Nuclear structure and dynamics in the neutron star crust

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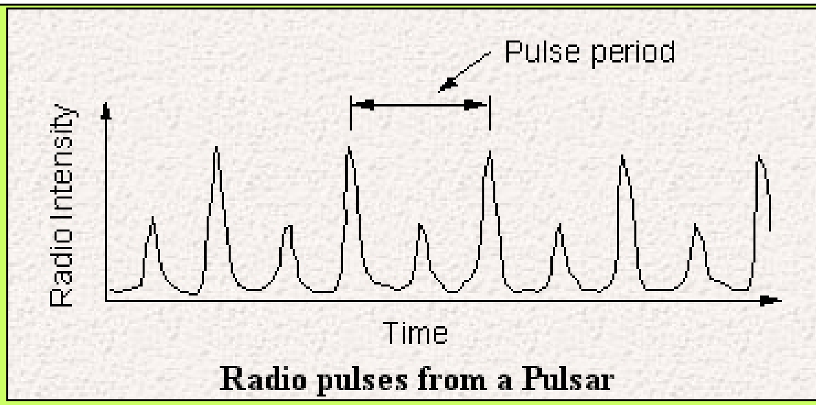
Neutron star discovery

-The existence of neutron stars was predicted by Landau (1932), Baade & Zwicky (1934) and Oppenheimer& Volkoff (1939).

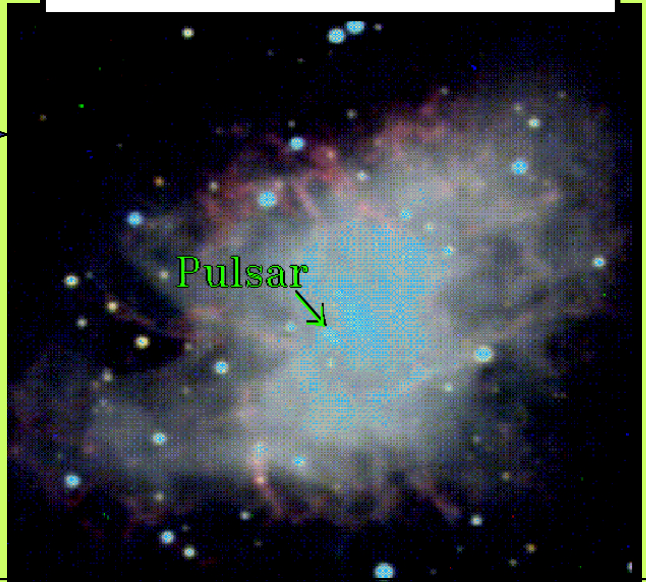
- On November 28, 1967, Cambridge graduate student Jocelyn Bell (now Burnell) and her advisor, Anthony Hewish discovered a source with an exceptionally regular pattern of radio flashes. These radio flashes occurred every 1 1/3 seconds like clockwork. After a few weeks, however, three more rapidly pulsating sources were detected, all with different periods. They were dubbed "pulsars."



Nature of the pulsars

pulse rate = 30/second
slowing down rate = 38 nanoseconds/day

Pulsar in the Crab Nebula

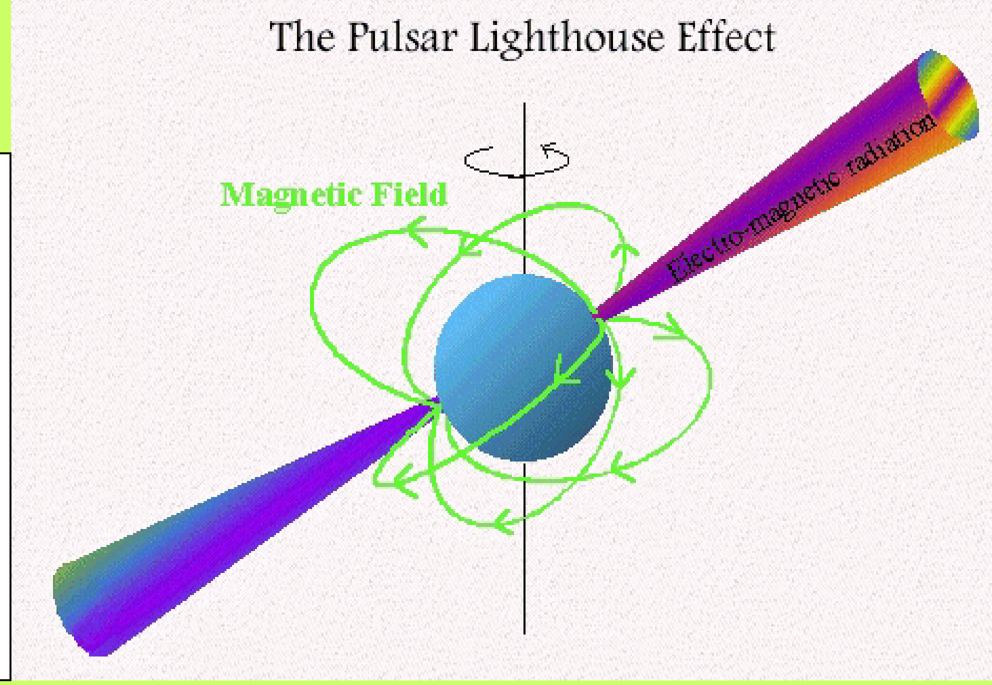


Calculated energy loss due to rotation of a possible neutron star $\approx$ Energy radiated

Conclusion: the pulses are produced by rotation!

Basic facts about neutron stars:

Radius: $\sim$ 10 km
Mass: $\sim$ 1-2 solar masses
Average density: $\sim 10^{14}\ g/cm^3$
Magnetic field: $\sim 10^8 - 10^{12}$ G
Magnetars: $\sim 10^{15}$ G
Rotation period: 1.5 msec. – 5 sec.



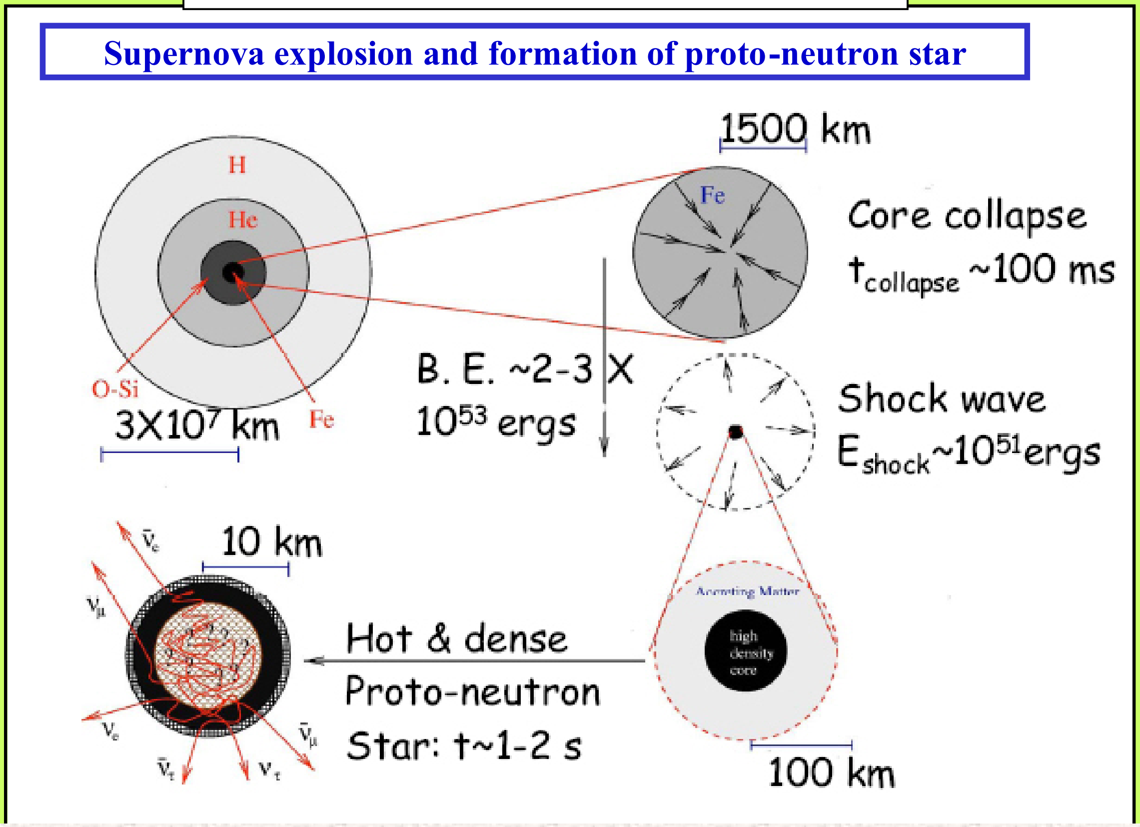
Number of known pulsars: > 1000
Number of pulsars in our Galaxy: $\sim 10^8$

Gravitational energy of a nucleon at the surface of neutron star $\sim$ **100 MeV**

Binding energy per nucleon in an atomic nucleus: $\sim$ **8 MeV**

Neutron star is bound by gravitational force

Birth of a neutron star



Summary: End Points of Stellar Evolution						
Remnant	**Progenitor Mass**	**Remnant Mass**	**Size**	**Density**	**Means of Support**	**Final Stage**
White Dwarf	$M_* < 8M_\odot$	$M_{WD} < 1.4M_\odot$	$R_{WD} \sim R_{earth}$	1 ton/cm^3	e^- degeneracy	Planetary Nebula
Neutron Star	$8M_\odot < M_* < 20M_\odot$	$M_{NS} < 3M_\odot$	$R_{NS} \sim 10$ km	200 million ton/cm^3	n degeneracy	Supernova
Black Hole	$M_* > 20M_\odot$	$M_{BH} > 3M_\odot$	0 $R_{grav} = 2GM/c^2$	∞	none	?

