Nonequilibrium properties of ultracold atomic gases

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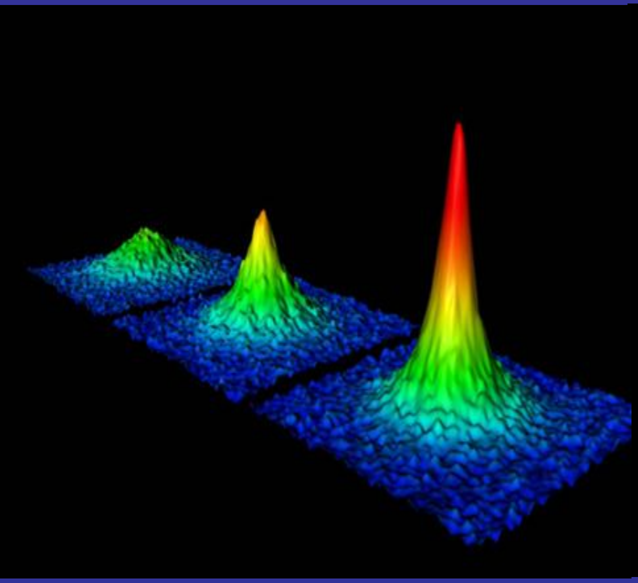
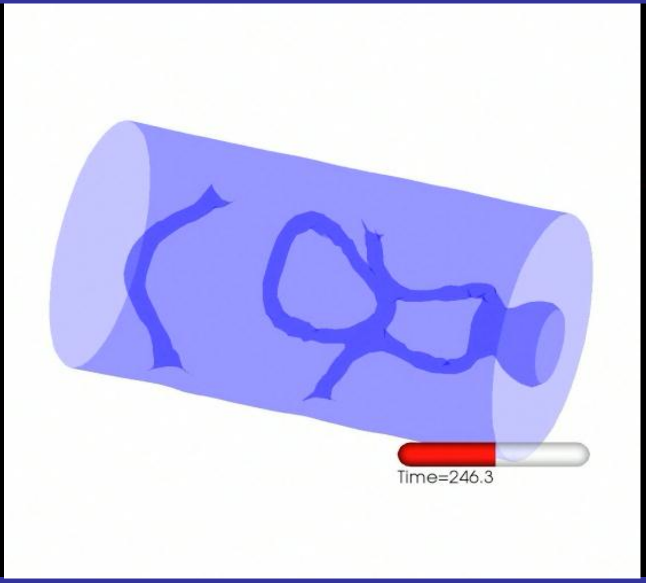

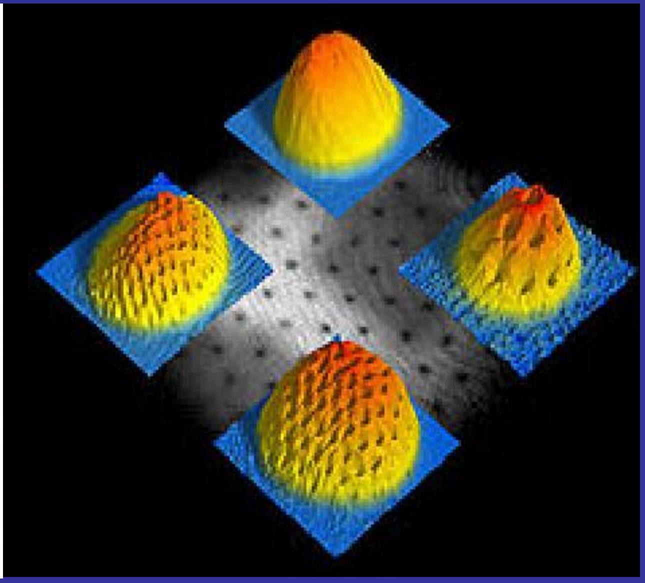

Collaborators:

A. Bulgac - Seattle
J.E. Drut - LANL
T. Lähde - Helsinki
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K.J. Roche - PNNL
G. Wlazłowski - Seattle/Warsaw
Y. Yu - Wuhan

What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$\boxed{n\, r_0^3 \ll 1}$ $\boxed{n\, |a|^3 \gg 1}$

n - particle density
a - scattering length
r_0 - effective range

$$\boxed{\text{i.e. } r_0 \to 0, a \to \pm\infty}$$

NONPERTURBATIVE REGIME

System is dilute but strongly interacting!

Universality: $E = \xi_0 E_{FG} \quad \text{for } T = 0$

$\xi_0 = 0.376(5)$ - **Exp. estimate**

E_{FG} - **Energy of noninteracting Fermi gas**

$$\text{ENERGY:}\quad E(x)=\frac{3}{5}\xi(x)\varepsilon_F N;\quad x=\frac{T}{\varepsilon_F}$$

$$\text{PRESSURE:}\quad P=-\frac{\partial E}{\partial V}=\frac{2}{5}\xi(x)\varepsilon_F\frac{N}{V}$$

$$PV=\frac{2}{3}E$$

Note the similarity to the ideal Fermi gas

$$\lim_{k\to\infty} n(k)=\frac{C}{k^4},\qquad C\ \text{- contact}$$

$$E=\sum_{\sigma=\pm1}\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}\left(n_\sigma(k)-\frac{C}{k^4}\right)$$

Total energy of the system

$$\left(\frac{dE}{d(1/a_s)}\right)_S=-\frac{\hbar^2}{4\pi m}C$$

Adiabatic relation

C and 1/a are conjugate thermodynamic variables
1/a – „generalized force"
C – „generalize displacement" – capture physics at short length scales.

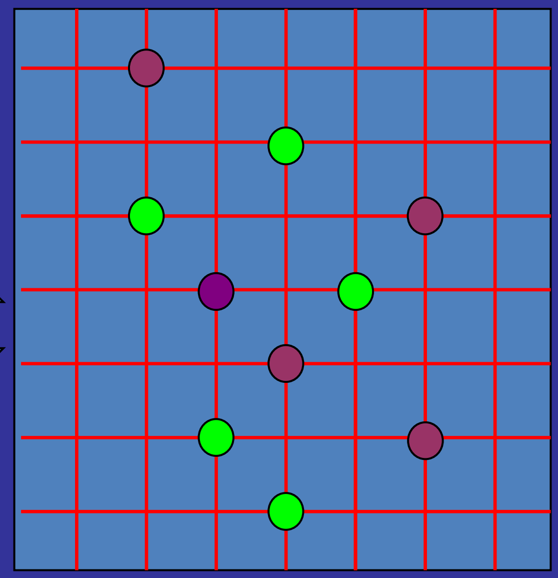
Shina Tan, Ann.Phys.323,2971(2008), Ann.Phys.323,2952(2008)
Other theory papers: Tan, Leggett, Braaten, Combescot, Baym, Blume, Werner, Castin, Randeria,Strinati,…

Theoretical approach: Fermions on 3D lattice

Coordinate space

L –limit for the spatial correlations in the system

$$k_{cut} = \frac{\pi}{\Delta x} \; ; \; \Delta x$$



$$Volume = L^3$$

$$lattice\ spacing = \Delta x$$

- Spin up fermion:

- Spin down fermion:

External conditions:

T - temperature

μ - chemical potential

Periodic boundary conditions imposed

$$\text{UV momentum cutoff} \quad \Lambda_{\text{UV}} = \frac{\pi}{\Delta x}$$

$$\text{IR momentum cutoff} \quad \Lambda_{\text{IR}} = \frac{2\pi}{L}$$

$$\frac{\hbar^2 {\Lambda_{IR}}^2}{2m} << \varepsilon_F, \; \Delta << \frac{\hbar^2 {\Lambda_{UV}}^2}{2m}$$

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \;\; \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{m k_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

Grand-canonical ensemble:

