay line [31], a compression based on induced phase modulation [31], four-wave mixing [32] and sum frequency generation [33]; and finally a compression using a semiconductor optical amplifier operating in the pump-probe configuration [34]. The presence of an auxiliary pulse can be useful, as well, for a compression in the configuration of a bulk medium. Such a compression can be realized, e.g. by intensity-dependent spatial deflection of a pulse and its subsequent spatial filtering [35, 36]; or by using a grating-pair dispersive delay line together with the two-beams interference technique, due to which two-dimensional spatial bright solitons are formed [37, 38].

In this paper we will show that the two-pulse configuration is beneficial also for a compressor based on a Kerr-type planar waveguide. We will study properties of such a compressor, and try to optimize it. We will assume that the wavelengths of the pulses are chosen in such a way that the pulse, which is the subject of the compression, propagates in the anomalous dispersion regime, while the auxiliary pulse propagates in the normal regime. As the model equation we will take two coupled $(2+1)$-dimensional nonlinear Schrödinger equations (NSEs):

$$\mathrm{i}\frac{\partial}{\partial\zeta}\psi_1 + \tfrac{1}{2}\sigma_1\frac{\partial^2}{\partial\tau^2}\psi_1 + \frac{1}{2}\frac{\partial^2}{\partial\xi^2}\psi_1 + (|\psi_1|^2 + 2|\psi_2|^2)\psi_1 = 0, \qquad (1\,a)$$

$$\mathrm{i}\frac{\partial}{\partial\zeta}\psi_2 + \tfrac{1}{2}\sigma_2\frac{\partial^2}{\partial\tau^2}\psi_2 + \mu\frac{1}{2}\frac{\partial^2}{\partial\xi^2}\psi_2 + r(|\psi_2|^2 + 2|\psi_1|^2)\psi_2 = 0, \qquad (1\,b)$$

where ζ, τ, ξ denote, respectively, the longitudinal coordinate, the local time, and the transverse spatial coordinate. The subscript $j = 1$ ($j = 2$) corresponds to the pulse referred to as the anomalous (normal) pulse, i.e. propagating in the anomalous (normal) dispersion regime; it means that $\sigma_1 > 0$ and $\sigma_2 < 0$ (the limiting case of vanishing dispersion, $\sigma_2 = 0$, will also be considered in this paper). Here σ_j, μ, r stand, respectively, for the dispersion-to-diffraction ratio of the jth pulse, $j = 1, 2$, the ratio of the Fresnel diffraction length of the anomalous pulse to the Fresnel diffraction length of the normal pulse; and finally the ratio of the carrier frequency of the anomalous pulse to the carrier frequency of the normal pulse (further details regarding the notation used in equations (1 *a*) and (1 *b*) can be found in [39]). The second and third terms in equations (1 *a*) and (1 *b*) are associated, respectively, with diffraction, which causes spreading of the pulse in space; and first-order group velocity dispersion, which leads to temporal broadening of the pulses. The term before last in each equation describes self-phase modulation; while the last one represents cross-phase modulation, a nonlinear effect through which the phase of one pulse is affected by the other pulse and, as a result, redistribution of energy within each pulse can occur. Terms describing four-wave mixing, such as fast oscillating, are neglected, for in the configuration of two pulses with different wavelength the phase-matching condition is not satisfied [29]; therefore, energy transfer between pulses is not taken into account. Terms proportional to the difference in group velocities of the pulses are also omitted. Such a treatment, which could be considered as inconsistent with the former assumption, should be treated as the first step of the analysis. In addition, we believe that an inclusion of a small difference in group velocities of the pulses would not cause qualitative changes in the results presented in this paper (see also the discussion in Appendix B of [39]). Note that equations (1 *a*) and (1 *b*) are valid only for pulses in the picosecond time domain, for shorter pulses higher-order time-derivative terms, such as higher-order GVD or self-steepening, should be included.

The analysis of the set of equations (1 *a*) and (1 *b*) will be carried out with the aid of the variational methods and numerical simulations (using the Split-Step spectral method). The variational method (regarding the accuracy of variational solutions see, e.g. [40] or the Appendix of this paper) will be used as an auxiliary tool to outline the main characteristics of the evolution of the pulses, while the essential part of the analysis will rely on exact numerical solutions. It will be assumed that the trial function in the variational method is given by the Gaussian function:

$$\psi_j(\zeta,\tau,\xi) = A_j(\zeta)\exp\left[-\frac{\tau^2(1+\mathrm{i}C_{\tau j}(\zeta))}{2w_{\tau j}(\zeta)}\right]\exp\left[-\frac{\xi^2(1+\mathrm{i}C_{\xi j}(\zeta))}{2w_{\xi j}(\zeta)}\right]\exp(\mathrm{i}\phi_j), \qquad (2)$$

which depends on 12 parameters, the temporal (spatial) width, $w_{\tau j}$ ($w_{\xi j}$), the temporal (spatial) chirp, $C_{\tau j}$ ($C_{\xi j}$), the amplitude, κ_j and the phase, ϕ_j, of the jth pulse, $j = 1, 2$. As the initial condition we will take function (2) with