Thermal evolution of a neutron star:

Temperature: 50 MeV → 0.1 MeV $(t \sim \text{min.})$

URCA process:

$$p + e \to n + \nu_e$$
$$n \to p + e + \bar{\nu}_e$$

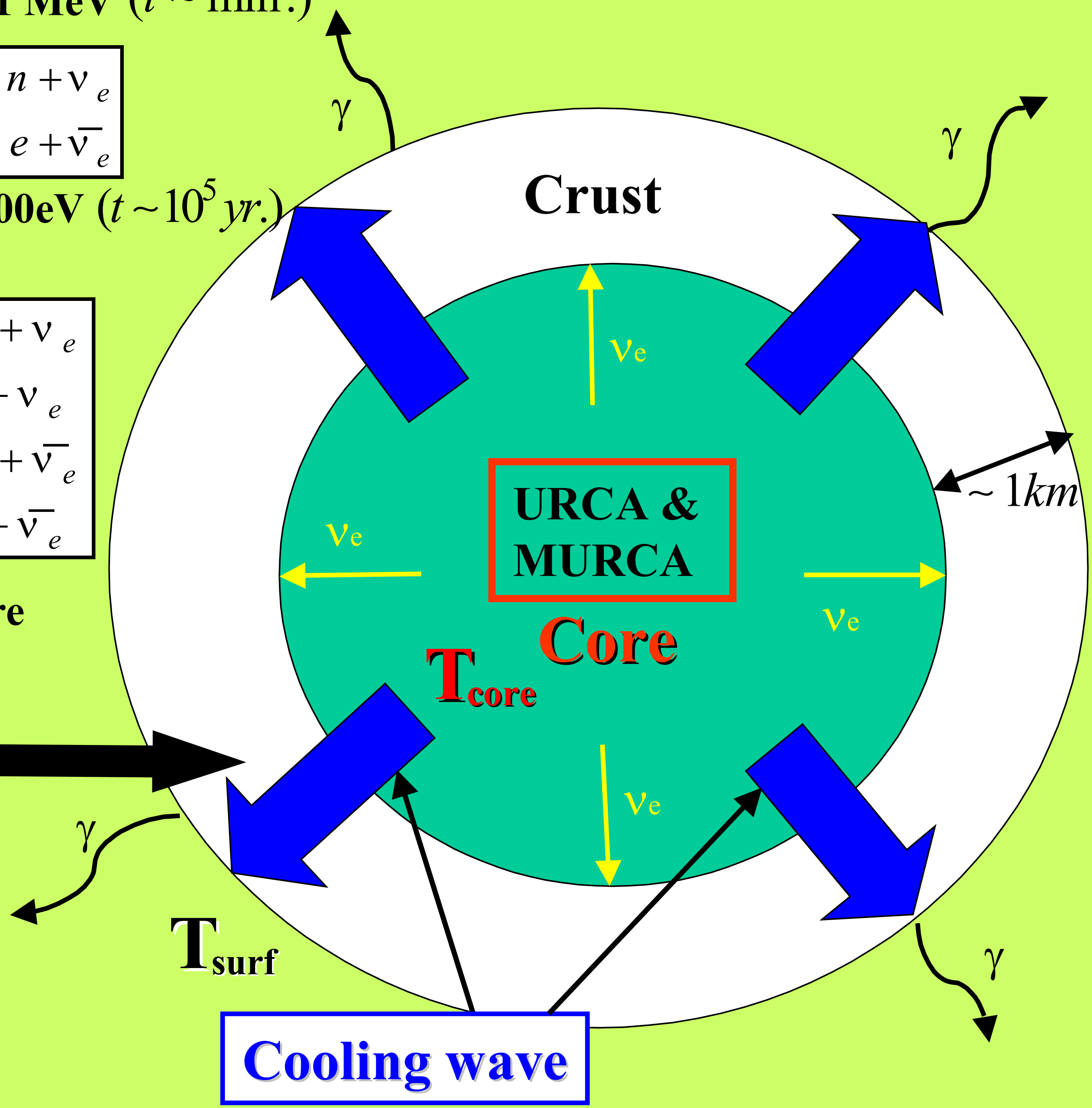
Temperature: 0.1 MeV → 100eV $(t \sim 10^5\, yr.)$

MURCA process:

$$p + p + e \to p + n + \nu_e$$
$$n + p + e \to n + n + \nu_e$$
$$p + n \to p + p + e + \bar{\nu}_e$$
$$n + n \to n + p + e + \bar{\nu}_e$$

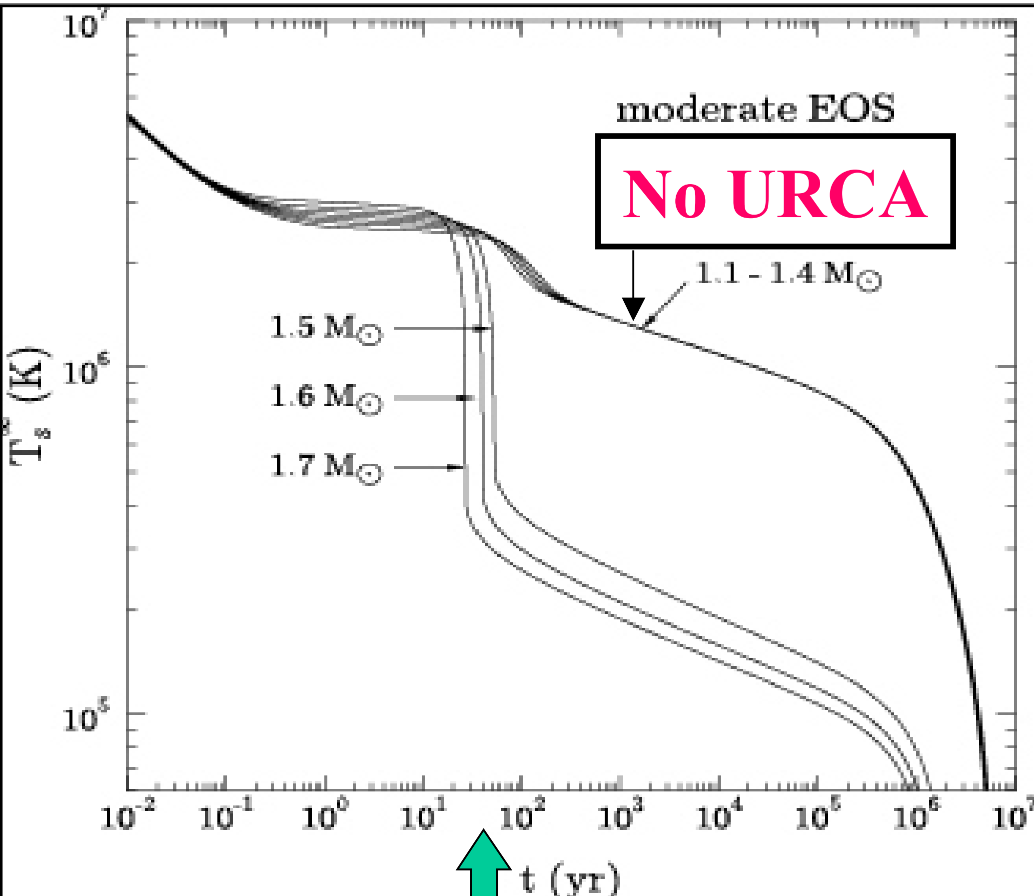
Energy transfer between core and surface:

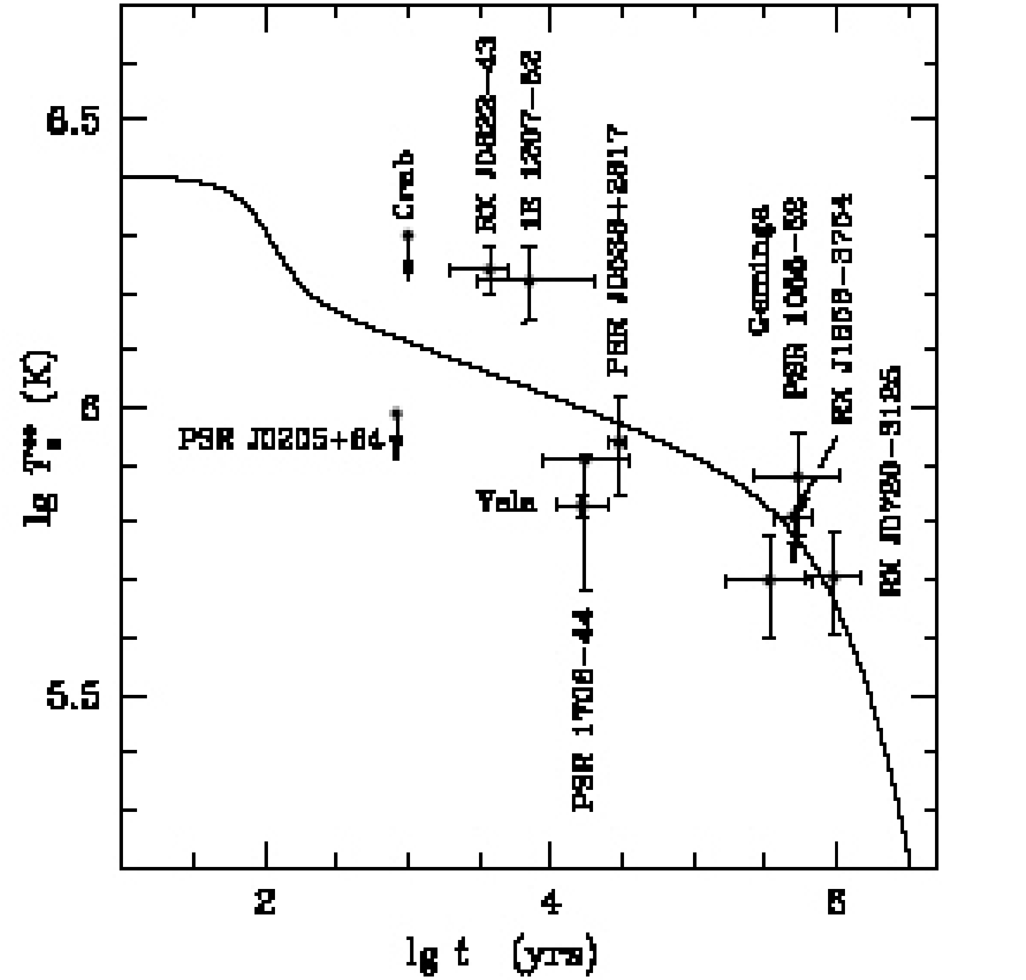
$$\frac{\partial T}{\partial t} = D\nabla^2 T; \quad D = \frac{\kappa}{C_v}$$