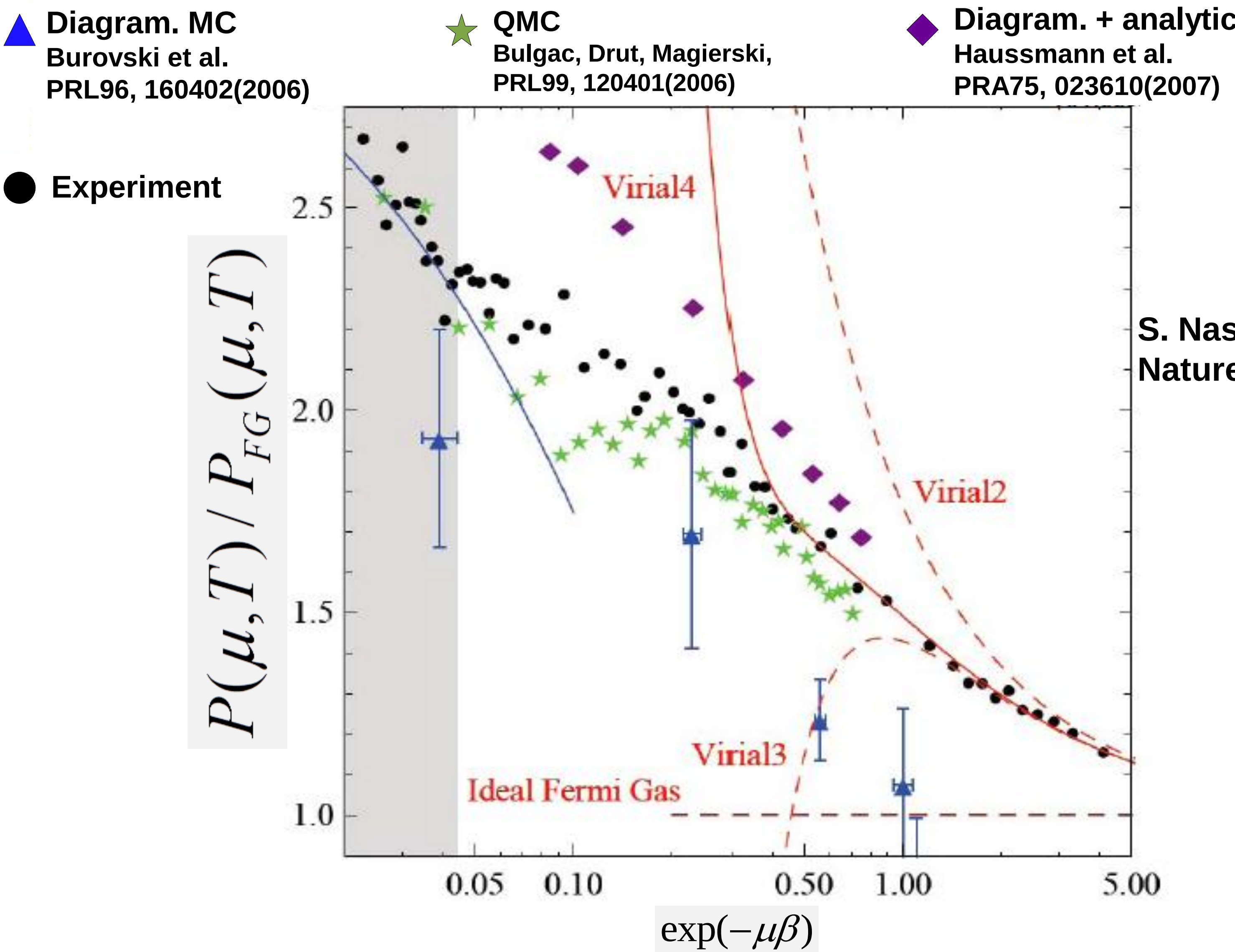
$$E(T) = \left\langle \hat{H} \right\rangle = \frac{1}{Z(T)} Tr \left\{ \hat{H} \rho(\hat{H}, \hat{N}, T) \right\}$$

$$Z(T) = Tr \left\{ \rho(\hat{H}, \hat{N}, T) \right\}; \; \rho(\hat{H}, \hat{N}, T) = e^{-\frac{1}{kT}\left(\hat{H} - \mu \hat{N} \right)}$$

$$Z(\beta) = \int D[\sigma(\vec{r}, \tau)] \; e^{\ln\{\det[1+\hat{U}(\{\sigma\})]\}}$$

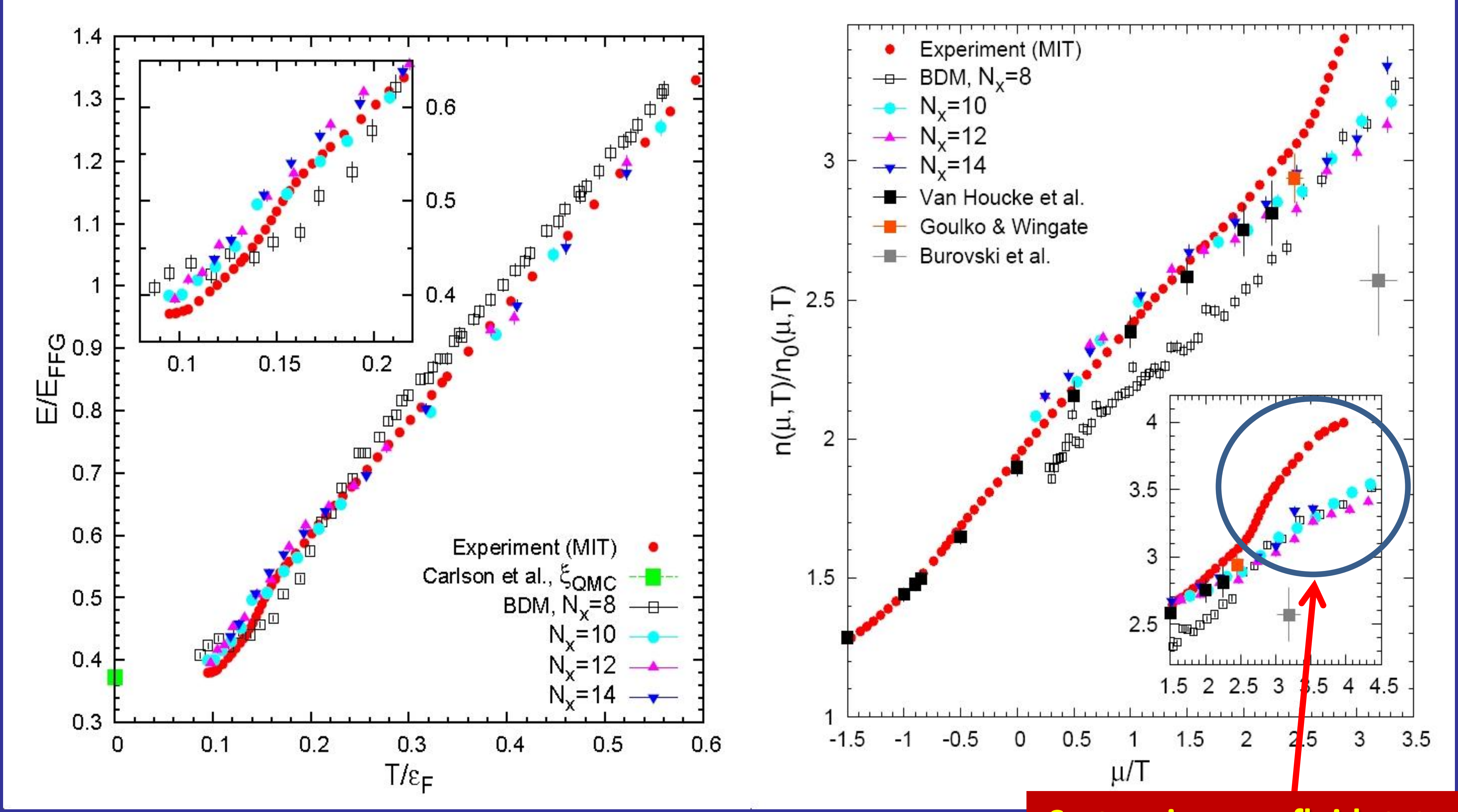
$$S[\sigma(\vec{r}, \tau)] = -\ln\{\det[1 + \hat{U}(\{\sigma\})]\}$$ - action

Comparison with Many-Body Theories (1)



S. Nascimbene et al.
Nature 463, 1057 (2010)

Equation of state of the unitary Fermi gas - current status



Experiment: M.J.H. Ku, A.T. Sommer, L.W. Cheuk, M.W. Zwierlein , Science 335, 563 (2012)

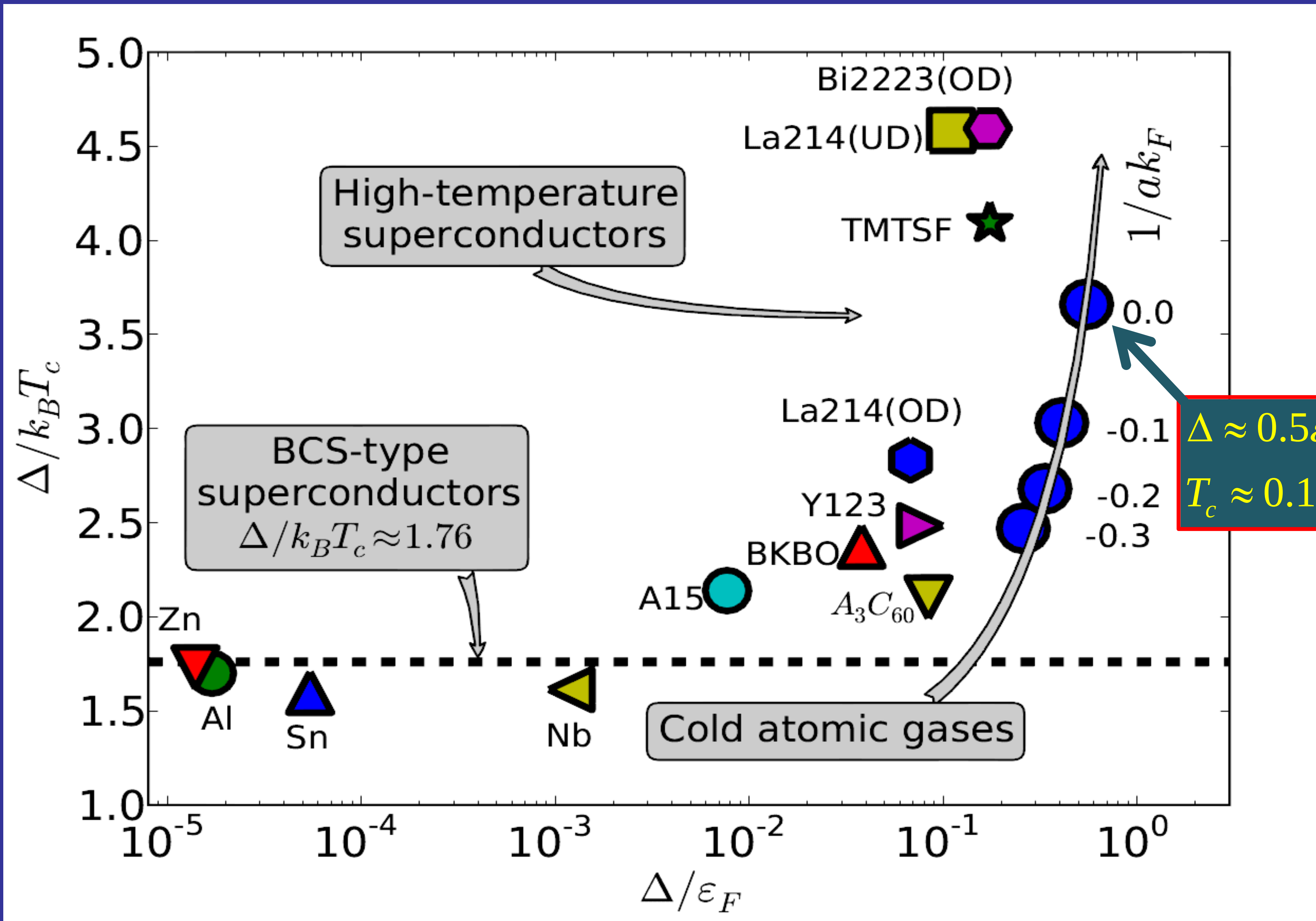
QMC (PIMC + Hybrid Monte Carlo): J.E.Drut, T.Lähde, G.Wlazłowski, P.Magierski, Phys. Rev. A 85, 051601 (2012)

