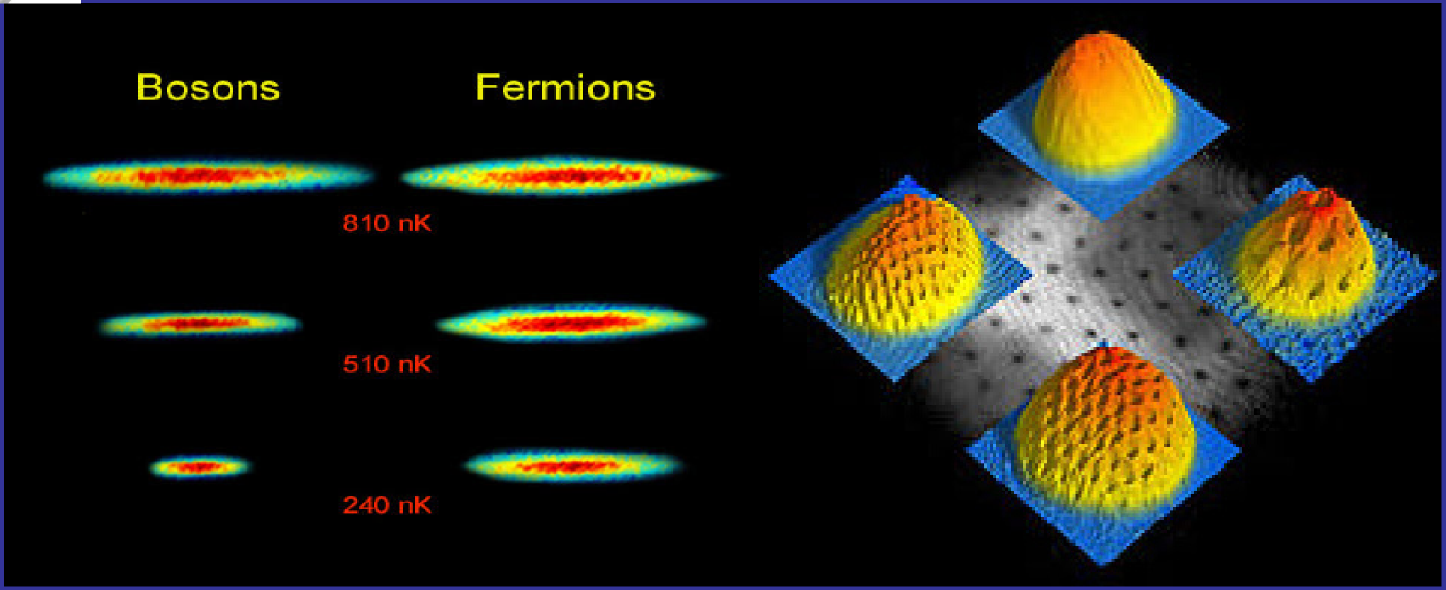
Nuclear physics with ultra cold atomic gases



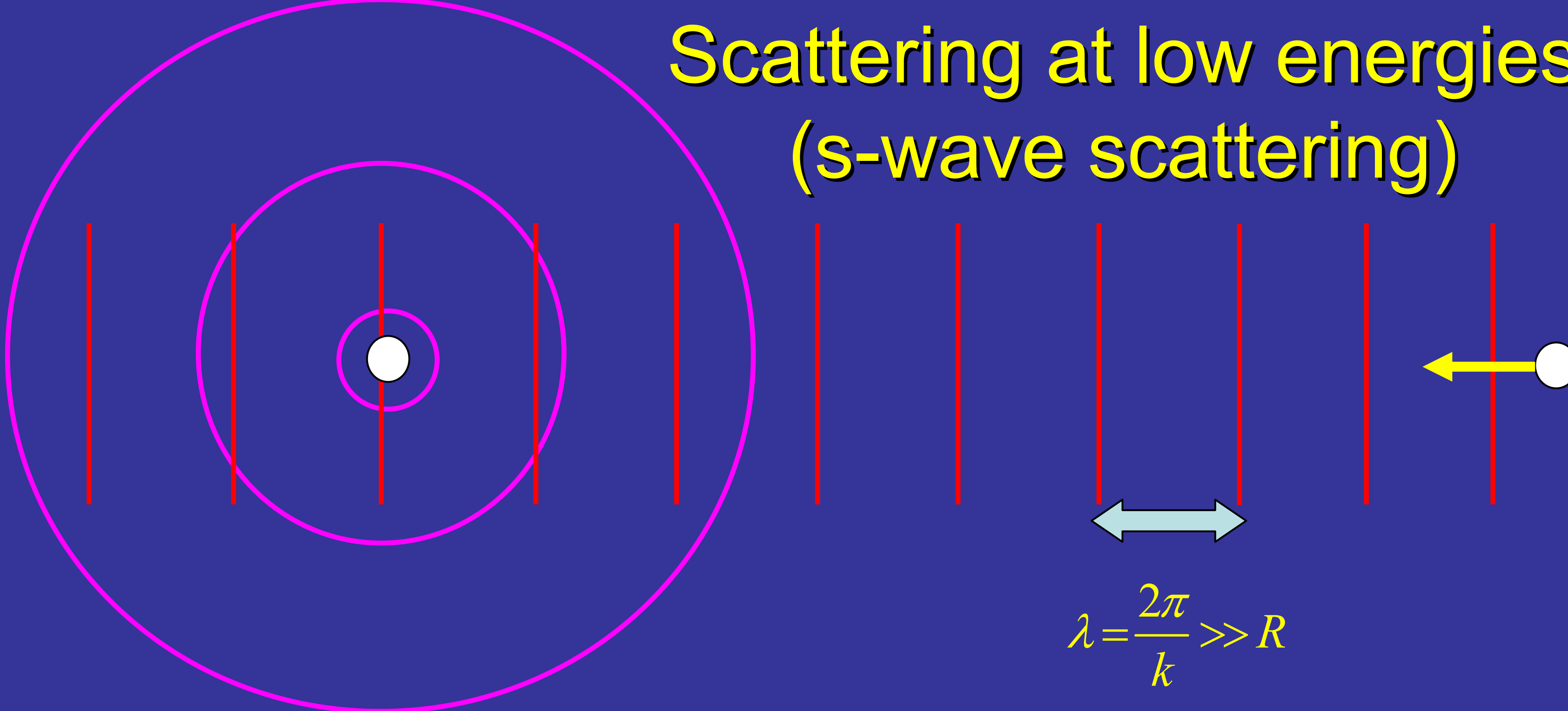
Piotr Magierski (Seattle, Warsaw)