$w_{\tau j}(0) = w_{\xi j}(0) = 1$, $C_{\tau j}(0) = C_{\xi j}(0) = 0$, $\phi_j(0) = 0$, where $j = 1, 2$. We will assume that $\rho = \mu = 1$. Magnitudes of the dispersion-to-diffraction ratio, σ_j, and the parameter $\kappa_j := A_j(0)$, referred here to as the strength of nonlinearity of the jth pulse, $j = 1, 2$, will be varied in the analysis (note that the choice of name *strength of nonlinearity* is motivated by the fact that κ is proportional to energy of the jth pulse and to the nonlinear part of the refractive index of the Kerr medium in which it propagates, see Appendix A of [39]).

Details concerning derivation of ODE in the variational method for all 12 parameters of ansatz (2) can be found in [39]; here we will rewrite only those equations which refer to the temporal and the spatial widths of the pulses:

$$\frac{d^2\omega_{\tau 1}}{d\zeta^2} = \frac{\sigma_1^2}{w_{\tau 1}^3} - \sigma_1 \frac{1}{2}\frac{\mathcal{I}_1}{w_{\tau 1}^2 w_{\xi 1}} - \frac{4\mathcal{I}_2 w_{\tau 1}\sigma_1}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{(3/2)}(w_{\xi 1}^2 + w_{\xi 2}^2)^{1/2}}, \tag{3 a}$$

$$\frac{d^2\omega_{\xi 1}}{d\zeta^2} = \frac{1}{w_{\xi 1}^3} - \frac{1}{2}\frac{\mathcal{I}_1}{w_{\tau 1} w_{\xi 1}^2} - \frac{4\mathcal{I}_2 w_{\xi 1}}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{(1/2)}(w_{\xi 1}^2 + w_{\xi 2}^2)^{3/2}}, \tag{3 b}$$

$$\frac{d^2\omega_{\tau 2}}{d\zeta^2} = \frac{\sigma_2^2}{w_{\tau 2}^3} - \sigma_2 \frac{1}{2}\frac{\mathcal{I}_2 r}{w_{\tau 2}^2 w_{\xi 2}} - \frac{4\mathcal{I}_1 w_{\tau 2}\sigma_2 r}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{(3/2)}(w_{\xi 1}^2 + w_{\xi 2}^2)^{1/2}}, \tag{3 c}$$

$$\frac{d^2\omega_{\xi 2}}{d\zeta^2} = \frac{\mu^2}{w_{\xi 2}^3} - \mu \frac{1}{2}\frac{\mathcal{I}_2 r}{w_{\tau 2} w_{\xi 2}^2} - \frac{4\mathcal{I}_1 w_{\xi 2}\mu r}{(w_{\tau 1}^2 + w_{\tau 2}^2)^{(1/2)}(w_{\xi 1}^2 + w_{\xi 2}^2)^{3/2}}, \tag{3 d}$$

where $\mathcal{I}_j := w_{\tau j} w_{\xi j} |A_j(\zeta)|^2 = \kappa_j$ represents a constant of the motion. Equations (3 *a*)–(3 *d*) can be solved analytically only in the special case when $I_1 = I_2$ and $\sigma_1 = \sigma_2 = \mu = 1$. In a more general case they should be solved numerically; we will use for this purpose the Runge–Kutta method.

2. Single-pulse configuration

In this section we will study a compression of a single pulse propagating in a Kerr-type planar waveguide. To model this configuration we will use equation (1 *a*) but neglecting the last term. The properties of such an equation were studied, e.g. in [39], where it was demonstrated that the spatio-temporal dynamics of the pulse depends, to a high degree, on the sign of the parameter σ (in order to distinguish the one-pulse configuration from the two-pulse one we will substitute σ_1 by σ and κ_1 by κ).

In the case of anomalous dispersion some solutions can develop into a singularity of the electric field in the self-focus point. This phenomenon, known as catastrophic self-focusing, occurs simultaneously in space and time when values of the parameters describing the system are above the threshold of catastrophic self-focusing, which is usually computed with the aid of the method of moments [41–43], the variational method [44, 45] and also numerical simulations [39, 46, 47]. The threshold obtained in the variational method is the same as in the method of moments,

$$\bar{\kappa}_{\mathrm{catV}} = 1 + \sigma, \tag{4}$$

however, it is lower than the one given in the numerical simulations,

$$\bar{\kappa}_{\text{catN}} = 0.885 + \sigma. \tag{5}$$

Note that the occurrence of catastrophic self-focusing is not only non-physical, it also prevents examination of the pulse behaviour behind the self-focus, for it emerges just as an artefact of approximations made when deriving the NSE. However, the NSE can still serve as the model equation for self-focusing when values of the parameters describing the system are below the threshold of catastrophic self-focusing [48]. It is also expected that below this threshold filamentation of the pulse does not develop in a real physical system, since neither variational method nor numerical simulations predict its existence.