System in superfluid state

Visible discrepancy between experiment and theory.

Challenge for theory!

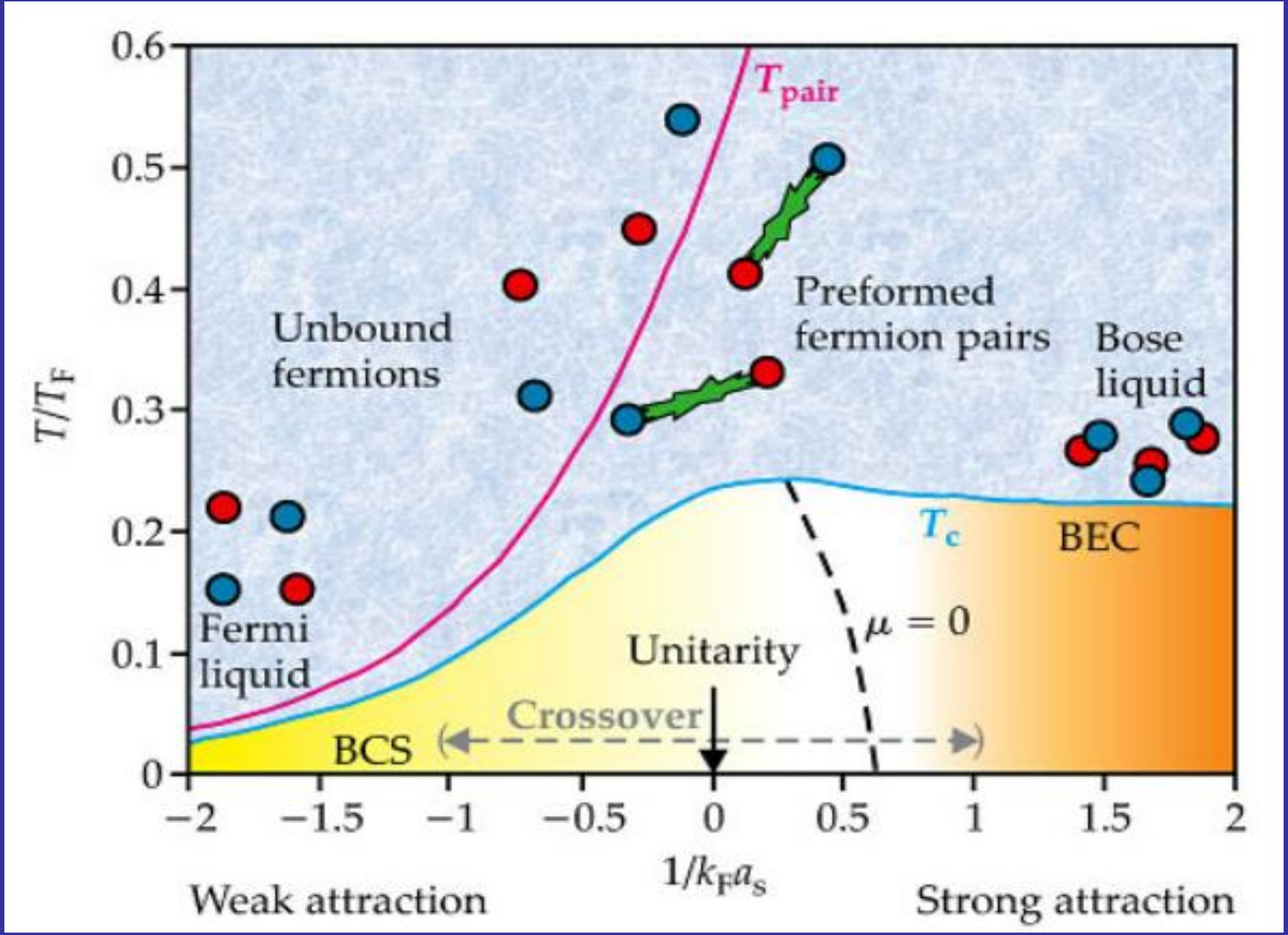
Cold atomic gases and high Tc superconductors



$\Delta \approx 0.5\varepsilon_F$

$T_c \approx 0.16\varepsilon_F$

From Fischer et al., Rev. Mod. Phys. 79, 353 (2007) & P. Magierski, G. Wlazłowski, A. Bulgac, Phys. Rev. Lett. 107, 145304 (2011)



From Sa de Melo, Physics Today (2008)

<u>Pairing pseudogap:</u> suppression of low-energy spectral weight function due to incoherent pairing in the normal state ($T > T_c$)

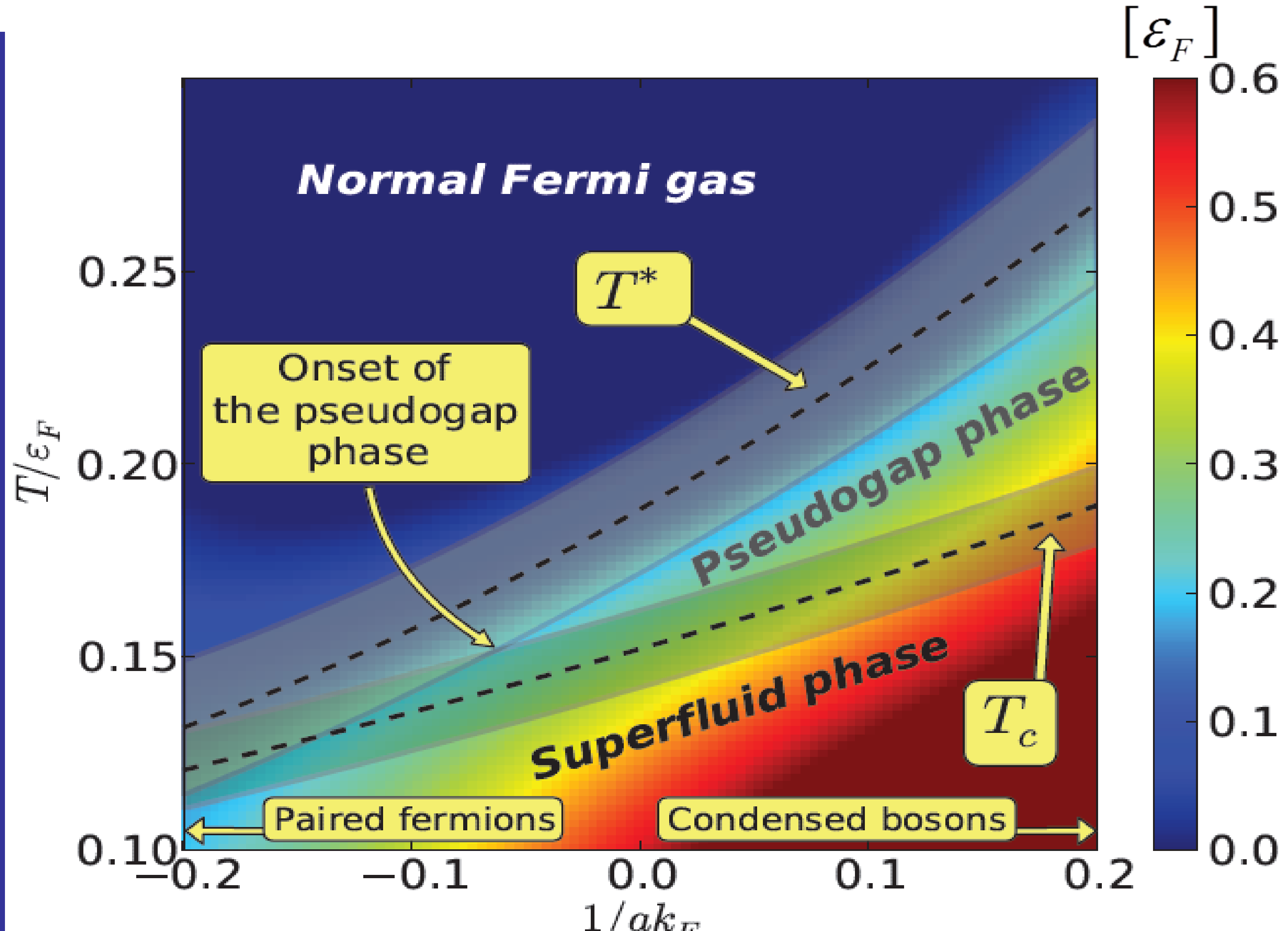
<u>Important issue related to pairing pseudogap:</u>