Collaborators: Aurel Bulgac (Seattle), Joaquin E. Drut (Seattle)

Outline

- Particle scattering at low energies.
- BCS-BEC crossover. What is the unitary regime?
  Neutron matter.
- How one can manipulate the two-body interaction in experiments with atomic gases?
- Theoretical approach: path integral description of strongly interacting Fermi gases.
- Equation of state for the Fermi gas in the unitary regime. Critical temperature.
- Conclusions and open questions.

Scattering at low energies (s-wave scattering)



$$\lambda = \frac{2\pi}{k} >> R$$

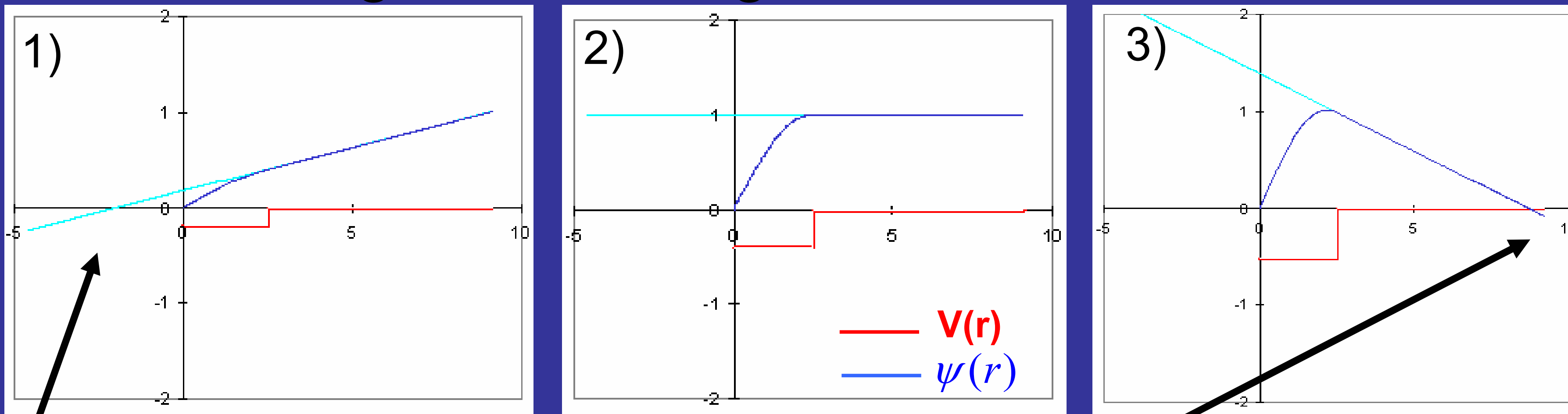
R - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f\frac{e^{ikr}}{r}; \quad f \text{ - scattering amplitude}$$

$$f = \frac{1}{-ik - \frac{1}{a} + \frac{1}{2} r_0 k^2},$$ a - scattering length, r_0 - effective range

If $k \to 0$ then the interaction is determined by the scattering length alone.