Another situation occurs in the case of normal dispersion, namely, the terms describing dispersion and diffraction have different signs and two different effects, spatial self-focusing and temporal self-defocusing, simultaneously influence the propagation of the pulse. This means that in the solution of the NSE neither singularity [49] nor localized steady-states occur [50] and three different types of a pulse evolution, which we will classify here with respect to increasing nonlinearity (for a fixed value of the dispersion-to-diffraction ratio), can be distinguished. They can be characterized as follows: (i) both widths of the pulse spread monotonically without initial focusing; (ii) the spatial width of the pulse decreases initially, reaching a minimum on a certain distance of propagation, and then it spreads, while the temporal widths spread monotonically without initial focusing; and finally (iii) both widths of the pulse focus initially, the distance of propagation on which the spatial width of the pulse reaches its minimum and the value of this minimum is, in general, larger than in the case of the spatial transverse variable. In particular, for $\sigma = -1$ the numerical simulations demonstrate that (ii) occurs when $\kappa \gtrsim 0.9$ and (iii) when $\kappa \gtrsim 28$; while in the variational method (ii) occurs when $\kappa \gtrsim 2$ and no behaviour described by (iii) is observed. A common feature which concerns all above-mentioned categories of the pulse evolution is the fact that for all ζ, the temporal width of the pulse is larger than the spatial one. Moreover, from the results of the numerical simulations it follows that breaking of spatio-temporal symmetry and the uniform structure of the pulse takes place. This process can finally lead to an occurrence of several humps in the field distribution [50], splitting of the pulses into two sub-pulses [51], or splitting into several sub-pulses [52]. Worthy of notice is the fact that when splitting of the pulse occurs, it occurs for a propagation distance larger than the distance on which initial focusing of the pulse takes place. Note also that every time when the existence of spatio-temporal splitting of pulses has been reported, numerical simulations were used.

Now, let us turn to the problem of a pulse compression and assume that values of the parameters describing the system are below the threshold of catastrophic self-focusing. Using the variational method and numerical simulations it has been shown in [28, 53] that spatio-temporal compression of the pulse, which propagates either in anomalous or normal dispersion, can occur when the strength of non-linearity is sufficiently large; however, only the temporal aspect of the compression has been considered. The compressor has been characterized by two parameters; one of them, the maximal compression factor, is defined as follows:

$$c_{\max} := \frac{w_\tau(0)}{w_{\tau\min}(\zeta_{\min})}, \quad \delta \ll 1,$$

where $\omega_\tau(0)$ is the initial temporal width of the pulse, and $w_{\tau\min}$ is the minimal temporal width, which can be achieved during the propagation of the pulse down the waveguide. From the above definition it follows that the minimal temporal width of the pulse takes place at the propagation distance $\zeta_{\min}$, which actually constitutes the second parameter of the compressor, referred to here as the optimal length of the compressor. Let us introduce also a third parameter which can describe the compressor: namely the compression length, defined as the interval of the propagation distance,

$$\Delta\zeta := \{\zeta : w_\tau(\zeta) - \delta w_{\tau\min} < w_{\tau\min}\},$$

within which the evolution of the pulse does not change significantly, i.e. its temporal width is approximately constant and equal to the minimal width.

From the results of the numerical simulations presented in [28, 53] it follows that for both types of dispersion, anomalous and normal, the maximal compression factor, $c_{\max}$, increases with increasing strength of nonlinearity, κ, and with decreasing dispersion-to-diffraction ratio, $|\sigma|$. However, in the case of anomalous dispersion much smaller values of the strength of nonlinearity are necessary to obtain the same maximal compression factor as in the case of normal dispersion; for instance, the temporal compression of a pulse with anomalous dispersion, $\sigma = 1$, occurs already for $\kappa \approx 0.5$, while in the case of normal dispersion, $\sigma = -1$, the required strength of nonlinearity is much larger, $\kappa \approx 28$. Another advantage of the configuration with anomalous dispersion, from the point of view of a compression, is the fact that spatio-temporal splitting of the pulse, which can occur in the case of normal dispersion, does not develop. In view of the above, let us now assume that the pulse which is the subject of the compression propagates in the anomalous dispersion regime. From table 1 it is evident that in order to optimize such a compressor, i.e. to obtain $c_{\max}$ and $\Delta\zeta$ as large as possible, values of the parameters describing the system should be just below the threshold of catastrophic self-focusing. Indeed, when the above-mentioned condition is satisfied, the compression length is quite large, $\Delta\zeta \approx 1$. Owing to this fact the adjustment of the length of the compressor should not be problematic. However, the maximal

Table 1. Dependence of the maximal compression factor, $c_{\max}$, and the dispersion length, $\Delta\zeta$, on the strength of nonlinearity of the anomalous pulse, $\kappa(\kappa_1)$ for the compressor operating in the single-pulse (two pulses) configuration. It is assumed that in both cases the strength of nonlinearity is just below the threshold of catastrophic self-focusing, $\bar{\kappa}_{\mathrm{Ncat}} = 1.885$ (in the single-pulse configuration with $\sigma = 1$) and $\bar{\bar{\kappa}}_{\mathrm{Ncat}} = 1.985$ (in the two-pulses configuration with $\sigma_1 = 1$, $\kappa_2 = 1$, $\sigma_2 = -1$).