- Are there sharp gapless quasiparticles in a normal Fermi liquid

  YES: Landau's Fermi liquid theory;

  NO: breakdown of Fermi liquid paradigm

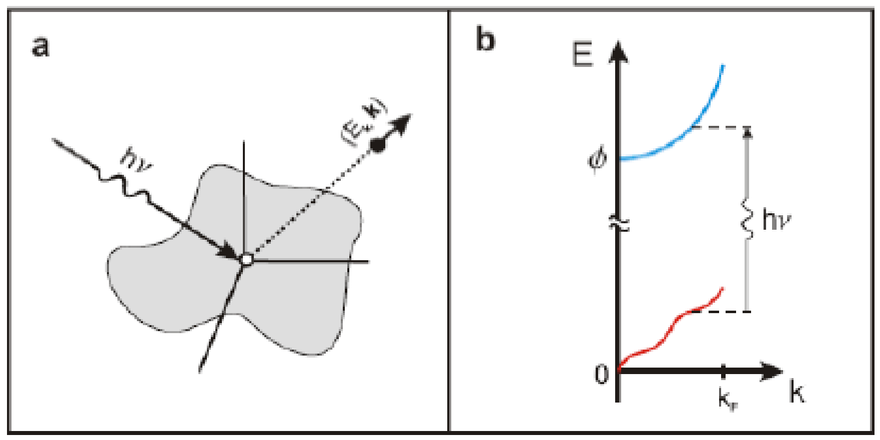
Gap in the single particle fermionic spectrum - theory



Ab initio result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.

Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.* 107,145304(2011)
Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.* 103,210403(2009)

RF spectroscopy in ultracold atomic gases

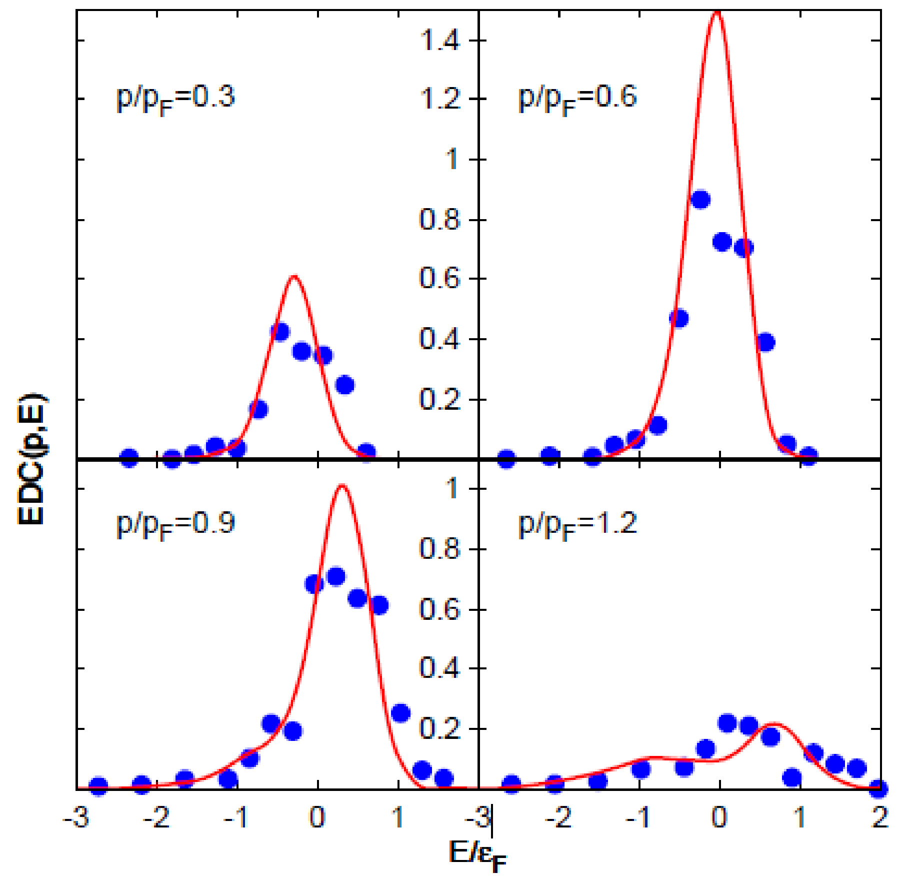


$$-E_s + h\nu = \frac{\hbar^2 k^2}{2m} + \phi$$

$$E(N) = E(N-1) + E_s$$

Stewart, Gaebler, Jin, Using photoemission spectroscopy to probe a strongly interacting Fermi gas, Nature, 454, 744 (2008)

$$\mathrm{EDC}(p,E,T) = \mathcal{C}p^2 \int_0^\infty dr\, r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E-\mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E-\mu(r)),$$



Experiment (blue dots): D. Jin's group
Gaebler et al. *Nature Physics 6, 569(2010)*
Theory (red line):
Magierski, Wlazłowski, Bulgac, Phys.Rev.Lett.107,145304(2011)

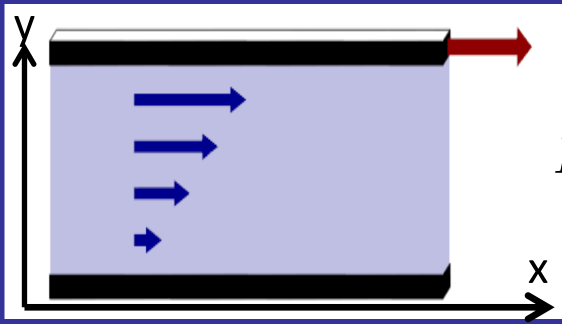
Hydrodynamics at unitarity

Scaling: $\psi_i(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N) \to \frac{1}{\lambda^{3N/2}}\psi_i\left(\vec{r}_1/\lambda,\vec{r}_2/\lambda,\ldots,\vec{r}_N/\lambda\right); E_i \to E_i/\lambda^2$

No intrinsic length scale $\Rightarrow$ **Uniform expansion keeps the unitary gas in equilibrium**

<u>Consequence:</u>
uniform expansion does not produce entropy = bulk viscosity is zero!

<u>Shear viscosity:</u>



$$F = A\eta\frac{\partial v_x}{\partial y}$$

For any physical fluid:

$$\frac{\eta}{S} \geq \frac{\hbar}{4\pi k_B}$$

KSS conjecture

Kovtun,Son,Starinets, Phys.Rev.Lett. 94, 111601, (2005)
from AdS/CFT correspondence

Maxwell classical estimate: $\eta \sim$ mean free path

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: unitary Fermi gas, quark-gluon plasma

Shear viscosity from Kubo linear response formalism

$$\eta(\omega) = \pi \frac{\rho_{xy,xy}(\vec{q}=0,\omega)}{\omega}$$

Shear viscosity

$$G_{xy,xy}(\vec{q},\tau) = \int_0^\infty \rho_{xy,xy}(\vec{q},\omega) \frac{\cosh(\omega(\tau-\beta/2))}{\sinh(\omega\beta/2)}$$

Analytic continuation

$$G_{xy,xy}(\vec{q},\tau) = \int d^3 r e^{-i\vec{q}\cdot\vec{r}} \left\langle \hat{\Pi}_{xy}(\vec{r},\tau)\hat{\Pi}_{xy}(0,0) \right\rangle$$

Stress tensor operator:

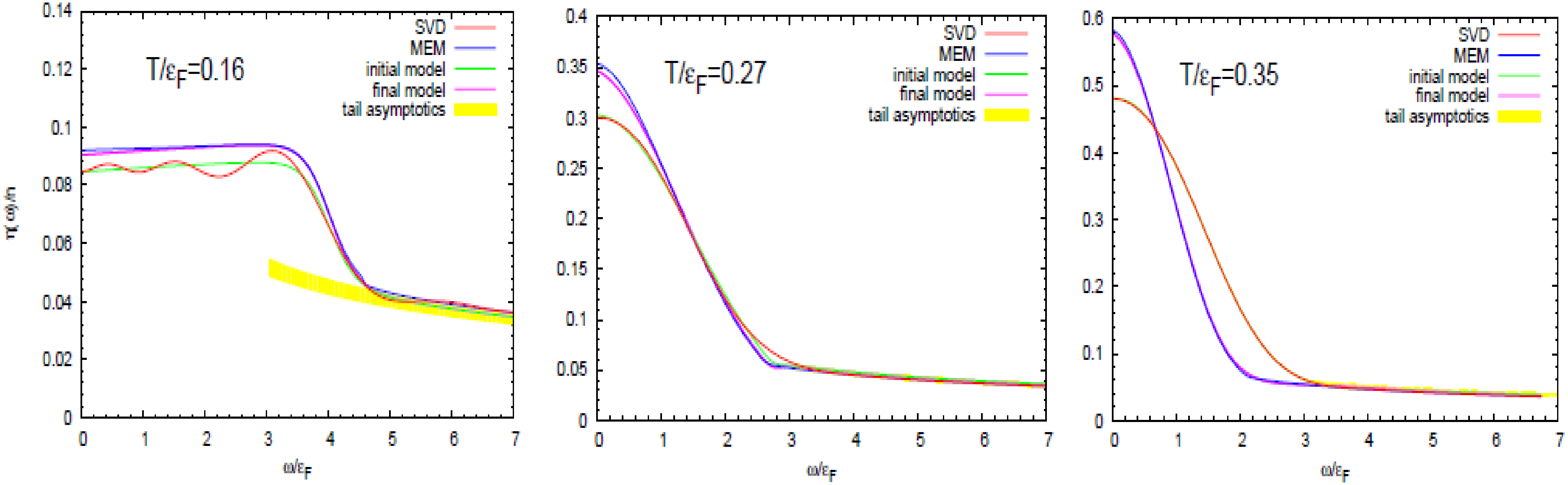
$$\partial_t \hat{j}_k(\boldsymbol{r},t) = -i[\hat{j}_k(\boldsymbol{r},t),\hat{H}] = -\partial_l \hat{\Pi}_{kl}(\boldsymbol{r},t).$$

$$i[\hat{j}_k(\boldsymbol{r}),\hat{T}] = \partial_l \hat{\Pi}^{(T)}_{kl}(\boldsymbol{r})$$