Scattering at low energies: attractive interaction



$a<0$ there is no bound state $\qquad a=\pm\infty \qquad a>0$ a bound state exists

What is the energy of the dilute Fermi gas? $\qquad E(k_F a) = ?$

$(k_F r_0 << 1)$

$\varepsilon_F = \frac{\hbar^2 k_F^{\,2}}{2m}; \quad n = \frac{k_F^{\,3}}{3\pi^2}$ **- particle density**

Perturbation series:

$$E/E_{FG} = 1 + \frac{10}{9\pi}(k_F a)\left[1 + \frac{6}{35\pi}(k_F a)(11 - 2\ln 2) + \ldots\right]$$

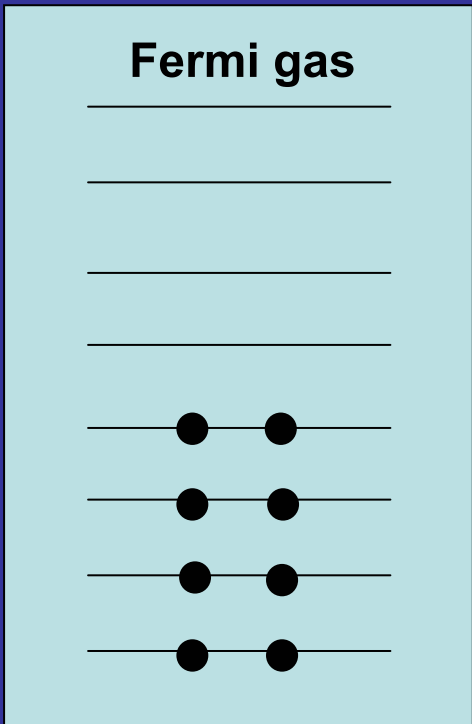
$$E_{FG} = \frac{3}{5}\varepsilon_F N$$ - Energy of the noninteracting Fermi gas

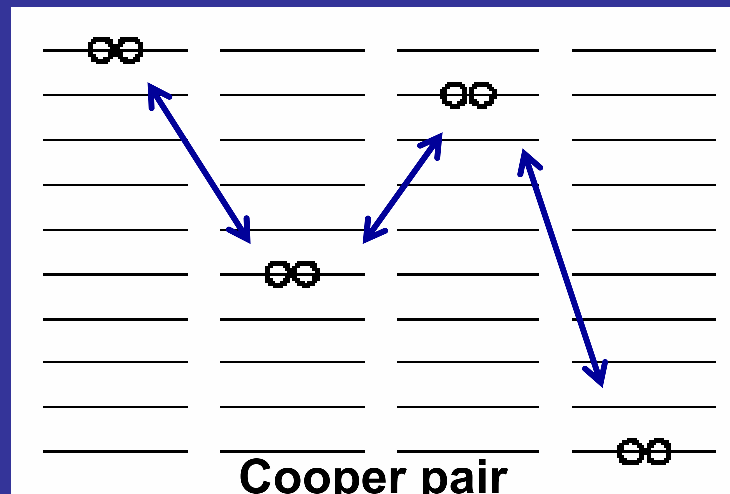
PAIRING NOT INCLUDED YET!

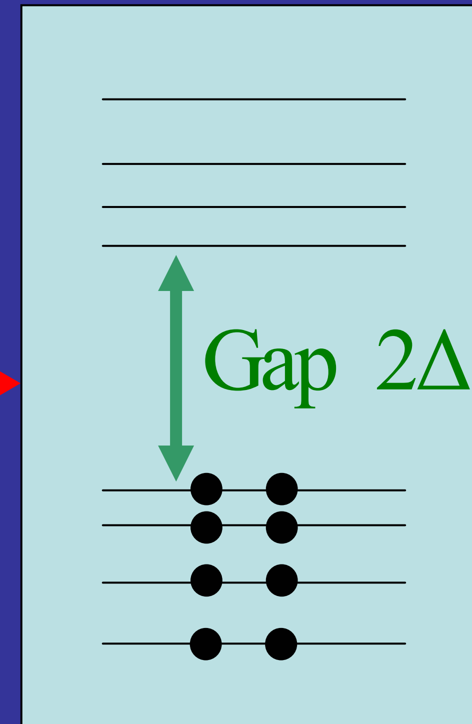
How pairing emerges?