Single-pulse case										
	$\bar{\kappa}_{\mathrm{catN}}$									
κ	1.885	1.88	1.87	1.86	1.85	1.84	1.83	1.82	1.81	1.80
$c_{\max}$	1.56	1.55	1.52	1.50	1.49	1.47	1.46	1.45	1.44	1.43
$\Delta\zeta$	1.25	1.06	0.89	0.81	0.76	0.70	0.65	0.61	0.57	0.53
Two-pulses case										
	$\bar{\bar{\kappa}}_{\mathrm{catN}}$									
κ_1	1.985	1.98	1.97	1.96	1.95	1.94	1.93	1.92	1.91	1.90
$c_{\max}$	4.64	4.30	3.98	3.77	3.61	3.48	3.36	3.26	3.15	3.03
$\Delta\zeta$	0.25	0.27	0.30	0.35	0.36	0.41	0.38	0.41	0.35	0.38

compression factor in this configuration is rather small, $c_{\max} \approx 1.5$. Besides, meeting the optimization condition would be, as a matter of fact, quite difficult in practice. This can be explained as follows: for a given medium which constitutes the basic element of the compressor (with a fixed value of GVD and nonlinear index of refraction), and for a fixed wavelength of the pulse, the only free parameter of the system is energy of the pulse. It means that such a compressor cannot be considered as an universal device, for its operation is limited to pulses with energy belonging to a narrow interval of values.

In the next sections of this paper we will try to give an answer to the question whether or not it is possible to improve the operation of the above-described compressor, i.e. based on a Kerr-type planar waveguide in which a single pulse with anomalous dispersion propagates. For this purpose we will examine such a configuration with an additional pulse launched into the waveguide.

3. Two-pulses configuration

In this section we will study properties of the compressor with two simultaneously propagating pulses, the anomalous one and the normal one. We will assume that the first pulse is the subject of the compression, while the second one will be treated as an auxiliary pulse. As the model equation we will use two coupled $(2+1)$-dimensional NSEs (the set of equations (1 *a*) and (1 *b*)), which will be solved with the aid of the variational method and numerical simulations. We chose values of the parameters describing the anomalous pulse in such a way that: (i) it focuses catastrophically or (ii) it focuses without an occurrence of a singularity. The single normal pulse will undergo the following evolution: initial temporal or spatio-temporal focusing, taking place on the propagation distance shorter than the distance corresponding to catastrophic self-focusing of the single anomalous pulse, and on larger propagation distances defocusing, with the possible occurrence of spatio-temporal splitting. Note that case (i) occurs for $\kappa_1 > \bar{\kappa}_{\mathrm{catV}}$ $(\kappa_1 > \bar{\kappa}_{\mathrm{catN}})$, where $\bar{\kappa}_{\mathrm{catV}}$ $(\bar{\kappa}_{\mathrm{catV}})$ is the threshold of catastrophic self-focusing given in the variational method (numerical simulations) by condition (4) (condition (5)).

First, let us consider the evolution of the pulses in case (i) with an additional assumption that catastrophic self-focusing of two simultaneously propagating pulses does not develop, i.e. the strength of nonlinearity of the normal pulse (for given values of $\kappa_1 > \bar{\kappa}_{\mathrm{catX}}$ and σ_1) is sufficiently large,

$$\kappa_2 > \kappa_{\mathrm{Xlow}}(\kappa_1, \sigma_1), \tag{6 a}$$

and the dispersion-to-diffraction ratio satisfies the following relation

$$|\sigma_{\mathrm{Xlow}}(\kappa_2)| < |\sigma_2| < |\sigma_{X\mathrm{upp}}(\kappa_2)|, \tag{6 b}$$

where $X \equiv V$ $(X \equiv N)$ represents results obtained with the aid of the variational (numerical) method. We observed that for the initial stage of propagation the normal pulse can cause an additional focusing of the anomalous pulse; however, for larger propagation distances, it experiences substantial spatio-temporal defocusing, which causes, through cross-phase modulation, a deceleration and ever a total arresting of catastrophic self-focusing of the anomalous pulse.