Only the kinetic part of the stress tensor
Contributes to shear viscosity

$$\hat{\Pi}^{(T)}_{kl}(\boldsymbol{r}) = \frac{1}{4mV} \sum_{\boldsymbol{p}\boldsymbol{q}\lambda} (2p_k + q_k)(2p_l + q_l) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \times \hat{a}^\dagger_\lambda(\boldsymbol{p})\, \hat{a}_\lambda(\boldsymbol{p}+\boldsymbol{q}).$$

Combined Maximum Entropy and SVD methods were applied



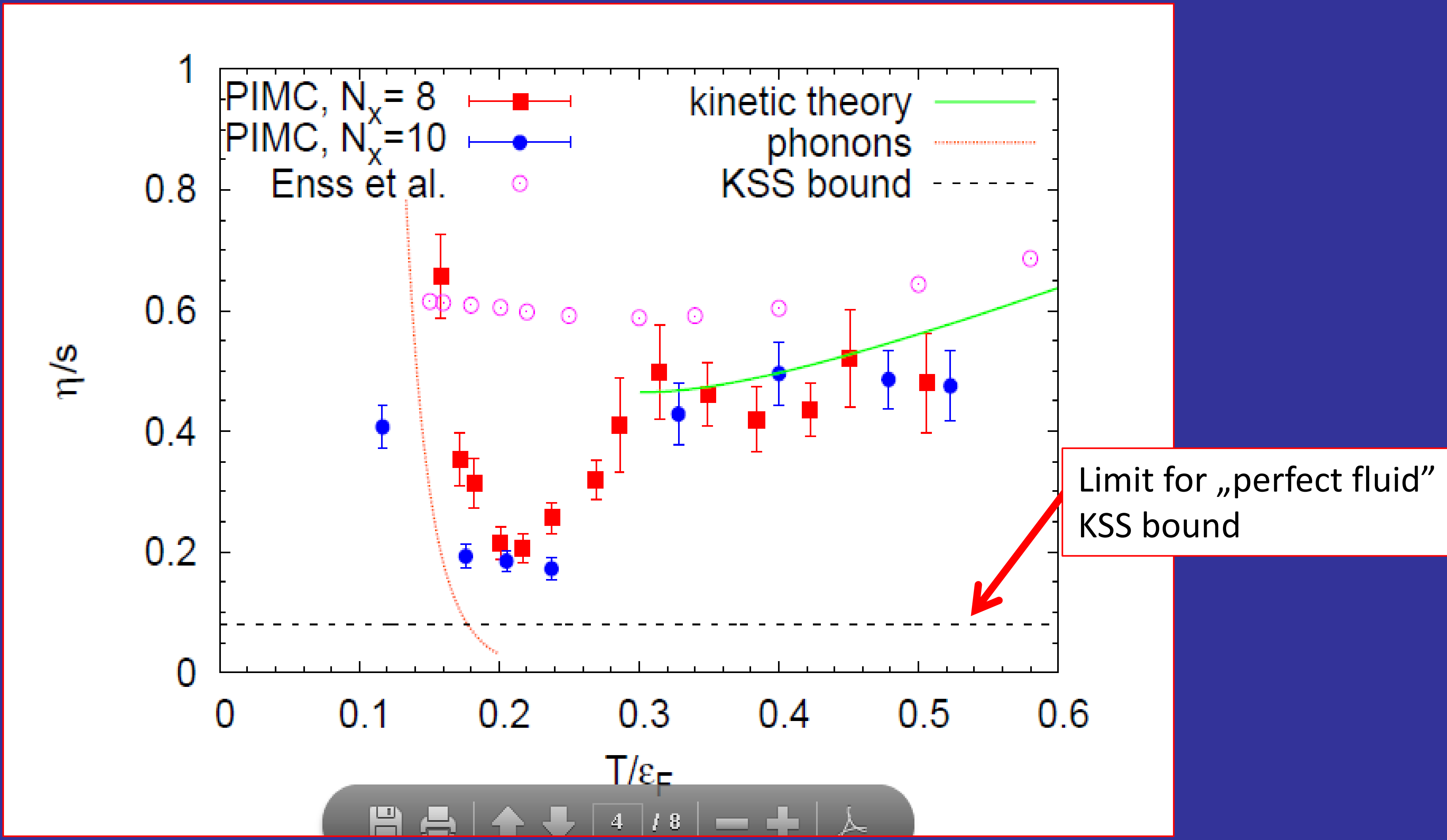
Additional symmetries and sum rules:

$$G(\tau) = G(\beta - \tau)$$

$$\frac{1}{\pi}\int_0^\infty d\omega \left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}}\right] = \frac{\varepsilon}{3},$$ *ε – energy density*

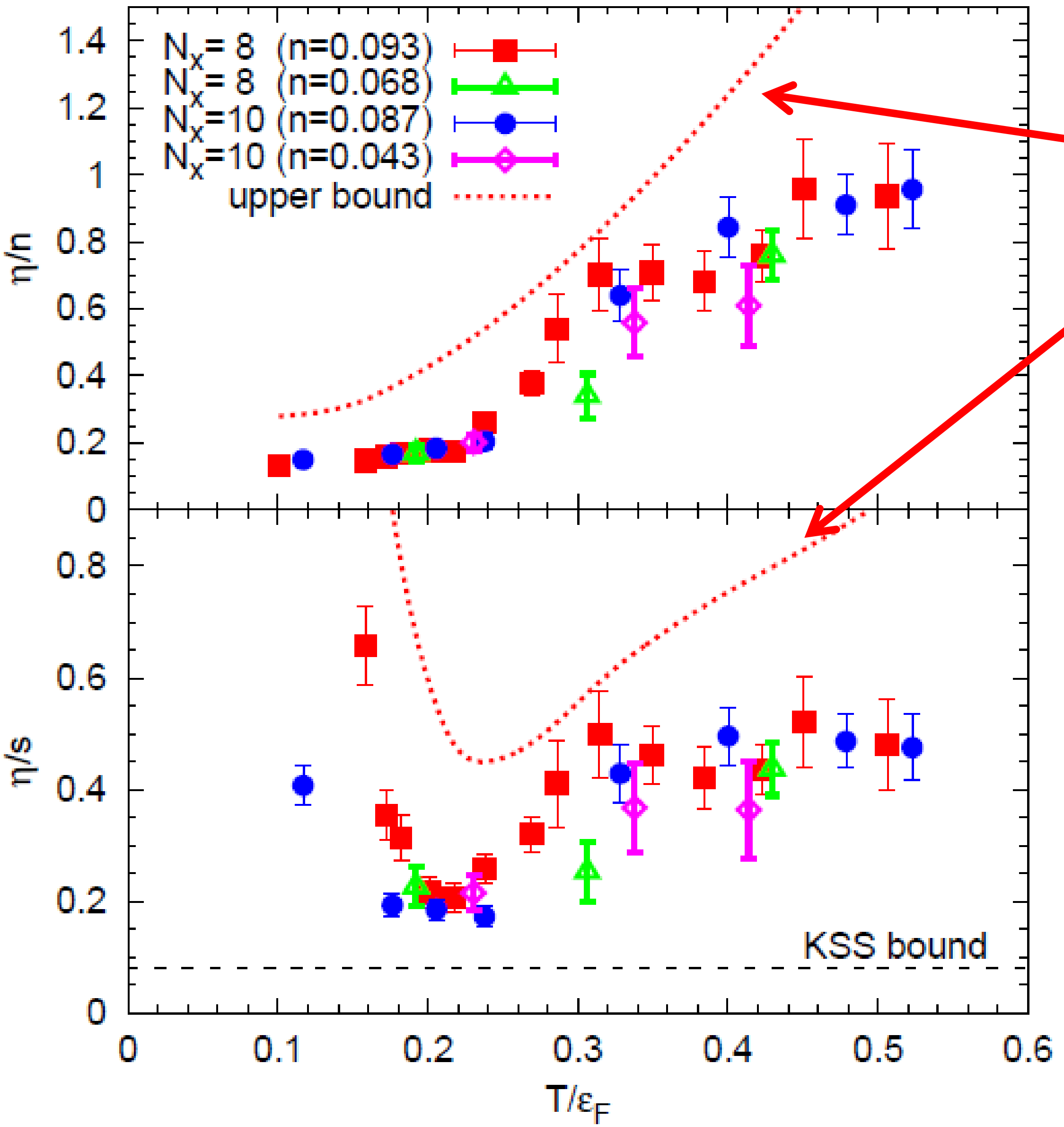
$$\eta(\omega \to \infty) \simeq \frac{C}{15\pi\sqrt{m\omega}}.$$