For τ < 100 years:

$$T_{core} < T_{surf}$$

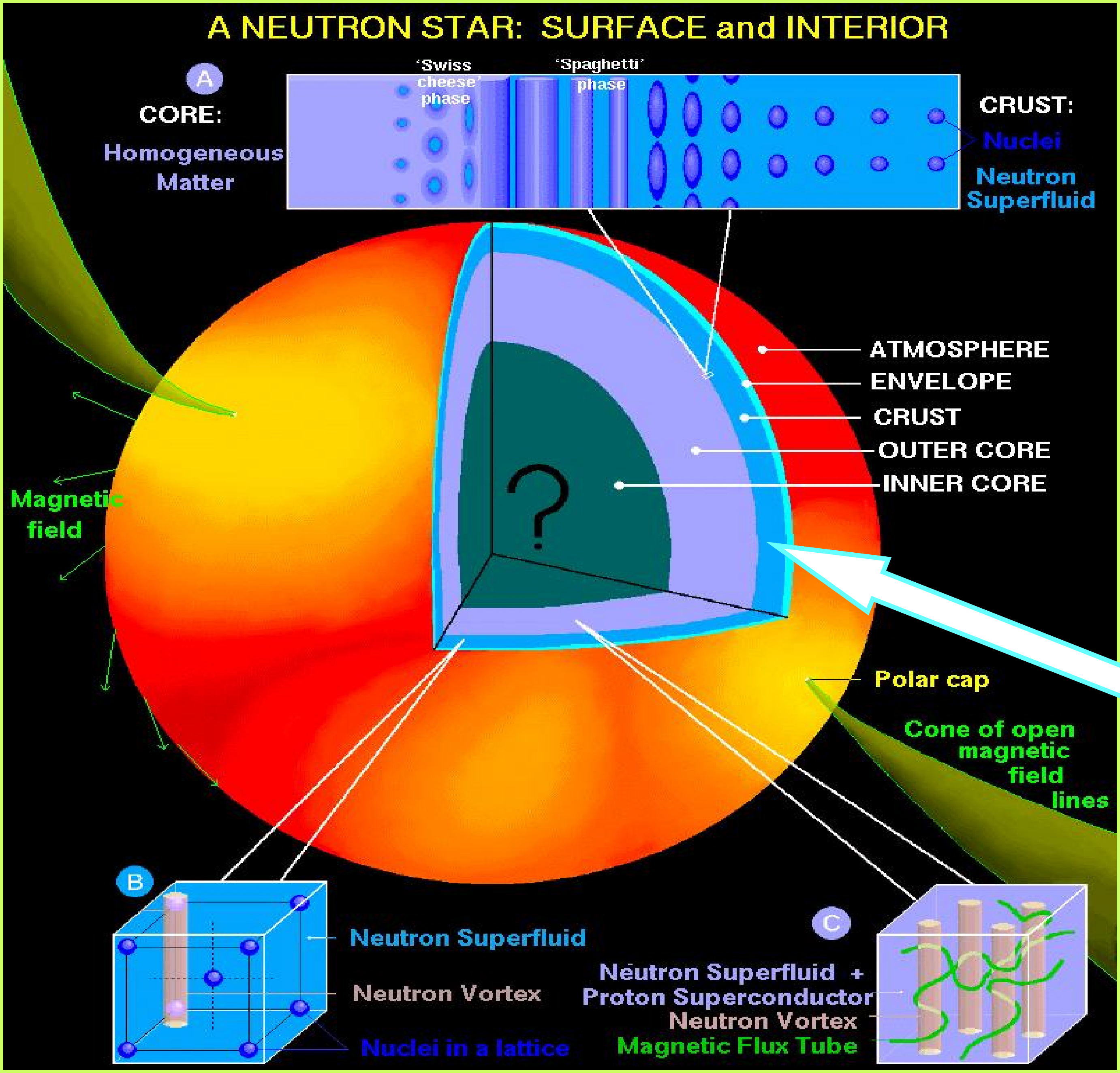




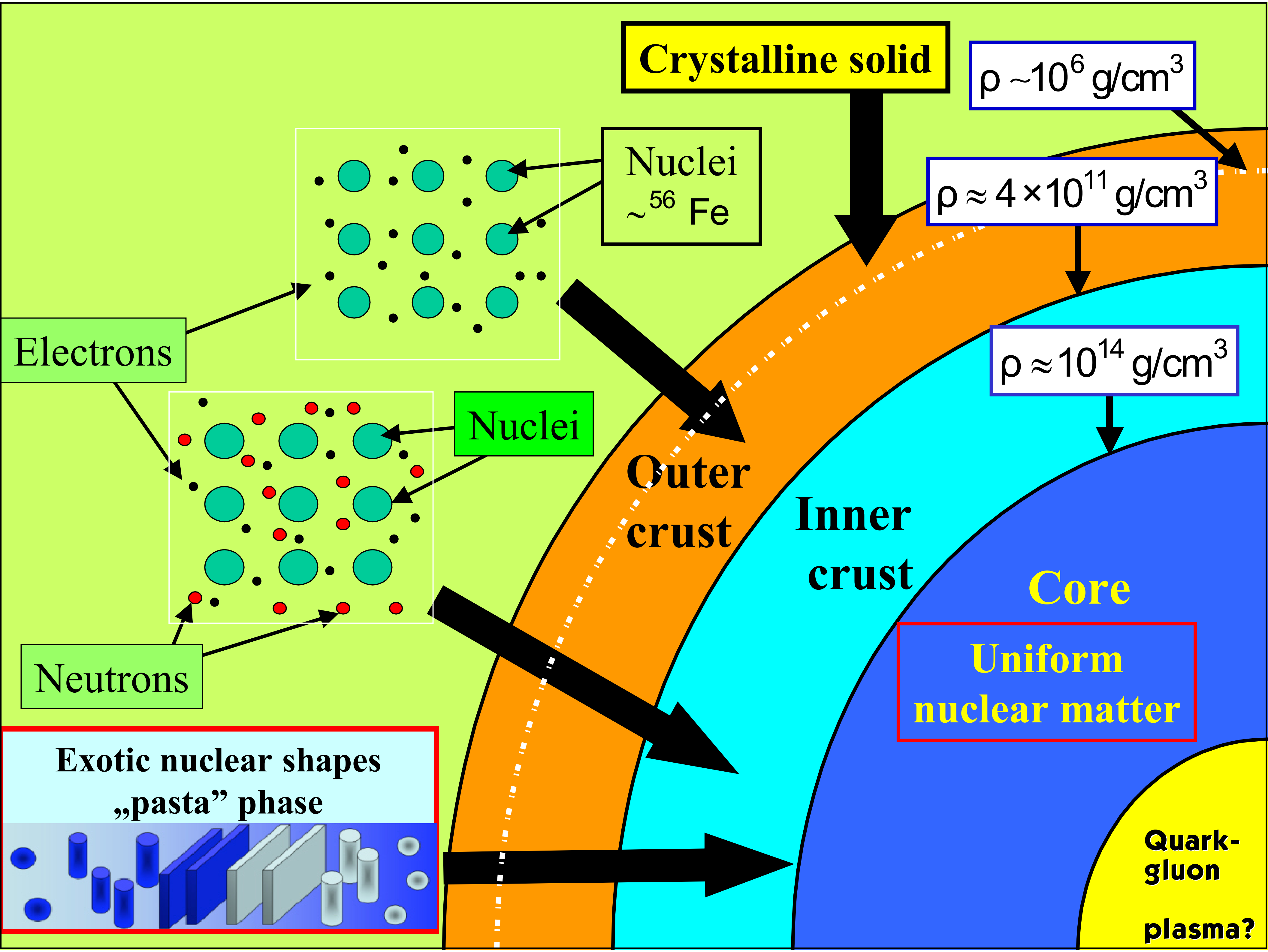
M.E. Gusakov, A.D. Kaminker, D.G. Yakovlev, and O.Y. Gnedin, Astron. Astrophys. 423 (2004) 1063.

Relaxation time: a typical time for the cooling wave to reach the surface

Structure of a neutron star

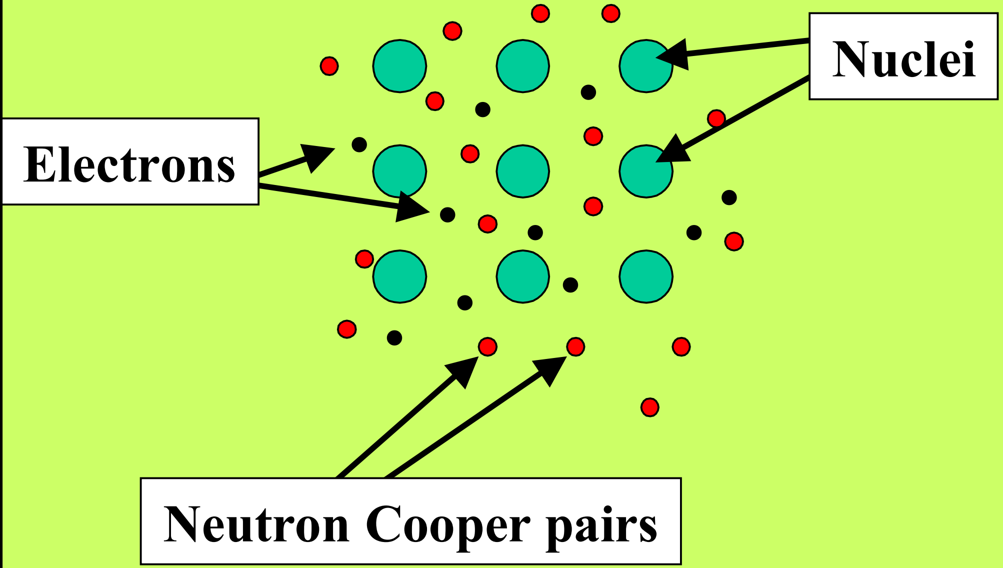


~ 0.01 - 0.02 M_{NS}

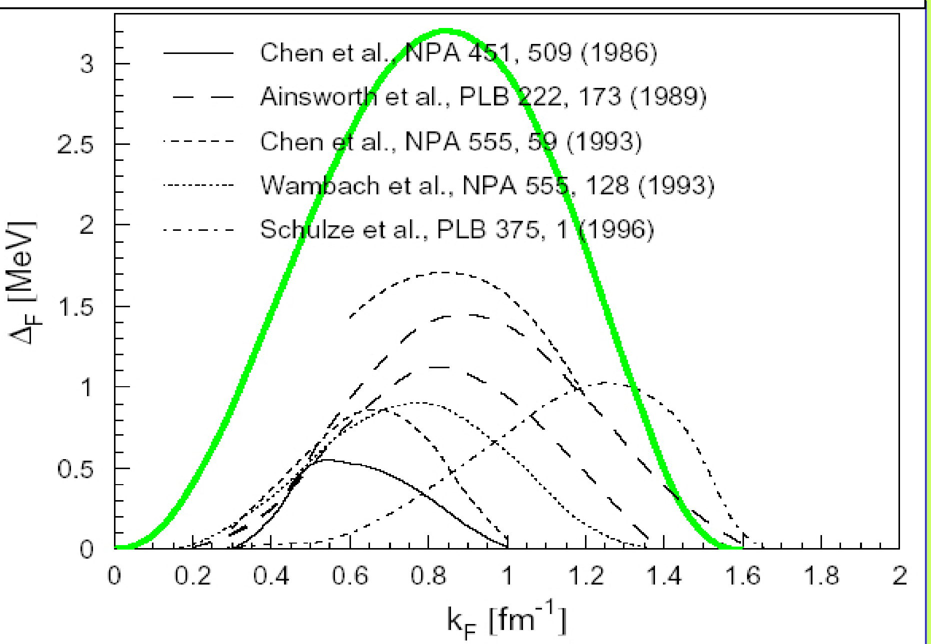


Crystalline solid
ρ ~10^6 g/cm^3
Nuclei ~56 Fe
ρ ≈ 4×10^11 g/cm^3
Electrons
ρ ≈ 10^14 g/cm^3
Nuclei
Outer crust
Inner crust
Core
Uniform nuclear matter
Neutrons
Exotic nuclear shapes „pasta" phase
Quark-gluon plasma?