Let us consider now case (ii) with the following parameters: $\kappa_1 = 1.88$, $\sigma_1 = 1$, $\kappa_1 = 1$, $\sigma_2 = -1$. A comparison of the evolution of the normal pulse, which propagates in the presence of the anomalous pulse, with the evolution of the single

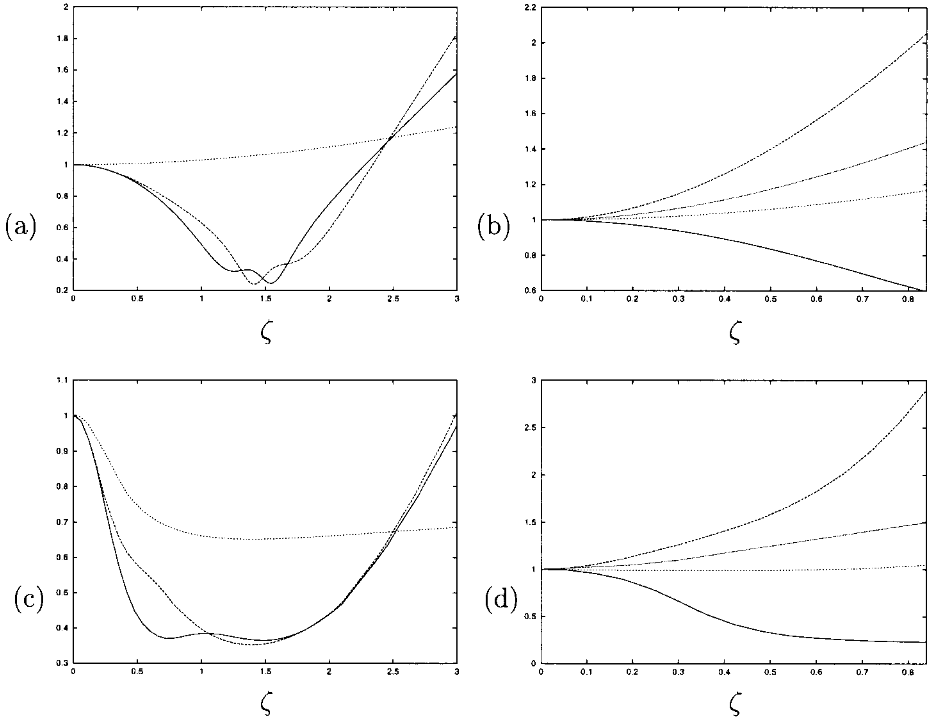

Figure 1. The results obtained with the aid of the variational method, (*a*) and (*b*), and numerical simulations, (*c*) and (*d*), showing the dependence of the spatial (full curve), w_ξ, and the temporal width (long-dashed curve), w_τ, of the anomalous and the normal pulse propagating simultaneously ($\kappa_1 = 1.88$, $\sigma_1 = 1$, $\kappa_2 = 1$, $\sigma_2 = -1$). For comparison, the dependence of the spatial (dotted curve), w_ξ, and the temporal width (short-dashed curve), w_τ, of the anomalous pulse ($\kappa = 1.88$, $\sigma = 1$) and the normal pulse ($\kappa = 1$, $\sigma = -1$) propagating as single pulses, is also included. The anomalous pulse is represented in (*a*) and (*c*), while the normal one in (*b*) and (*d*).

normal pulse leads to the conclusion that the anomalous pulse strengthens initial focusing of the normal pulse in the spatial transverse domain, but in the temporal domain it gives rise to additional defocusing. This observation concerns the results of the variational method and the numerical simulations, shown, respectively, in figures 1 (*b*) and (*d*). Note that for larger propagation distances, $\zeta > 0.85$, a qualitative difference between results of both methods arises. It is associated with the fact that in the numerical simulations spatio-temporal splitting of the pulse takes place, which, however, is not predicted in the variational method (note that this discrepancy is not evident from figures 1 (*b*), and (*d*), since they concern propagation distances shorter than the distance on which splitting takes place). Regarding the anomalous pulse, from the results of the variational method and the numerical simulations, displayed, respectively, in figures 1 (*a*) and (*c*), it is evident that the spatial widths of the pulse decreases initially, reaching the minimum on a certain distance of propagation, $\zeta_{\min}$, and then it starts to increase. The evolution of the temporal width is similar, but the distance of propagation on which the width reaches its minimum and the magnitude of this minimum are, in general, larger than the corresponding values of the spatial width. Moreover, the compar-

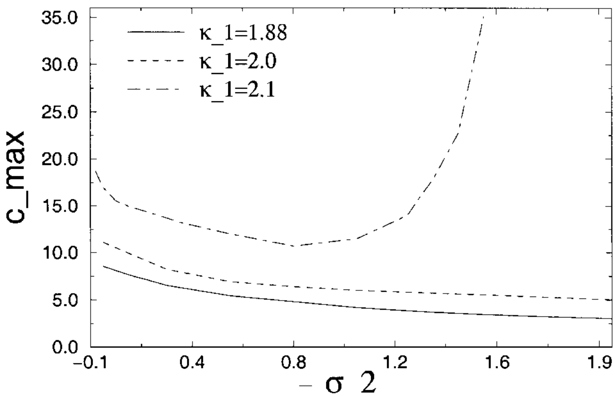

Figure 2. The maximal compression factor of the anomalous pulse, $c_{\max}$, versus the dispersion-to-diffraction ratio of the normal pulse, σ_2, obtained with the aid of the variational method. Three different values of the strength of nonlinearity of the anomalous pulse are considered: $\kappa_1 = 1.88 < \bar{\kappa}_{\mathrm{Vcat}}$ (full curve), $\kappa_1 = 2.0 = \bar{\kappa}_{\mathrm{Vcat}}$ (dashed curve), and $\kappa_1 = 2.1 = \bar{\kappa}_{\mathrm{Vcat}}$ (chain curve), where $\bar{\kappa}_{\mathrm{Vcat}}$ is the threshold of catastrophic self-focusing obtained in the variational method for the single-pulse configuration.