Shear viscosity from PIMC



Wlazłowski, Magierski, Drut, Phys. Rev. Lett. 109, 020406 (2012) -- **PIMC**

Enss, Haussman, Zwerger, Ann. Phys. 326, 770 (2011) – **T-matrix**

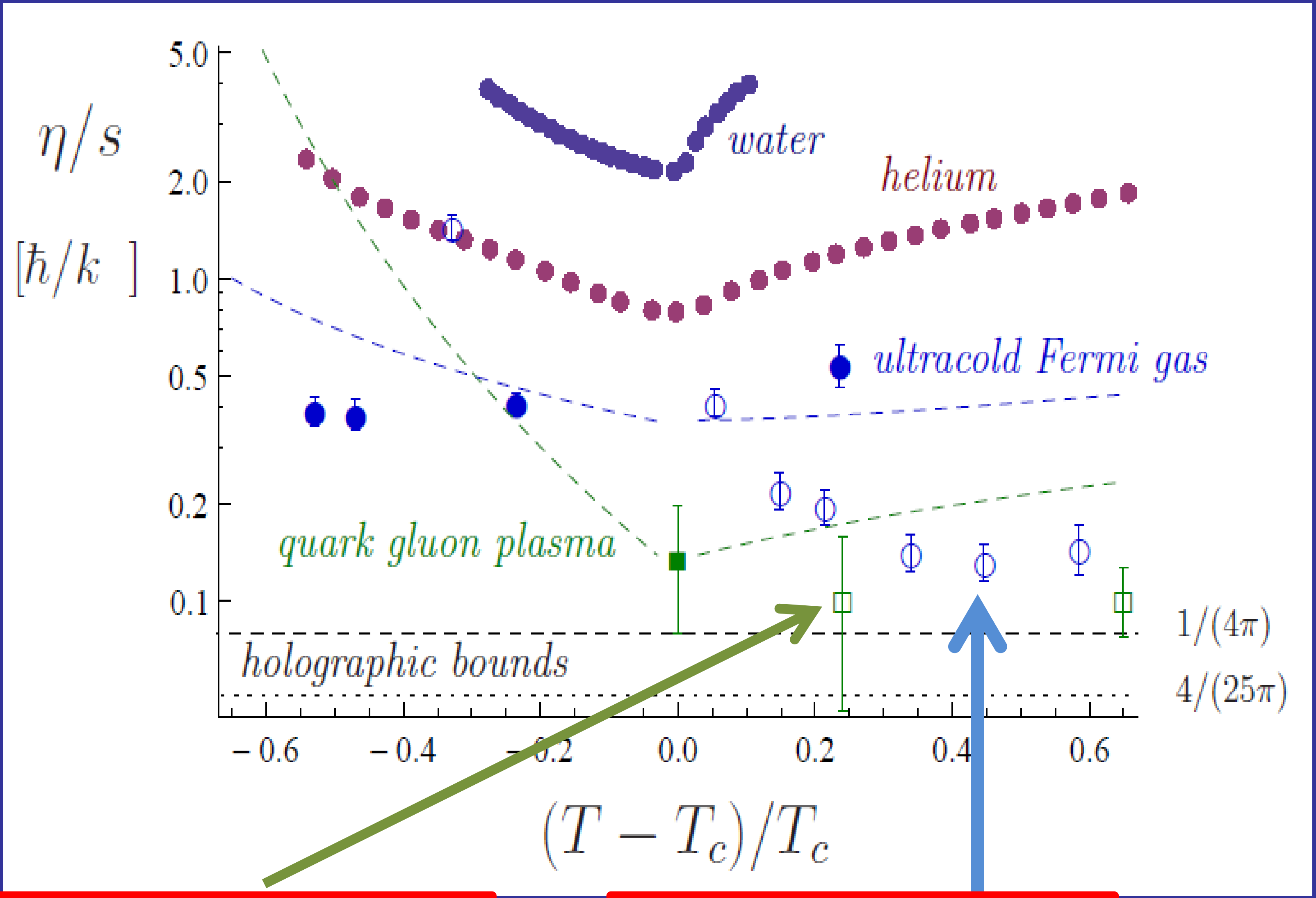


Uncertainties related to numerical analytic continuation

G.Wlazłowski, P.Magierski,J.E.Drut, Phys. Rev. Lett. 109, 020406 (2012)

Shear viscosity to entropy ratio – experiment vs. theory

(from A. Adams et al.1205.5180)



Lattice QCD (SU(3) gluodynamics):
H.B. Meyer, Phys. Rev. D 76, 101701 (2007)

QMC calculations for UFG:
G. Wlazłowski, P. Magierski, J.E. Drut,
Phys. Rev. Lett. 109, 020406 (2012)

Formalism for Time Dependent Phenomena

The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered.

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

http://www.tddft.org

$$E(t) = \int d^3r \, \left[\; \varepsilon(n(\vec{r},t),\tau(\vec{r},t),\nu(\vec{r},t),\underline{\vec{j}(\vec{r},t)}) + V_{ext}(\vec{r},t)n(\vec{r},t) + \ldots \right]$$

$$\begin{cases} [h(\vec{r},t)+V_{ext}(\vec{r},t)-\mu]\mathrm{u}_i(\vec{r},t)+[\Delta(\vec{r},t)+\Delta_{ext}(\vec{r},t)]\mathrm{v}_i(\vec{r},t) = i\hbar\dfrac{\partial \mathrm{u}_i(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t)+\Delta^*_{ext}(\vec{r},t)]\mathrm{u}_i(\vec{r},t)-[h(\vec{r},t)+V_{ext}(\vec{r},t)-\mu]\mathrm{v}_i(\vec{r},t) = i\hbar\dfrac{\partial \mathrm{v}_i(\vec{r},t)}{\partial t} \end{cases}$$

SLDA - Extension of Kohn-Sham approach to superfluid Fermi systems

$$E_{gs} = \int d^3 r \varepsilon(n(\vec{r}), \tau(\vec{r}), \nu(\vec{r}))$$

$$n(\vec{r}) = 2\sum_k |\mathrm{v}_k(\vec{r})|^2, \quad \tau(\vec{r}) = 2\sum_k |\vec{\nabla}\mathrm{v}_k(\vec{r})|^2$$

$$\nu(\vec{r}) = \sum_k \mathrm{u}_k(\vec{r})\mathrm{v}_k^*(\vec{r})$$

$$\begin{pmatrix} T + U(\vec{r}) - \mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -(T + U(\vec{r}) - \mu) \end{pmatrix} \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix} = E_k \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix}$$

Mean-field and pairing field are both local fields!
(for sake of simplicity spin degrees of freedom are not shown)

There is a little problem!
The pairing field diverges.

$$\begin{cases} [h(\vec{r}) - \mu]\mathrm{u}_i(\vec{r}) + \Delta(\vec{r})\mathrm{v}_i(\vec{r}) = E_i \mathrm{u}_i(\vec{r}) \\ \Delta^*(\vec{r})\mathrm{u}_i(\vec{r}) - [h(\vec{r}) - \mu]\mathrm{v}_i(\vec{r}) = E_i \mathrm{v}_i(\vec{r}) \end{cases} \qquad \begin{cases} h(\vec{r}) = -\vec{\nabla}\dfrac{\hbar^2}{2m(\vec{r})}\vec{\nabla} + U(\vec{r}) \\ \Delta(\vec{r}) = -g_{\mathrm{eff}}(\vec{r})\nu_c(\vec{r}) \end{cases}$$

$$\boxed{\frac{1}{g_{eff}(\vec{r})} = \frac{1}{g[n(\vec{r})]} - \frac{m(\vec{r})k_c(\vec{r})}{2\pi^2\hbar^2}\left\{1 - \frac{k_F(\vec{r})}{2k_c(\vec{r})}\ln\frac{k_c(\vec{r}) + k_F(\vec{r})}{k_c(\vec{r}) - k_F(\vec{r})}\right\}}$$

$$\rho_c(\vec{r}) = 2\sum_{E_i \geq 0}^{E_c} |\mathrm{v}_i(\vec{r})|^2, \qquad \nu_c(\vec{r}) = \sum_{E_i \geq 0}^{E_c} \mathrm{v}_i^*(\vec{r})\mathrm{u}_i(\vec{r})$$

$$E_c + \mu = \frac{\hbar^2 k_c^2(\vec{r})}{2m(\vec{r})} + U(\vec{r}), \qquad \mu = \frac{\hbar^2 k_F^2(\vec{r})}{2m(\vec{r})} + U(\vec{r})$$

One has to introduce position and momentum dependent running coupling constant.