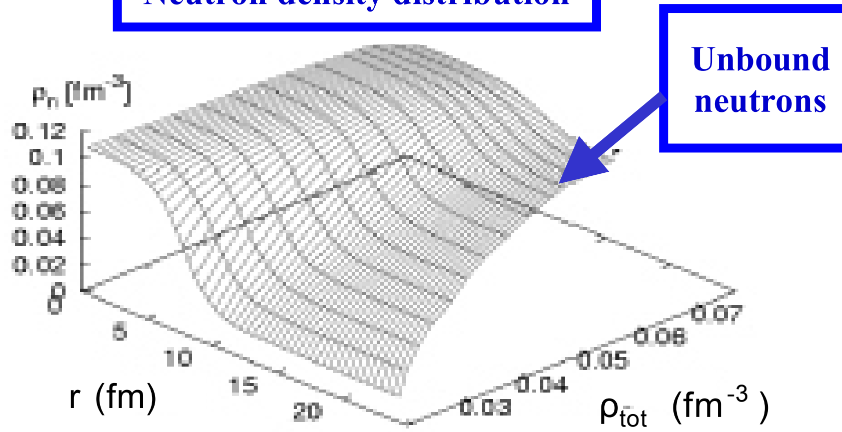
Structure of the inner crust



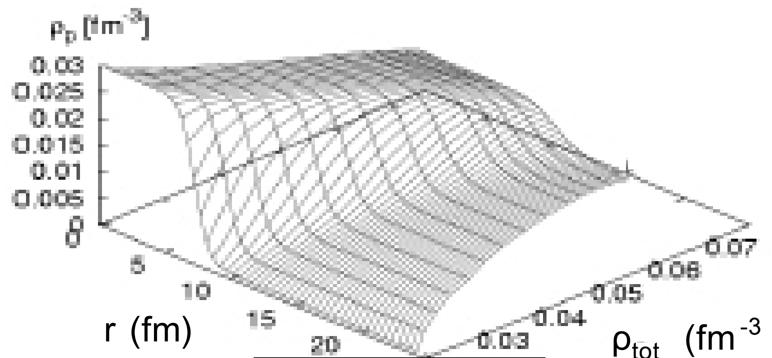
s-wave pairing gap in infinite neutron matter with realistic NN-interactions



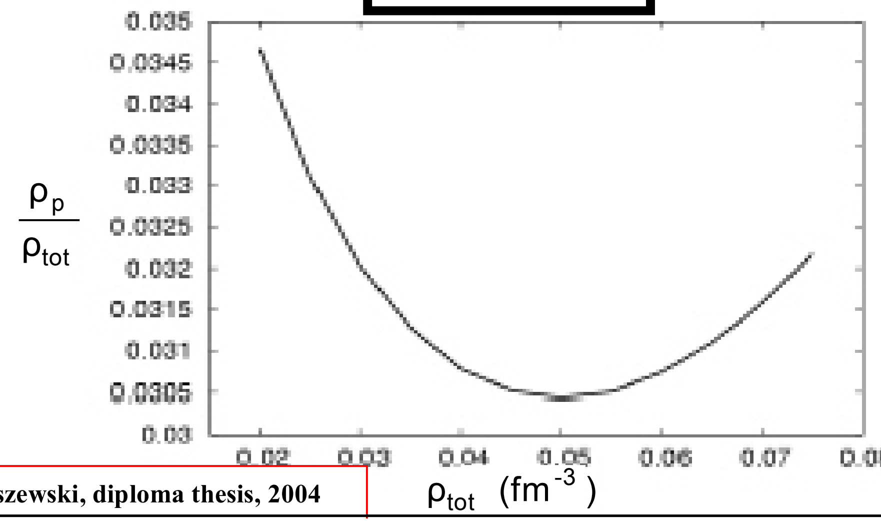
Neutron density distribution



Proton density distribution



Proton fraction



K.Klimaszewski, diploma thesis, 2004

Dynamics of a nucleus immersed in a neutron superfluid

$$H = \sum_m \left(\frac{|\hat{\pi}_m|^2}{2M} + \frac{C}{2} |\hat{\alpha}_m|^2 \right)$$

$$M = m\rho_{in} \frac{(\gamma - 1)^2}{2(\gamma + 1)} R_N^5; \quad \gamma = \frac{\rho_{out}}{\rho_{in}}$$

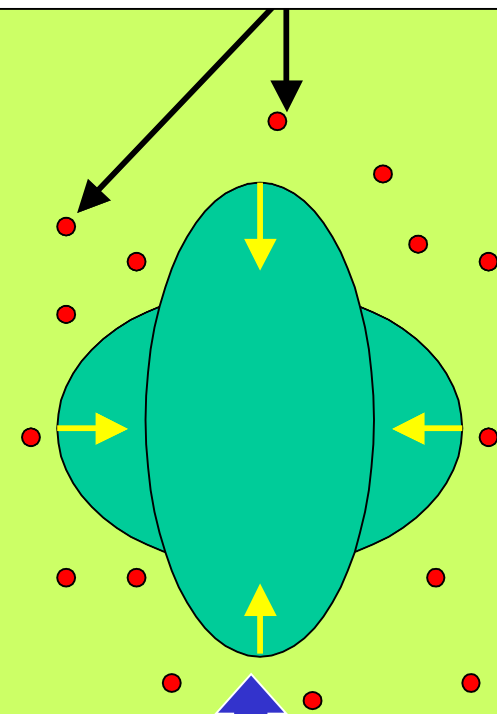
$$C = C^{surf} + C^{coul}$$

R_N - nuclear radius

ρ_{in} - nuclear density

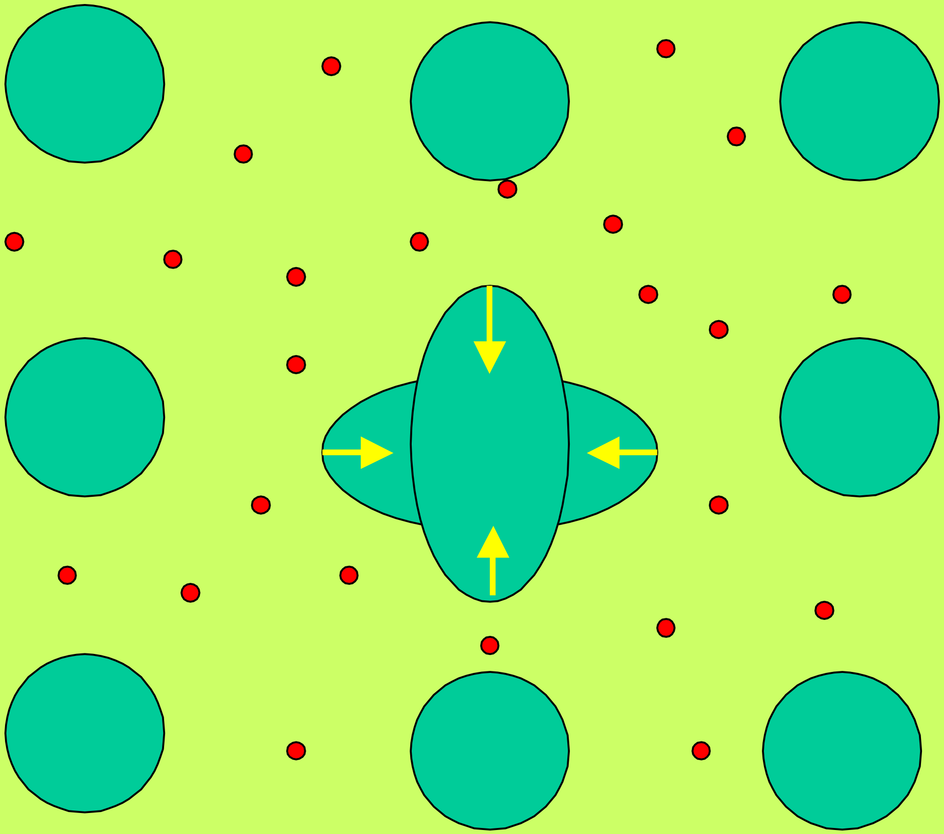
ρ_{out} - density of unbound neutrons

Neutrons Cooper pairs



Vibrating nucleus

Dynamics of a nucleus immersed in a neutron superfluid: correction due to the coupling to the lattice



Vibrational energy depends on the orientation with respect to the lattice

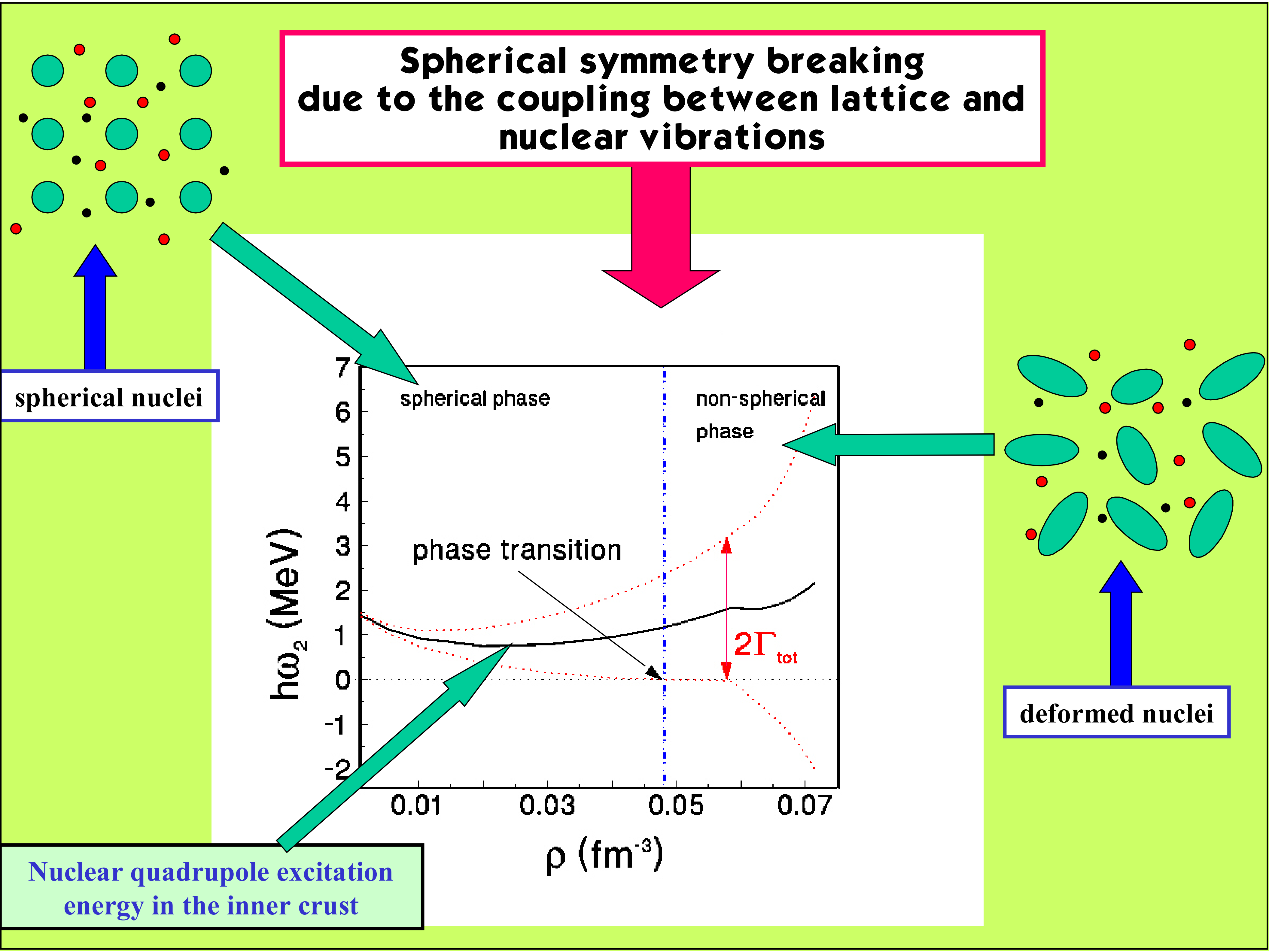
Splitting of a quadrupole vibrational multiplet:

$$\Gamma_{tot} \sim V_{Coul} \times V_{WS}{}^{-1} \times \alpha$$

V_{Coul} - Coulomb interaction energy of the nucleus with the lattice

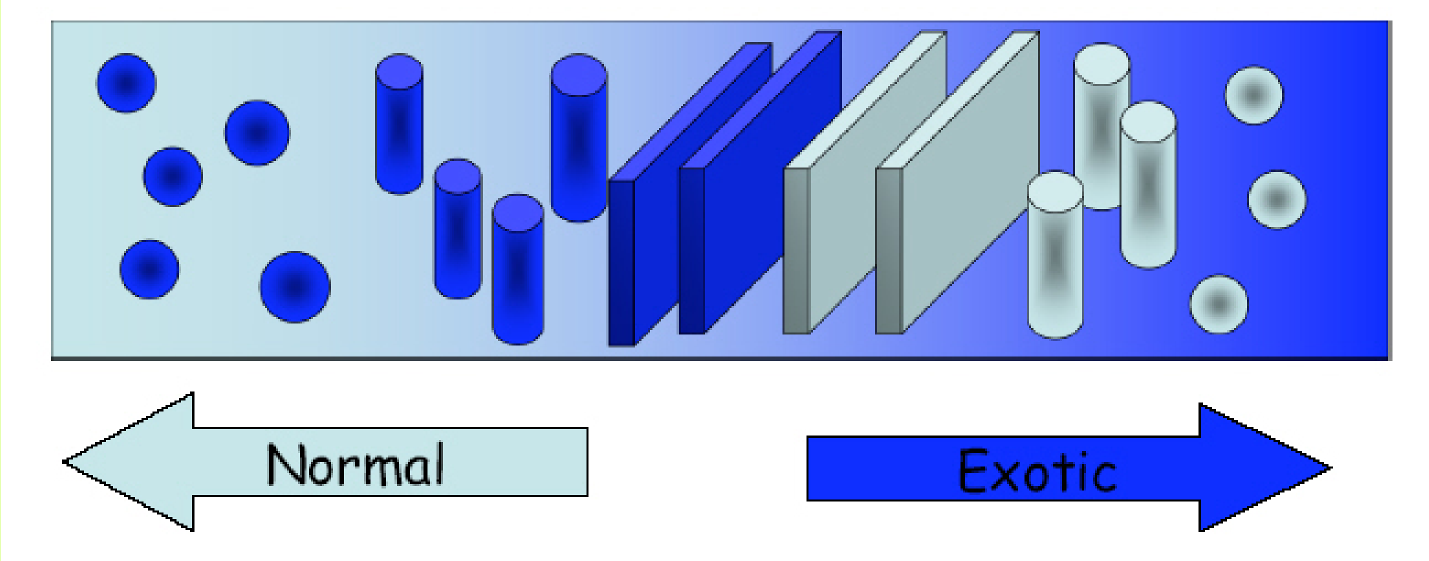
$V_{WS} = \dfrac{\text{Total volume}}{\text{Number of nuclei}}$ - Wigner - Seitz cell volume

α - amplitude of vibration



Spherical symmetry breaking
due to the coupling between lattice and
nuclear vibrations
spherical nuclei
spherical phase
non-spherical
phase
phase transition
$2\Gamma_{tot}$
$h\omega_2$ (MeV)
ρ (fm^{-3})
7
6
5
4
3
2
1
0
-1
-2
0.01
0.03
0.05
0.07
deformed nuclei
Nuclear quadrupole excitation
energy in the inner crust

Structure of the bottom of the inner crust: „pasta" phases

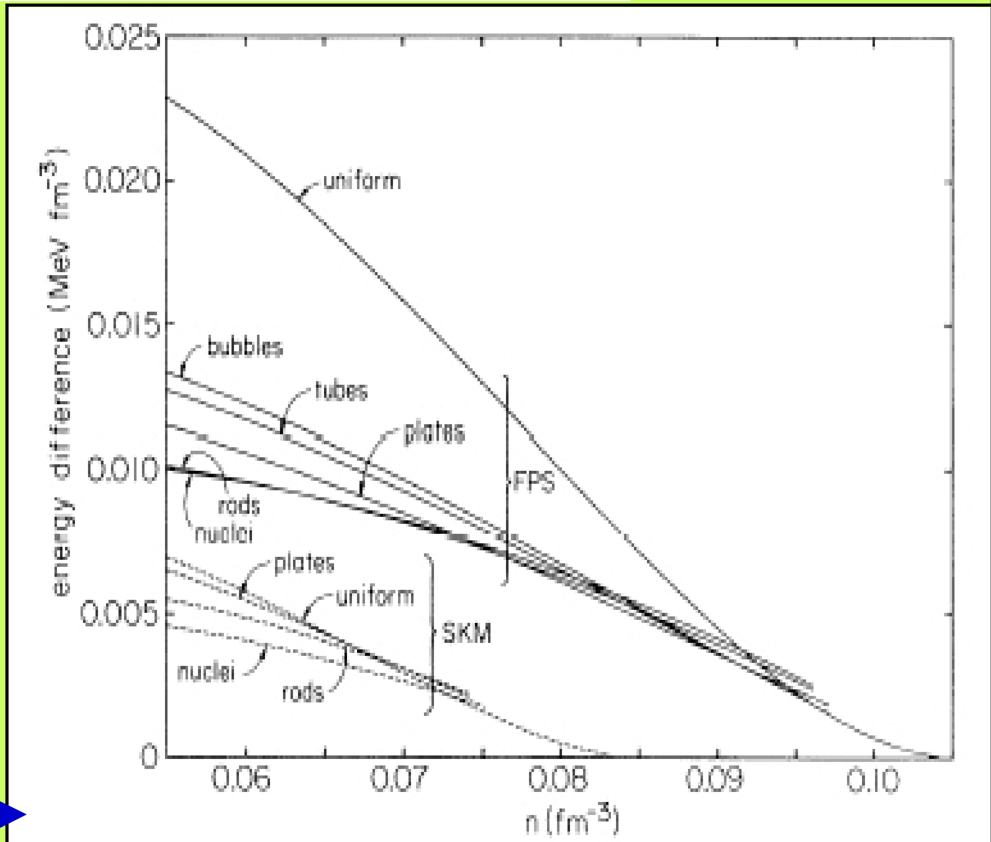


Nuclear Pasta

At densities of 10^{13} -10^{14} g/cm^3 competition between nuclear attraction and Coulomb repulsion leads to a very complex ground state that involve round (meat ball), rod (spaghetti), plate (lasagna), and other shapes.

This „nuclear pasta" is expected to have unusual properties and dynamics. It may be important for radio, x-ray, and neutrino radiation.

Simple semiclassical model predict a sequence of phase transitions:

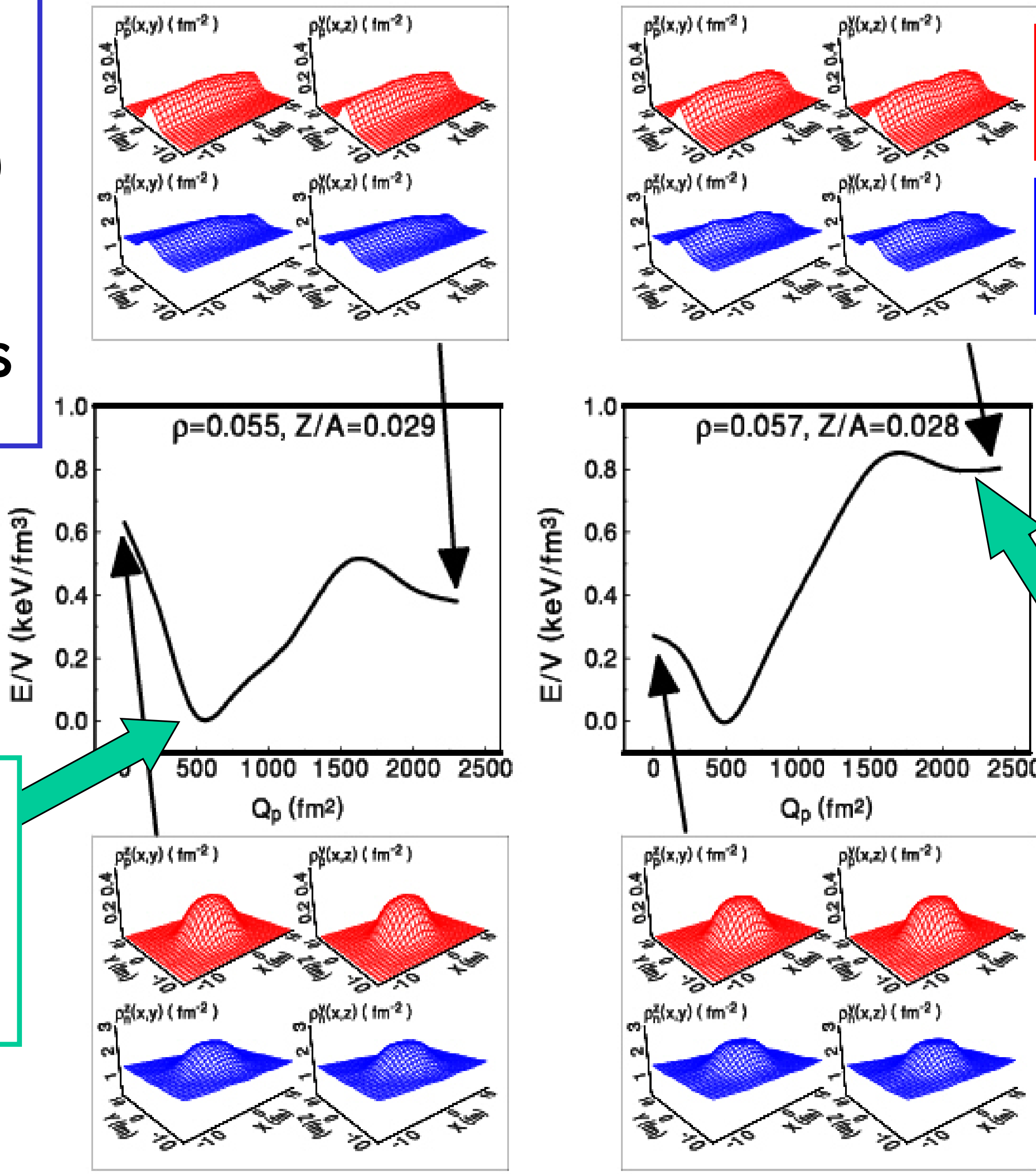


Lorenz, Ravenhall and Pethick, Phys. Rev. Lett. 70, 379 (1993)