ison of the two-pulse configuration and the single-pulse one leads to the conclusion that in the first case the minimal widths (spatial and temporal) of the anomalous pulse and the lengths of the distances of propagation, for which they are nearly constant and equal to the minimal widths, are considerably smaller than in the second case. Indeed, from the variational method it follows that temporal focusing of the single anomalous pulse does not develop at all, while in the configuration of two pulses it is rather strong and gives rise to the maximal compression factor of about 5. The numerical simulations demonstrate a slightly smaller difference, namely in the configuration of two pulses the maximal compression factor is approximately 2 times larger than in the case of the single anomalous pulse, $c_{\max} \approx 3$.

Let us concentrate now on the anomalous pulse and its temporal compression and try to find values of the parameters describing the system for which the maximal compression factor, $c_{\max}$, and the compression length, $\Delta\zeta$, are as large as possible. For this purpose let us consider the dependence $c_{\max}$ versus the dispersion-to-diffraction ratio of the normal pulse, σ_2. Such a dependence, obtained with the aid of the variational method, is displayed in figure 2, where three different values of the strength of nonlinearity of the anomalous pulse are considered: $\kappa_1 = 1.9 < \bar{\kappa}_{\mathrm{Vcat}}$, $\kappa_1 = 2.0 = \bar{\kappa}_{\mathrm{Vcat}}$ and $\kappa_1 = 2.1 > \bar{\kappa}_{\mathrm{Vcat}}$. It is evident that for the first two cases represented, respectively, by the full and dashed curves, the parameter $c_{\max}$ increases with decreasing σ_2, reaches a maximum at $\sigma_2 \to 0$, and then decreases. Another situation occurs for the strength of nonlinearity exceeding the critical value: namely in the dependence $c_{\max}$ versus σ_2, two maxima, one at $\sigma_2 \to \sigma_{\mathrm{Vlow}} \approx 0$ and the second one at $\sigma_2 \to \sigma_{\mathrm{Vupp}} \approx -1.5$, can be distinguished (see chain curve). Note that the parameters σ_{Vlow} and σ_{Vupp} occur also in condition (6 *b*) deliminating the threshold of catastrophic self-focusing. From figure 2 it follows also that the maximal compression factor

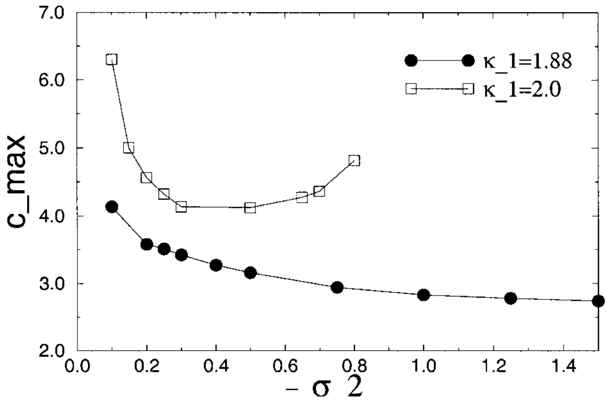

Figure 3. The maximal compression factor of the anomalous pulse, $c_{\max}$, versus the dispersion-to-diffraction ratio of the normal pulse, σ_2, obtained with the aid of the numerical simulations. Two different values of the strength of nonlinearity of the anomalous pulse are considered: $\kappa_1 = 1.88 < \bar{\kappa}_{\mathrm{Ncat}}$ (full circles), and $\kappa_1 = 2.0 > \bar{\kappa}_{\mathrm{Ncat}}$ (empty boxes), where $\bar{\kappa}_{\mathrm{Ncat}} = 1.885$ is the threshold of catastrophic self-focusing obtained numerically for the two-pulses configuration.

increases with increasing strength of nonlinearity of the anomalous pulse. We can, therefore, write that

$$\tilde{c}_{\max} > c_{\max}, \tag{7}$$

where $\tilde{c}_{\max} \equiv c(\kappa_1 > \bar{\kappa}_{\mathrm{Xcat}})$, $c_{\max} \equiv c(\kappa_1 < \bar{\kappa}_{\mathrm{Xcat}})$, $X \equiv V$.