SLDA energy density functional at unitarity

$$\varepsilon(\vec r) = \left[\alpha\frac{\tau_c(\vec r)}{2} - \Delta(\vec r)\nu_c(\vec r)\right] + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\vec r)}{5}$$

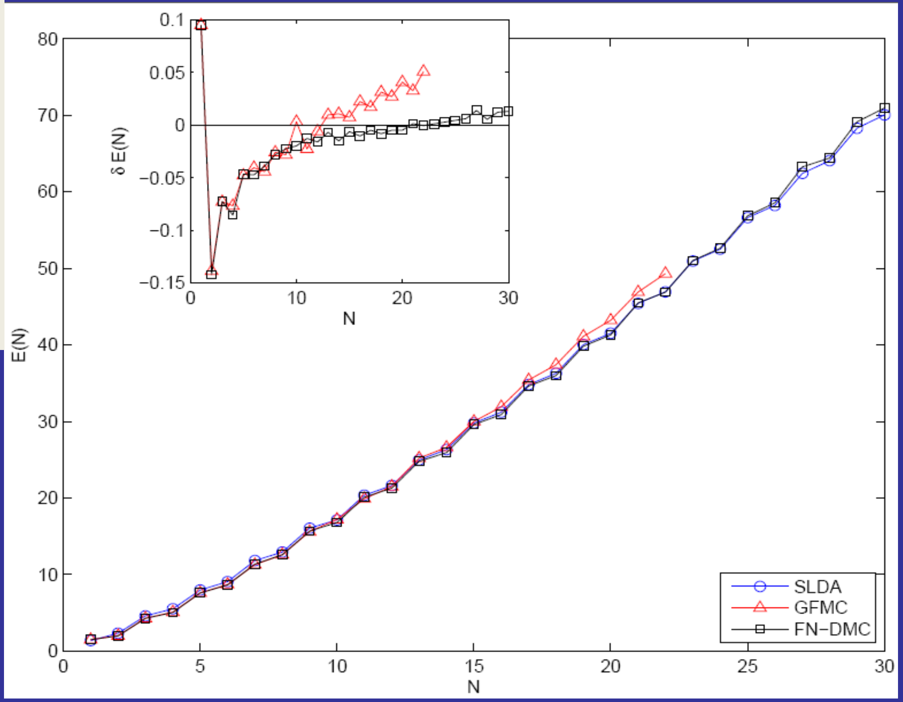
$$n(\vec r) = 2\sum_{0<E_k<E_c}\left|\mathrm{v}_k(\vec r)\right|^2, \quad \tau_c(\vec r) = 2\sum_{0<E_k<E_c}\left|\vec\nabla \mathrm{v}_k(\vec r)\right|^2,$$

$$\nu_c(\vec r) = \sum_{0<E<E_c}\mathrm{u}_k(\vec r)\mathrm{v}_k^*(\vec r)$$

$$U(\vec r) = \beta\frac{(3\pi^2)^{2/3}n^{2/3}(\vec r)}{2} - \frac{\left|\Delta(\vec r)\right|^2}{3\gamma n^{2/3}(\vec r)} + V_{ext}(\vec r)$$

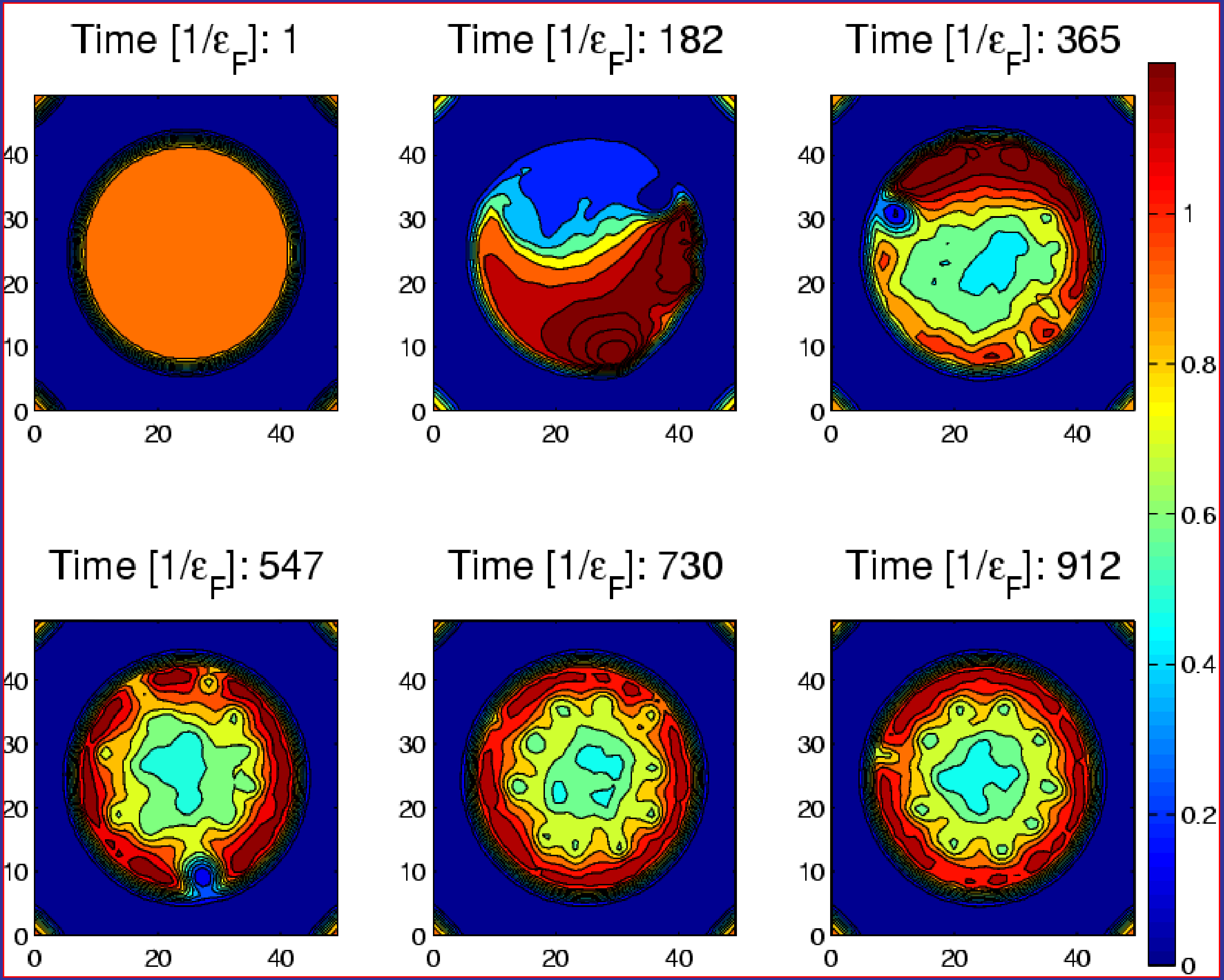
$$\Delta(\vec r) = -g_{eff}(\vec r)\nu_c(\vec r)$$

Fermions at unitarity in a harmonic trap
Total energies E(N)

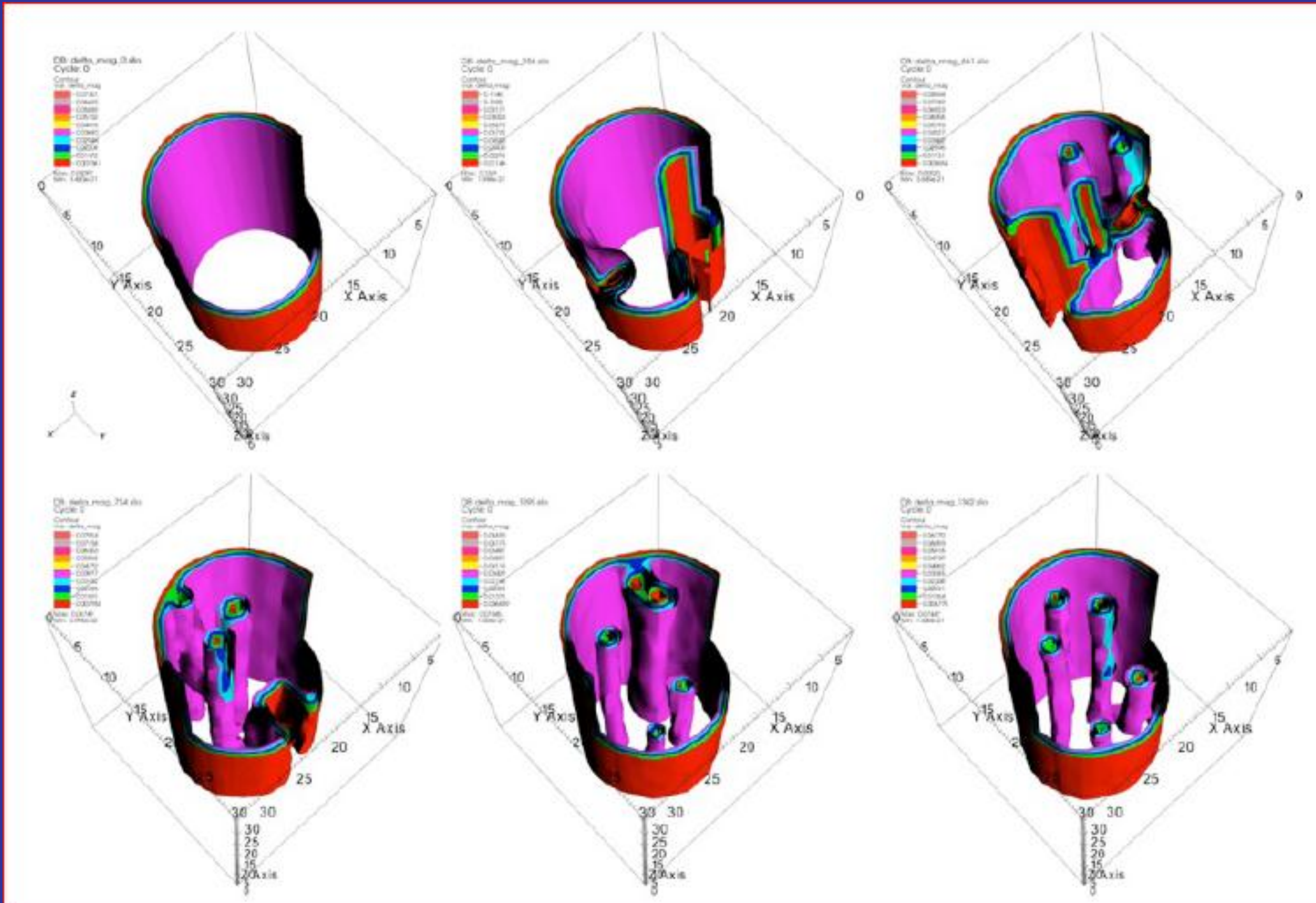


GFMC - Chang and Bertsch, Phys. Rev. A 76, 021603(R) (2007)
FN-DMC - von Stecher, Greene and Blume, PRL 99, 233201 (2007)
PRA 76, 053613 (2007)