First fully microscopic 3D calculations of pasta phases:

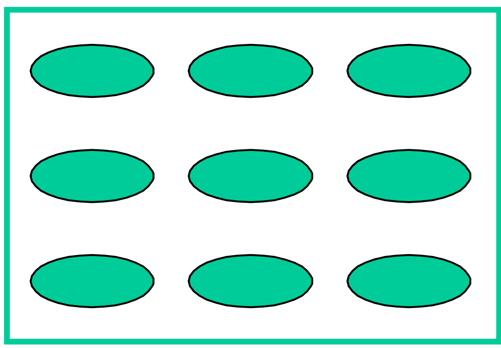
Hartree-Fock calcs. In coordinate space for 1000 nucleons in the Wigner-Seitz (WS) cell!

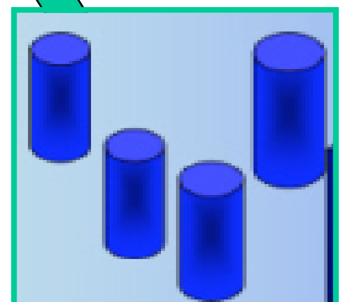
Coulomb interaction treated beyond the WS approximation!



Proton density distribution

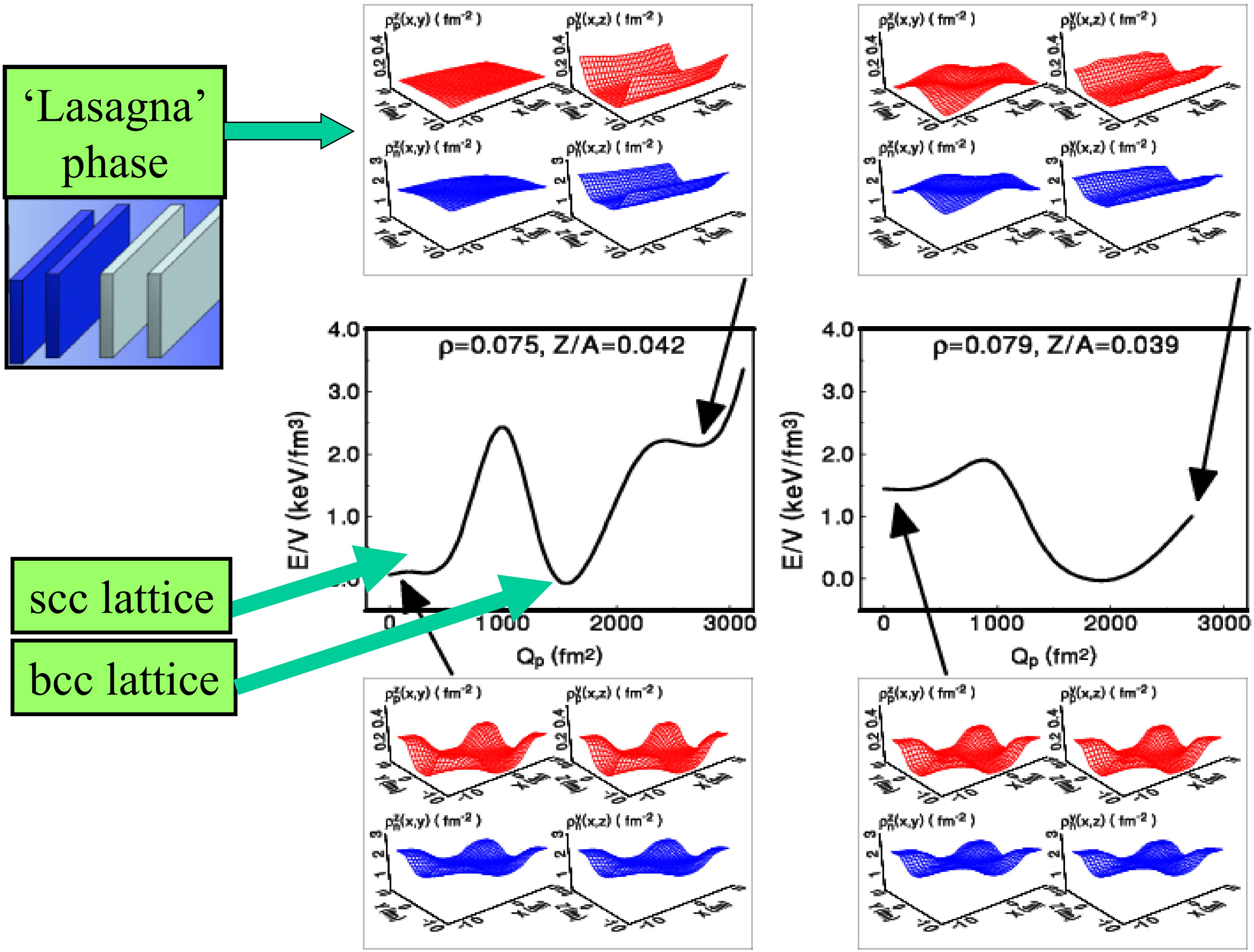
Neutron density distribution





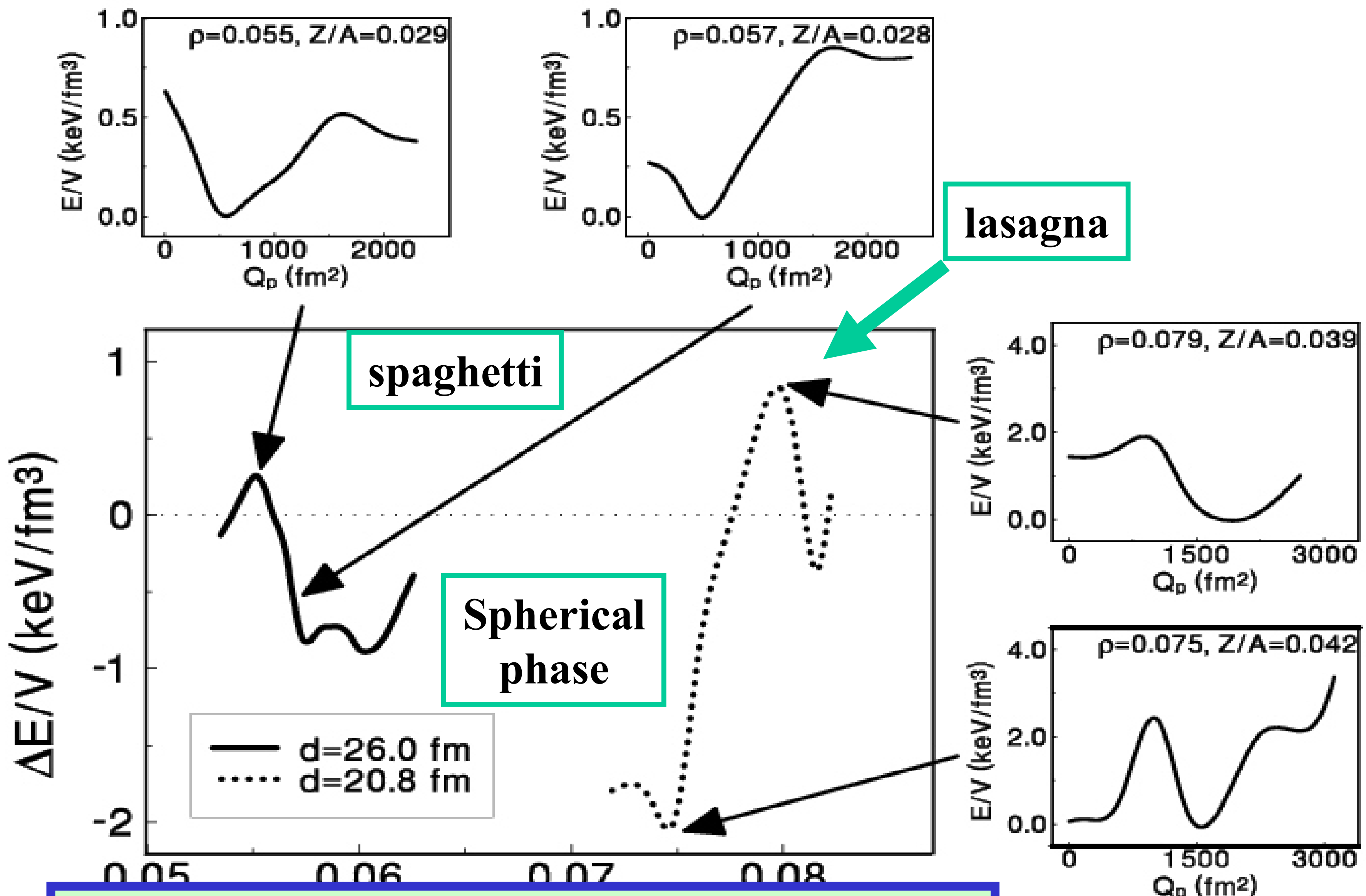
‘Spaghetti’ phase

P. Magierski and P.-H. Heenen, Phys.Rev.C65,045804 (2002)



P. Magierski and P.-H. Heenen, Phys.Rev.C65,045804 (2002)

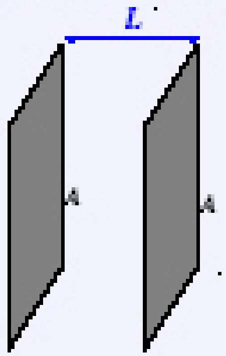
Energy difference between the spherical phase and the ‘spaghetti’ phase: ———
Energy difference between the spherical phase and the ‘lasagna’ phase: ....



What is the origin of these oscillations?