Similar conclusions can be drawn from figure 3, where results of the numerical simulations are displayed. Indeed, in case (i), i.e. for large nonlinearity, $\kappa_1 = 2.0 > \bar{\kappa}_{\mathrm{Ncat}} = 1.885$, two maxima occurring at $\sigma_2 \to \sigma_{\mathrm{Nlow}} \approx 0$ and $\sigma_2 \to \sigma_{\mathrm{Nupp}} \approx 0.8$ (see the empty circle points) can be distinguished for the dependence $c_{\max}$ versus σ_2; while in case (ii), i.e. for small nonlinearity, $\kappa_1 = 1.88 < \bar{\kappa}_{\mathrm{Ncat}}$, only one maximum, occurring at $\sigma_2 \to 0$ (see full-circle points), can be singled out (note that the parameters σ_{Nlow} and σ_{Nupp} are the same as in relation (6 *b*) deliminating the threshold of catastrophic self-focusing). From figure 3 it follows also that in case (i) the maximal compression factor is larger than in case (ii), i.e. condition (7) (with $X \equiv N$) is satisfied. Therefore, it can be concluded that the optimization of the compressor can be achieved either when values of the parameters describing this system are just below the threshold of catastrophic self-focusing (as in case (i)) or when dispersion of the normal pulse vanishes, $\sigma_2 \to 0$ (as in case (ii)). Now, let us consider these two cases in more detail.

3.1. *Case (i)*: $\kappa_1 > \bar{\kappa}_{\mathrm{cat}}(\sigma_2 < 0)$

Let us examine table 1, where the dependence $c_{\max}$ and $\Delta\zeta$ versus κ_1, obtained with the aid of the numerical simulations, is displayed. It is assumed that the strength of nonlinearity of the anomalous pulse is just below the threshold of catastrophic self-focusing, $\kappa_1 \lesssim \bar{\bar{\kappa}}_{\mathrm{Ncat}} = 1.985$, occurring in the two-pulse configuration with the following parameters: $\sigma_1 = 1$, $\kappa_2 = 1$, $\sigma_2 = -1$. Table 1 also contains a similar dependence for the case of the single anomalous pulse, whose strength of nonlinearity just below the threshold of catastrophic self-focusing, $\kappa \lesssim \bar{\kappa}_{\mathrm{Ncat}} = 1.885$. One can see that the maximal compression factor and the compression length are, respectively, 2.5–3 times larger and 1.5–5 times smaller than the corresponding values of the single pulse. Therefore, the compressor with two simultaneously propagating pulses, the anomalous one and the normal one, can be considered as partially advantageous over the configuration which operate with a single anomalous pulse. Still, there is room for improvement: namely one would like to have larger values of the compression length. Whether or not it is possible will be studied in the subsequent part of this paper.

3.2. *Case (ii):* $\kappa_1 < \bar{\kappa}_{\mathrm{cat}}(\sigma_2 \to 0)$

Let us investigate now the configuration of the compressor with values of the parameters describing the system chosen in such a way that condition (ii) is satisfied, i.e. catastrophic self-focusing of the anomalous pulse does not occur when it propagates as a single pulse. In order to satisfy the optimization condition let us assume that the strength of nonlinearity of the normal pulse vanishes, $\sigma_2 \to 0$. Such a configuration, from the physical point of view, can be realized when the duration of the normal pulse is so large that the term describing dispersion-to-diffraction ratio, σ_2, as inversely proportional to the temporal width of the pulse [39], is small and can be neglected. In this situation, the model equations given by (1 *a*) and (1 *b*), i.e. two coupled $(2+1)$-dimensional NSEs, can be reduced to a set of coupled $(2+1)$- and $(1+1)$-dimensional NSEs. An analysis of such a system of equations, done with the aid of the variational method and numerical simulations, has already been presented in [54], where it is shown that soliton-like solutions, which propagate down the waveguide with constant shapes, amplitudes and widths, can exist in the configuration under discussion. This can happen when values of the parameters describing the system are properly chosen, i.e. the strength of nonlinearity of the anomalous pulse is below the threshold of catastrophic self-focusing (note that this is exactly case (ii) considered here), and the strength of nonlinearity of the normal pulse is above the threshold of first-order soliton generation. It is important to note that both widths of the pulses, which constitute such a solution, are a few times smaller than the initial widths; it means that simultaneously with the formation of the soliton-like solution a spatio-temporal compression takes place.

The compressor operating in the proposed configuration, with values of the parameters describing the system given by the example considered in figures 2 (*c*) and (*d*) of [54], i.e. with $\kappa_1 = 1$, $\sigma_1 = 1$, $\kappa_2 = 2$, $\sigma_2 = -1$, would have a large (at least, when compared with other examples examined in this paper) maximal compression factor, $c_{\max} \approx 5$, a large optimal length, $\zeta_{\min} \approx 15$, and large compression length, $\Delta\zeta \gg 1$. Note that the optimal length of the compressor under discussion is equivalent to the distance of propagation necessary for formation of soliton-like solutions; the compression length, which is associated with the distance of propagation on which soliton-like solutions maintain their shapes, should be infinitely large, assuming that soliton-like solutions are stable against small perturbations (we hope to answer the question whether or not they are stable in a forthcoming publication). The maximal compression factor and the optimal length of such a compressor depend, in general, on the initial parameters of the pulses which are subject to the compression. However, since the compression length is considerably large, then it is possible to choose the length of the compressor in a way which can guarantee that on the output optimally compressed pulses, irrespective of their initial shapes, amplitudes and widths, are obtained.

