On the compression of ultrashort optical pulses beyond the slowly varying envelope approximation

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***Abstract*— The pulse compression of ultra-short few-cycle pulses in nonlinear optical fibers is studied using the multisymplectic integration of the short pulse equation.**

I. Introduction

The study of different mechanisms of nonlinear pulse compression is usually based on the nonlinear Schrödinger equation (NSE) and its modifications. However, the present day optical technologies and experiments are starting to use ultrashort femto- and atto-second optical pulses [1], whose duration is smaller than a few cycles of the corresponding electromagnetic wave. In this case the usual description of optical pulses using the NSE and the slowly varying envelope approximation is not valid . Therefore, an analysis of the pulse compression of ultra-short few-cycles optical pulses beyond the slowly varying approximation is needed.

An approach to the description of ultrashort few-cycles optical pulses is proposed in [2]. It leads (in properly normalized units) to the so-called *short pulse equation* (SPE):

$$u_{zt} = u + \frac{1}{6}(u^3)_{tt}, \tag{1}$$

where $u(z,t)$ represents the magnitude of the electric field. This equation can describe ultra-short infrared pulses in silica optical fibers [3]. SPE represents an opposite extreme to the slowly varying envelope approximation and NSE: as the pulse duration shortens, the description using the NSE becomes less accurate, while the SPE provides a better approximation to the corresponding solution of the Maxwell equations.

In [4] and [5] SPE is shown to be integrable. Multicomponent generalizations of SPE have been discussed in [7] and [8]. In [6] Sakovich & Sakovich constructed a first analytical solution of SPE which represents ultra-short pulses.

In this paper we use the SPE in order to study the compression of ultra-short pulses.

II. Multisymplectic integrator

The numerical study of SPE and its generalizations requires a robust, fast and stable numerical scheme. Such a scheme has been constructed [9] using the so-called multisymplectic (De Donder-Weyl Hamiltonian) approach. It has been developed in the context of classical field theory by many authors (see e.g.[10]). An application to the numerical integration of PDEs was suggested in [11]. The key idea is to use a specific discretization, the multisymplectic integrator, which preserves the so-called multisymplectic structure. This idea generalizes the technique of symplectic integrators to PDEs. Unlike the infinite dimensional symplectic structure associated with PDEs, the multisymplectic structure is defined over a finite dimensional analogue of the phase space and its preservation is easier to implement numerically.

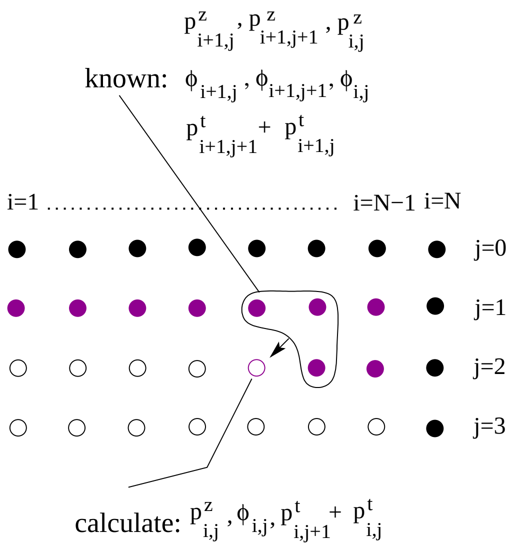

Fig. 1.

The multisymplectic integrator for SPE is constructed by the midpoint discretization of the De Donder-Weyl Hamiltonian form of SPE written in terms of the potential $\phi(t,z) : \phi_t = u$,

$$\begin{aligned}
&\frac{p^t_{i+1,j+\frac{1}{2}} - p^t_{i,j+\frac{1}{2}}}{\Delta t} + \frac{p^z_{i+\frac{1}{2},j+1} - p^z_{i+\frac{1}{2},j}}{\Delta z} = \phi_{i+\frac{1}{2},j+\frac{1}{2}},\\
&\frac{\phi_{i+1,j+\frac{1}{2}} - \phi_{i,j+\frac{1}{2}}}{\Delta t} = 2p^z_{i+\frac{1}{2},j+\frac{1}{2}},\\
&\frac{\phi_{i+\frac{1}{2},j+1} - \phi_{i+\frac{1}{2},j}}{\Delta z} = 2p^t_{i+\frac{1}{2},j+\frac{1}{2}} + \frac{8}{3}(p^z_{i+\frac{1}{2},j+\frac{1}{2}})^3,
\end{aligned} \tag{2}$$

where $\phi_{i,j} = \phi(i\Delta t, j\Delta z)$. The discretization mesh is presented on Fig. 1.