Skyrme HF with SLy4, P.Magierski and P.-H.Heenen, Phys.Rev.C 65, 045804 (2002)

- H.B.G. Casimir (1948): two parallel uncharged metallic plates attract each other in vacuum



$$\frac{F^{||}(L)}{A} = -\frac{\hbar c}{L^4}\frac{\pi^2}{240} \approx -1.3\times 10^{-7}\frac{1}{L^4}\mathrm{N}\frac{\mu\mathrm{m}^4}{\mathrm{cm}^2}$$

$$\mathcal{E}^{||}(L) = -\frac{\hbar c}{L^3}\frac{\pi^2}{720}A$$

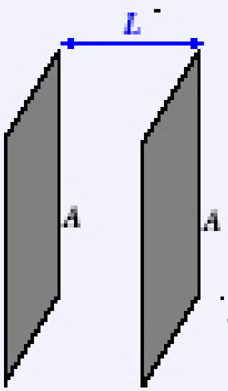
- Origin: zero-point fluctuations of e.m. field modified
  by the addition of the two plates relative to free case

  $\Rightarrow$ *change* in the energy of the vacuum: $\sum\!\!\!\!\!\!\int \hbar\omega_k|_{\mathrm{plates}(L)} - \sum\!\!\!\!\!\!\int \hbar\omega_k|_{\mathrm{free}}$

Casimir effect: *Mesoscopic manifestation of quantum fluctuations of the vacuum*

- experimental confirmation in the last decade (for the sphere-plate system!)
  - S. Lamoureaux, *Phys. Rev. Lett.* **78** (1997);
  - U. Mohideen & A. Roy, *Phys. Rev. Lett.* **81** (1998);
  - 9-Feb-2001 issue of *New York Times* about
    Casimir effect in *MicroElectroMechanical Systems*; etc.

Original Casimir effect:



1) "box" geometry
2) EM b.c.'s: $\mathbf{n}\cdot\mathbf{B}=0$ & $\mathbf{n}\wedge\mathbf{E}=0$.

$\Rightarrow$ 2 decoupled scalar fields:

TE: $\phi^{\mathrm{D}}(x,y,z,t)|_{z=0}=\phi^{\mathrm{D}}(x,y,z,t)|_{z=L}=0$ (Dirichlet b.c.'s)

$$\phi^{\mathrm{D}}_{nk_\perp}(x,y,z,t)=\sin(k_z z)\,e^{i(k_x x+k_y y)}e^{-i\omega_{n,k_x,k_y}t}$$

$$\boxed{k_z=\left(\frac{\pi}{L}\right)n\,,\quad n=1,2,3,\cdots;\quad \omega_{n,k_x,k_y}=c\sqrt{k_x^2+k_y^2+\left(\frac{n\pi}{L}\right)^2}}$$

TM: $\frac{\partial}{\partial z}\phi^{\mathrm{N}}(x,y,z,t)|_{z=0}=\frac{\partial}{\partial z}\phi^{\mathrm{N}}(x,y,z,t)|_{z=L}=0$ (Neumann b.c.'s)

$$\phi^{\mathrm{N}}_{nk_\perp}(x,y,z,t)=\cos(k_z z)\,e^{i(k_x x+k_y y)}e^{-i\omega_{n,k_x,k_y}t}$$

$$\boxed{k_z=\left(\frac{\pi}{L}\right)n\,,\quad n=0,1,2,3,\cdots.}$$

$$\begin{aligned}\mathcal{E}_C^{||\mathrm{EM}}(L) &= \lim_{\Lambda\to\infty} A\iint\frac{dk_x dk_y}{(2\pi)^2}\left(\sum_{n=1}^{\infty}\tfrac{1}{2}\hbar\omega_{n,k_x,k_y}+\sum_{n=0}^{\infty}\tfrac{1}{2}\hbar\omega_{n,k_x,k_y}\right)e^{-\frac{\hbar\omega_{n,k_x,k_y}}{\Lambda}}\\ &= \lim_{\Lambda\to\infty}\hbar c A L\left[\frac{3(\Lambda/\hbar c)^4}{\pi^2}+(-1+1)\frac{(\Lambda/\hbar c)^3}{4\pi L}-\frac{\pi^2}{720L^4}+\mathcal{O}\left((\hbar c)^2/\Lambda^2L^6\right)\right]\end{aligned}$$

"Generalization" of concept of Casimir energy:

1) *geometry dependence*

Casimir energy $\equiv$ vacuum energy from the *geometry-dependent* part of the density of states (d.o.s)

($\leftrightarrow$ *shell* correction energy in Nucl. Phys.)

$$\text{d.o.s.:} \qquad \rho(E) \equiv \sum_{E_k} \delta(E - E_k) = \rho_0(E) + \rho_{\text{bulk}}(E) + \delta\rho_C(E, \textit{geom.-dep.})$$

$$\text{N.o.s.:} \qquad \mathcal{N}(E) \equiv \sum_{E_k} \Theta(E - E_k) = \int_0^E dE'\, \rho(E')$$

$$\text{Casimir Energy:} \quad \mathcal{E}_C \equiv \int dE\, E\, \delta\rho_C(E, \textit{geom.-dep.}) = -\int dE\, \mathcal{N}_C(E, \textit{geom.-dep.})$$

2) *matter fields*

- *Here* : space not "filled" with *fluctuating* EM modes, but with gas of *non-interacting* (non-relativistic) fermions.
- *Similarity* : $\exists$ mode sums $\sum\!\!\!\!\!\!\int \hbar\omega_k$ with *constant* degeneracy factor, (because of Pauli's exclusion principle).
- *Difference* : $\exists$ of second scale: fermi energy = *chemical potential* μ (at $T \approx 0$) in addition to geometric size and distance scale(s).
- *Concrete* : Matter fields (fermions) in the space between voids build up a quantum pressure on the voids

$\exists$ effective *interaction* between empty regions of space in the background of *non-interacting* fermions

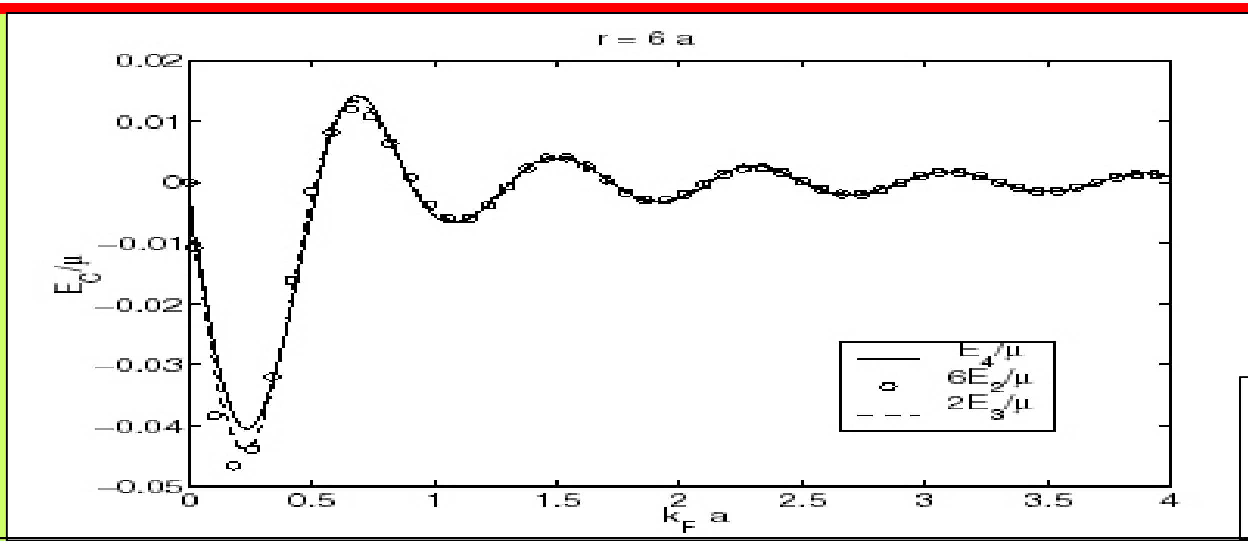
Quantum fluctuation effects

Let me create a caricature of a the “pasta phase” in the crust of a neutron star.

Question: [**What is the most favorable arrangement of these two spheres?**]

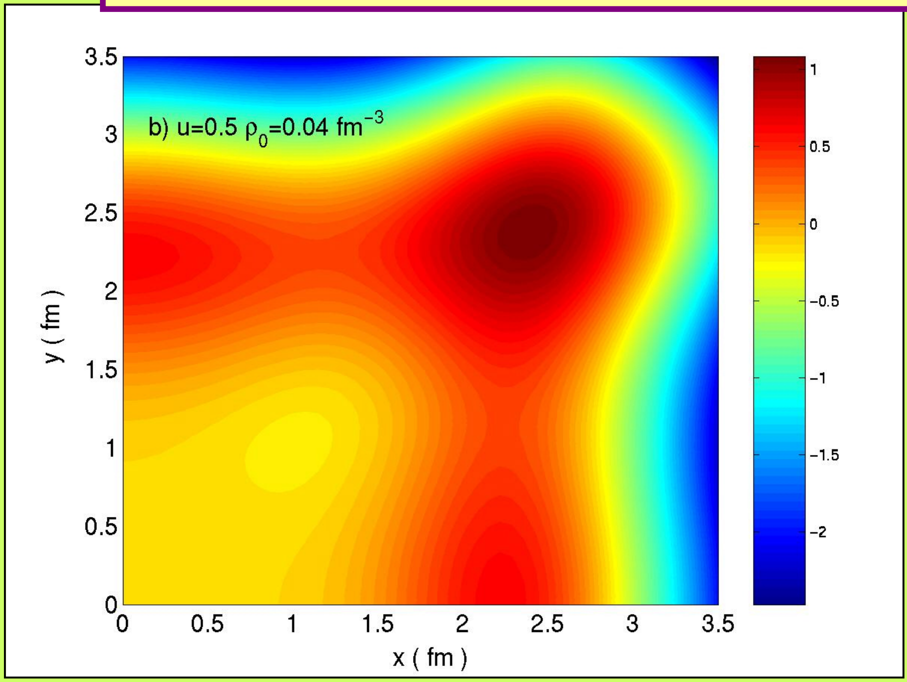
[Casimir Interaction among Objects Immersed in a Fermionic Environment]

$$E_C \approx \frac{\hbar^2 k_F \; a^2}{8\pi \; m \; r^3} \cos(2 k_F (r - 2a))$$

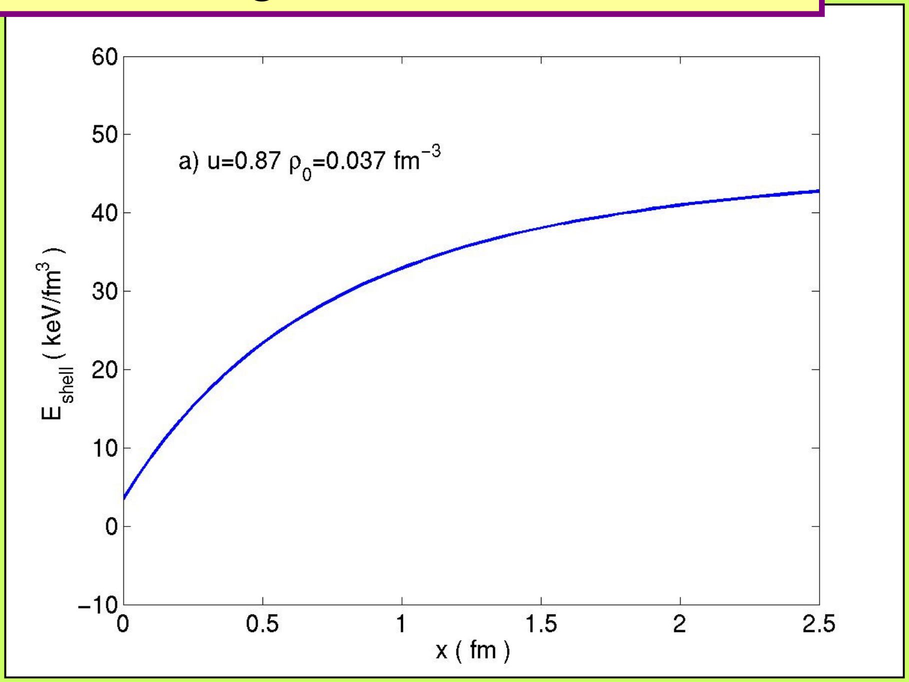


A.Bulgac, P. Magierski,
Nucl. Phys. A683, 695 (2001),
A.Bulgac, A. Wirzba,
Phys.Rev.Lett.87,120404(2001)