Such a compressor, therefore, can be considered as an universal device (in contradiction to other configurations and examples considered in this paper). Moreover, the maximal compression factor and the optimal length of the compressor is, respectively, more than 3 times larger and, at least, 10 times greater than the corresponding values for the compressor with a single anomalous pulse. There is only one fact which can be considered a disadvantage of the proposed configuration, namely the large value of the optimal length of the compressor (this could cause technical problems with preparing such a long planar waveguide).

4. Conclusions

We proposed in this paper a novel configuration of an optical (spatio-temporal) pulse compressor, whose operation is based on a Kerr-type planar waveguide into which two pulses, the one which is the subject of the compression with anomalous dispersion and the auxiliary one with normal dispersion, are simultaneously launched. The idea of using an additional pulse was inspired by the desire to include an additional degree of freedom in the system, which could make optimization of the compressor easier. The proposed configuration was modelled by two coupled $(2+1)$-dimensional nonlinear Schrödinger equations. The analysis was carried out with the aid of the variational method and numerical simulations (the variational method was used as an auxiliary tool to outline the main characteristics of the evolution of the pulses, while the essential part of the analysis was done based on numerical simulations). The temporal aspect of the compression was characterized by three parameters: the maximal compression factor, the optimal length of the compressor and the compression length. It was observed that the best parameters of the proposed compressor can be obtained when duration of the auxiliary pulse is so large that its dispersion can be neglected, while energy of the pulse is above the threshold of first-order soliton generation. In such a configuration a spatio-temporal compression occurs simultaneously with the generation of a soliton-like solution (formation of the soliton-like solution can be explained as follows: the pulse with negligible dispersion, since its dynamics can be modelled by integrable $(1+1)$-dimensional NSE, creates a waveguide in the medium in which it propagates and the other pulse gets trapped in this waveguide). It was also shown that the maximal compression factor and the optimal length of the proposed compressor is, respectively, more than 3 times larger and, at least, 10 times greater than the corresponding values for a compressor with a single pulse. Moreover, in such a configuration much smaller energies for the pulses subject to the compression can be used to obtain the same compression as in the single-pulse case. Besides, the compressor operating in the configuration under discussion can be considered, in contradiction to other configurations and examples investigated in this paper, as an universal device, because its operation depends only slightly on the initial parameters of the pulse subject to the compression (the compression length is relatively large, thus it is always possible to choose the length of the compressor in such a way that at the output optimally compressed pulses, irrespective of their initial shapes, amplitudes and widths, are obtained). To our knowledge, there is only one fact which can be considered as a disadvantage of such a configuration, namely the large optimal length of the compressor, thus long waveguides would be required and technical problems when preparing them could arise. However, we believe that the proposed configuration has the potential to be realized in practice. For this purpose AlGaAs structures, which have high nonlinear refractive index and large damage threshold, could be used. Note that we investigated in this paper only the temporal aspect of the compression; however, in the configuration under discussion one obtains pulses that are confined not only in time but also in space. Note that such pulses, compressed temporally and spatially, can be beneficial, e.g. for nanostructuring. Moreover, the configuration proposed in this paper could be used also as a basic element of an optical steering or switching device.

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Appendix

The variational method, known also as Whitham's method [55], was adopted to nonlinear partial differential equations (PDE) by Anderson [56]. It allows one to replace an infinite dimensional dynamical system by a finite one by choosing an appropriate multi-parametric ansatz for the solution. As a result, a set of ordinary differential equations (ODE) for the parameters of the trial function, coupled, in general, in a nonlinear manner, is obtained. In a special case analytical solutions of such a system of ODE can be given, however, in a general case, the use of numerical schemes is necessary. Still, even in the second case, it is profitable to apply the variational method, since it is much easier and consumes less time to solve ODE than to integrate numerically PDE. Moreover, the variational method is an universal device, suitable for equations in any dimension, with external forces and potentials. It is, therefore, widely used and accepted as a standard tool, especially in the nonlinear optics community.

The accuracy of the variational method depends, to a high degree, on the form of the trial function. When it is properly chosen it can even give exact solutions [57], very often it gives results which properly describe main characteristics of the evolution, i.e. are qualitatively similar to numerical solutions. However, in some cases, especially when during evolution substantial changes of the form of the solution occur, e.g. splitting of the pulse, radiation of energy, results following from the variational method may significantly diverge from the numerical solutions. Unfortunately, up to now, no general statement which could justify validity of the variational results and to estimate the error, has been given [40].

Therefore, results of the variational method can be used only to outline the main characteristics of the evolution, and one should always verify them numerically. Concerning the $(2+1)$-dimensional NSE containing terms describing both diffraction and (either anomalous or normal) dispersion, the accuracy of the variational method depends on the kind of dispersion. In the case of anomalous dispersion it gives results qualitatively similar to the numerical ones, up to the numerical factor (the threshold of catastrophic self-focusing in the variational method is approximately 5% larger than the numerical one). Larger discrepancy occurs in the case of normal dispersion, e.g. occurrence of spatio-temporal splitting, which was observed in numerical solutions, has not been predicted in the variational method.

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