Bulgac, PRA 76, 040502(R) (2007)



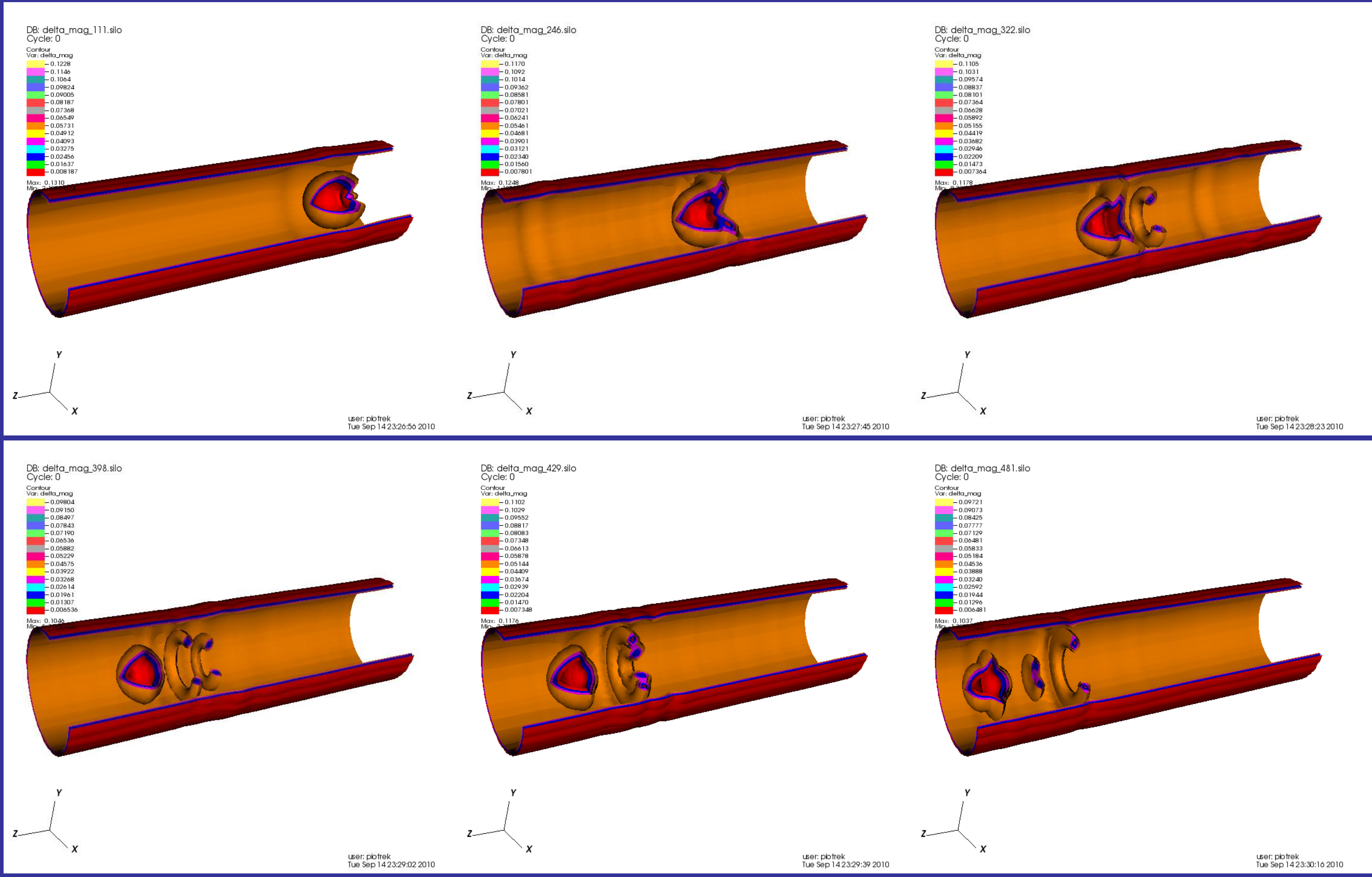
Density cut through a stirred unitary Fermi gas at various times.



Profile of the pairing gap of a stirred unitary Fermi gas at various times.

Exotic vortex topologies: dynamics of vortex rings

Heavy spherical object moving through the superfluid unitary Fermi gas



Road to quantum turbulence

Classical turbulence: energy is transfered from large scales to small scales where it eventually dissipates.

Kolmogorov spectrum: $E(k)=C\,\varepsilon^{2/3}\,k^{-5/3}$

E – kinetic energy per unit mass associated with the scale 1/k
ε - energy rate (per unit mass) transfered to the system at large scales.
k - wave number (from Fourier transformation of the velocity field).
C – dimensionless constant.

Superfluid turbulence (quantum turbulence): disordered set of quantized vortices. The friction between the superfluid and normal part of the fluid serves as a source of energy dissipation.

Problem: how the energy is dissipated in the superfluid system at small scales at T=0? - „pure" quantum turbulence

Possibility: vortex reconnections → Kelvin waves → phonon radiation

Vortex reconnections

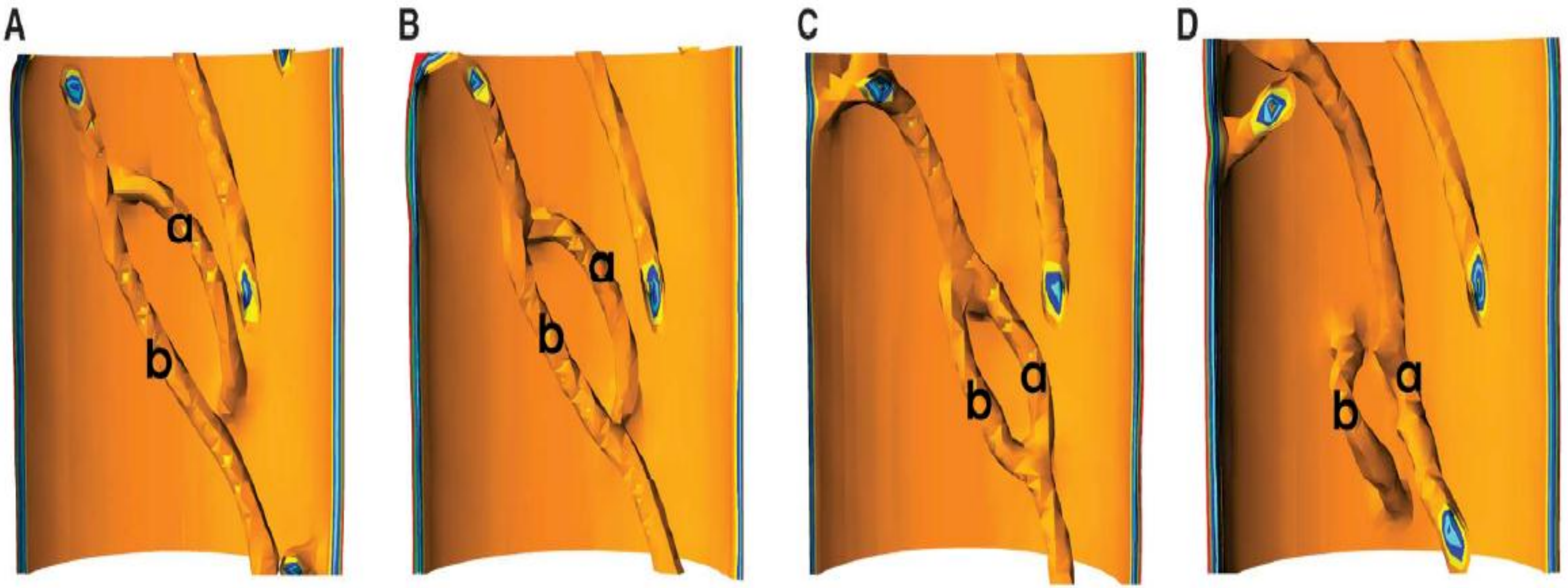


Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Summary

- The properties of the Unitary Fermi Gas (UFG) are defined by the number density and the temperature only.

- UFG demonstrates the pseudogap behavior (challenge for the finite temperature DFT).

- The first ab-initio determination of the shear viscosity for Fermi system indicates that UFG is close to being a perfect fluid.
  Precise experiment needed!

- In the time-dependent regime one finds an incredible rich range of new qualitative phenomena (e.g. vortex reconnections).

- Currently capable of treating large volumes with up to 40,000-50,000 fermions , and for long times, fully self-consistently and with no symmetry restrictions under the action of complex spatio-temporal external probes.