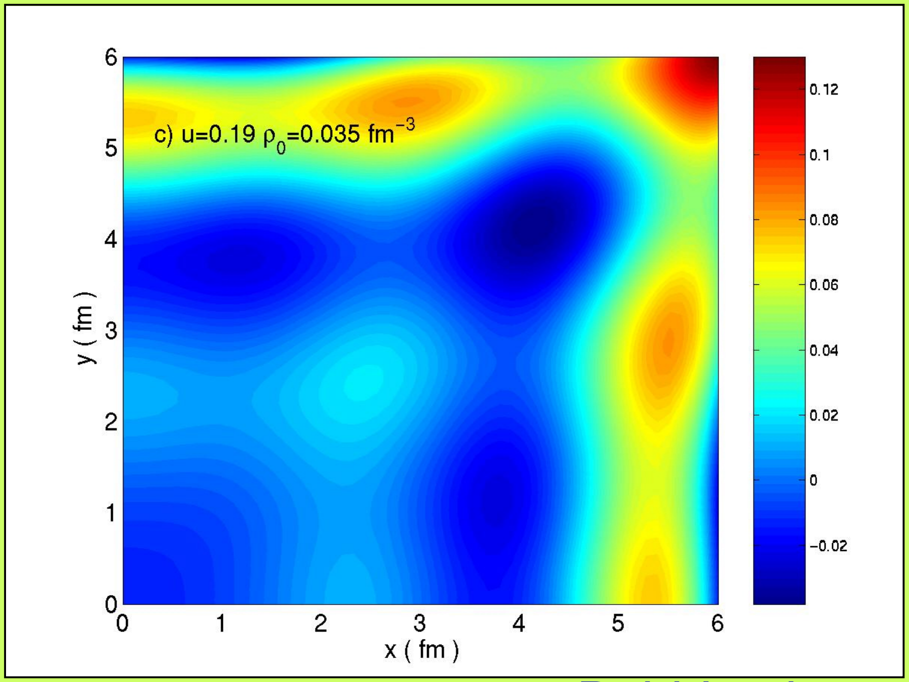
The Casimir energy for the displacement of a single void in the lattice



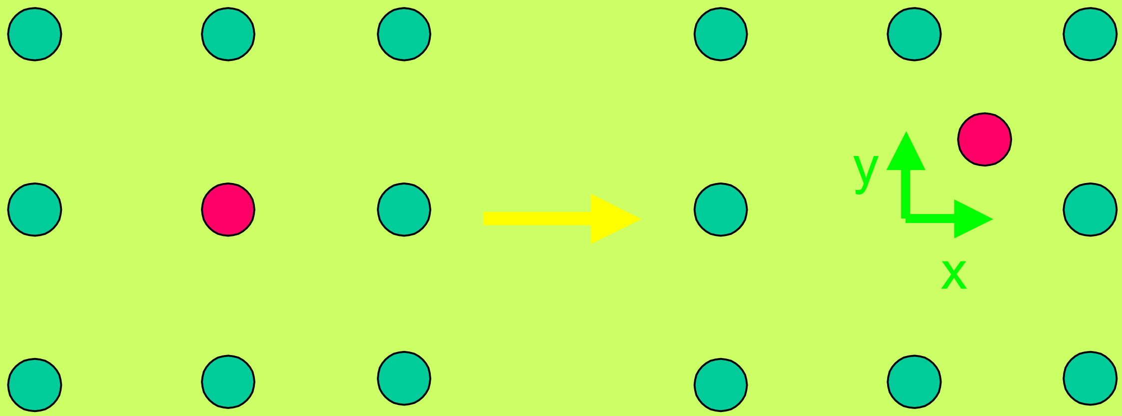
Rod phase



Slab phase



Bubble phase



A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)

Energy transfer between core and surface:

$$\frac{\partial T}{\partial t} = D\nabla^2 T; \quad D = \frac{\kappa}{C_V}$$

$$C_V = ?$$

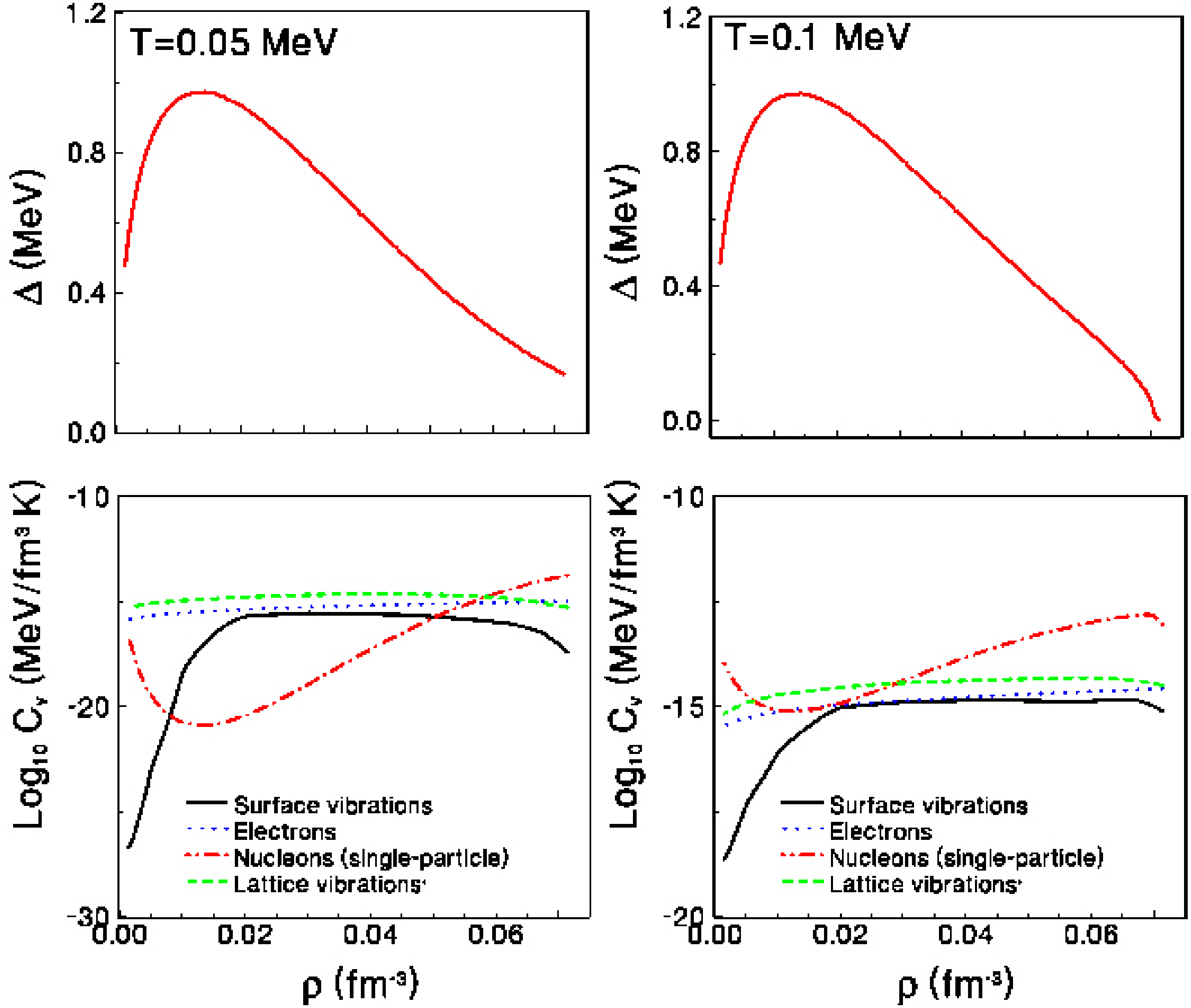
What are the basic degrees of freedom of the neutron-proton-electron matter at subnuclear densities?

Estimates of various contributions to the specific heat in the crust

Characteristic temperature: $T \sim 0.1\ MeV$

Electrons:	$C_V \approx T/\varepsilon_F$
Superfluid uniform nuclear liquid:	$C_V \approx (\Delta/T)^{3/2}\, \Delta \exp(-\Delta/T)$
Lattice vibrations:	$C_V \approx 3N$
Nuclear shape vibrations:	$C_V \approx (2l+1)N$

Contributions to the specific heat of the inner crust



Conclusions

- **There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.**
- **Thermal and electric conductivities of the inner crust are expected to be modified. In particular, the contributions coming from Umklapp processes have to be recalculated using the renormalized ion masses.**
- **Due to the coupling between the nuclear surface vibrations and the ion lattice part of the crust is filled with non-spherical nuclei. The phase transition takes place at densities far lower than the predicted density for the transition to the exotic „pasta phases".**
- **The contribution to the specific heat associated with nuclear shape vibrations seems to be important at densities around 0.02 fm $^{-3}$ where the pairing correlations are predicted to reach their maximum.**
- **Quantum corrections (Casimir energy) to the ground state energy of an` inhomogeneous neutron matter at the bottom of the crust are of the same magnitude or larger than the energy differencies between spherical, „spaghetti", and „lasagna" phases.**
- **The "pasta phase" might have a rather complex structure, various shape can coexist, and at the same time significant lattice distortions are likely and the bottom of the neutron star crust could be on the verge of a disordered phase.**

Open questions:

- Basic degrees of freedom of the „pasta phase”?
- Influence on the cooling curve of neutron stars?
- The role of isovector nuclear modes?
- Mechanical properties of the crust? Liquid crystal?
- The physics of superfluid vortices in the inner crust?