In [9] we have seen that the multisymplectic integrator is an order of magnitude more precise and approximately 25 times faster at long propagation times than the pseudo-spectral method and even more effective when compared with the split-step method. A comparison with the exact solution of SPE

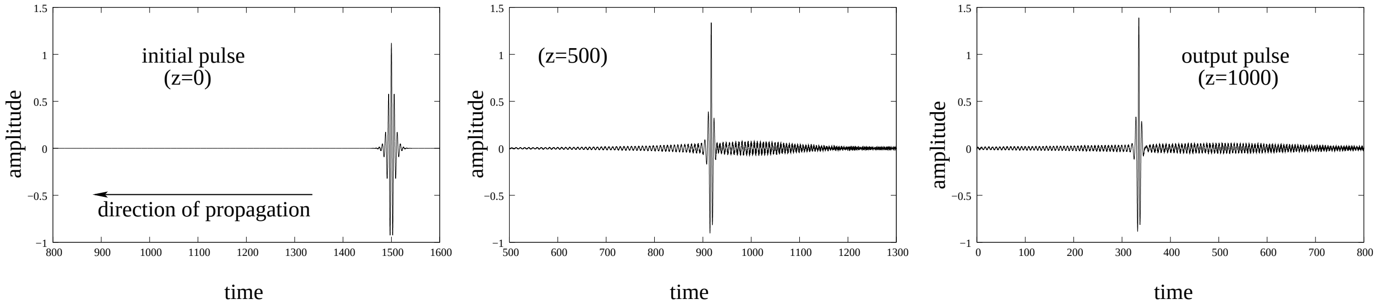

Fig. 2.

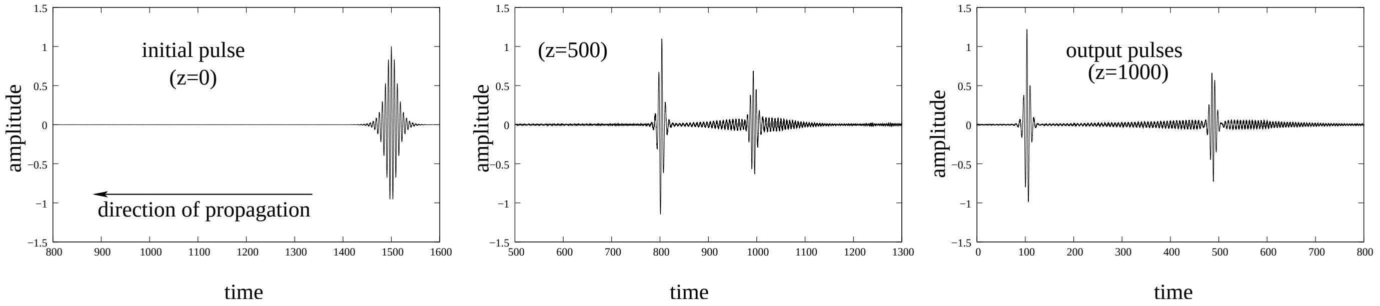

Fig. 3.

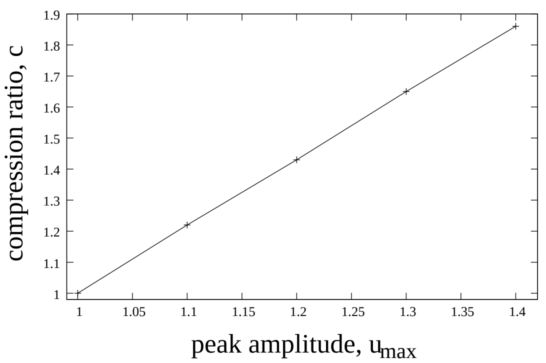

Fig. 4.

shows that our multisymplectic integration of SPE is stable and robust and preserves the energy functional.

III. Pulse Compression

The multisymplectic integrator enables us to study the behavior of the solutions of SPE at long propagation distances. Here we use it to study the compression of ultra-short pulses.

Fig. 2 shows how a few cycles ultra-short initial pulse evolves into the compressed pulse and radiation. This behaviour is observed only for a certain range of the amplitudes of the initial pulse with the same initial width. If the initial amplitude is too low the pulse actually broadens, and if the initial amplitude is too high the carrier-wave shock formation takes place. For the range of initial amplitudes which can be used for pulse compression we found a linear dependence of the compression ration c from the initial peak amplitude u_{max}, see Fig. 4.

For broader pulses we observe a phenomenon similar to the solition fission phenomenon [12] known from the perturbed NSE, where several compressed pulses are created, see Fig. 3.

Note that in both cases the initial pulse evolves to a Sakovich$\&$Sakovich' solitary wave solution [6] and the radiation waves.

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