$a < 0$

Cooper's argument (1956)







$$\Delta_{BCS} = \frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2k_F a}\right), \quad \text{iff} \quad k_F\,|a| \ll 1 \;\text{ and }\; \frac{1}{k_F} \ll \eta = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta} \;\text{ - size of the Cooper pair}$$

$$\frac{E_{HF+BCS}}{E_{FG}} = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{5}{8}\left(\frac{\Delta_{BCS}}{\varepsilon_F}\right)^2 = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{40}{e^4}\exp\left(\frac{\pi}{k_F a}\right)$$

Hartree-Fock term **BCS term**

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$i.e.\ \ r_0 \to 0,\ a \to \pm\infty$$

NONPERTURBATIVE REGIME

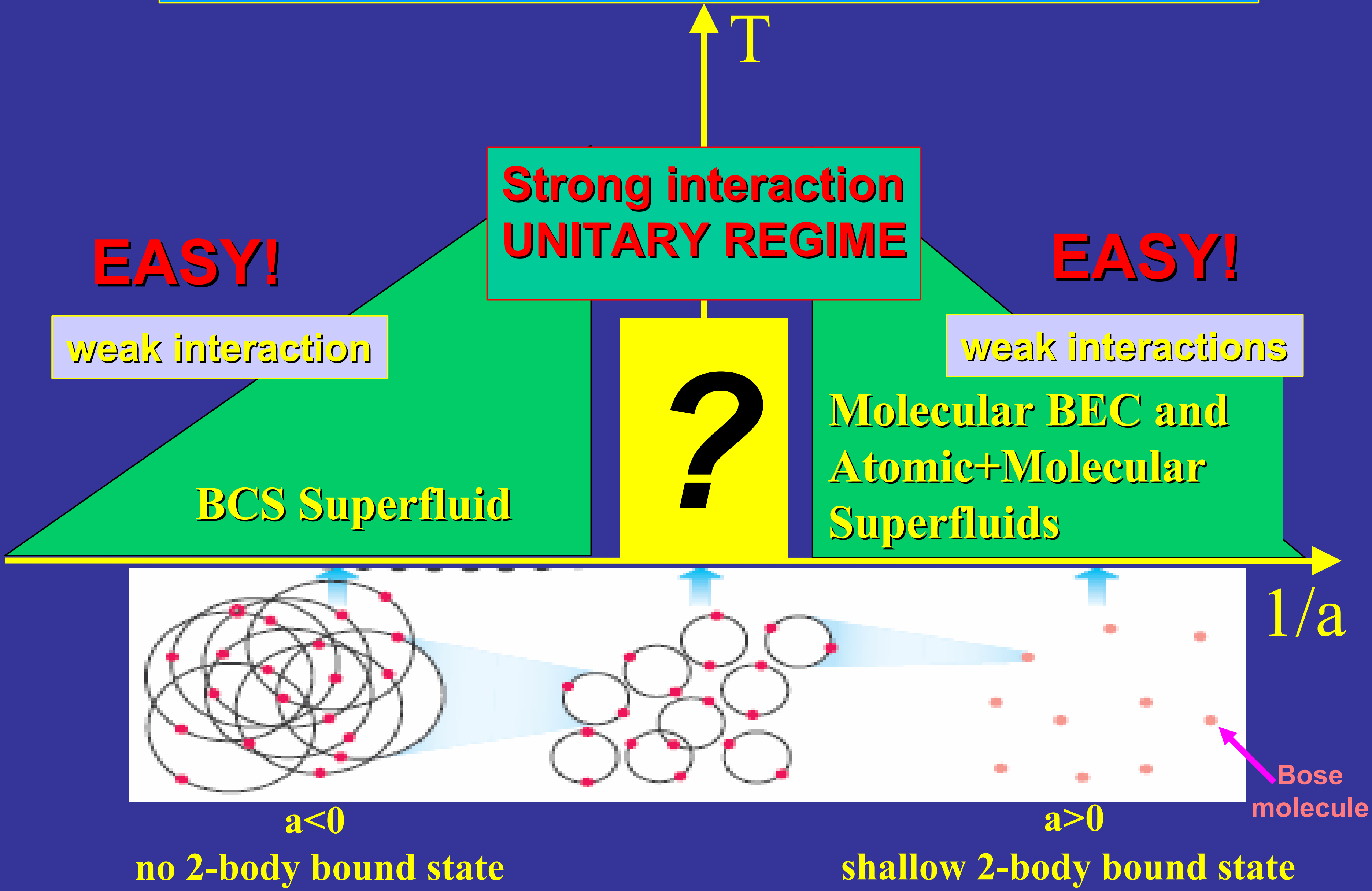
System is dilute but strongly interacting!

The only scale: $E_{FG}/N = \frac{3}{5}\varepsilon_F$

UNIVERSALITY: $$E(T) = \xi\left(\frac{T}{\varepsilon_F}\right) E_{FG}$$

QUESTIONS: **What is the shape of** $\xi(T/\varepsilon_F)$ **?**
What is the critical temperature for the superfluid-to-normal transition?
...

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



A little bit of history

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

<u>In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.</u>

- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***

- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**

- **Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems:** $\xi(T=0) \approx 0.44$

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

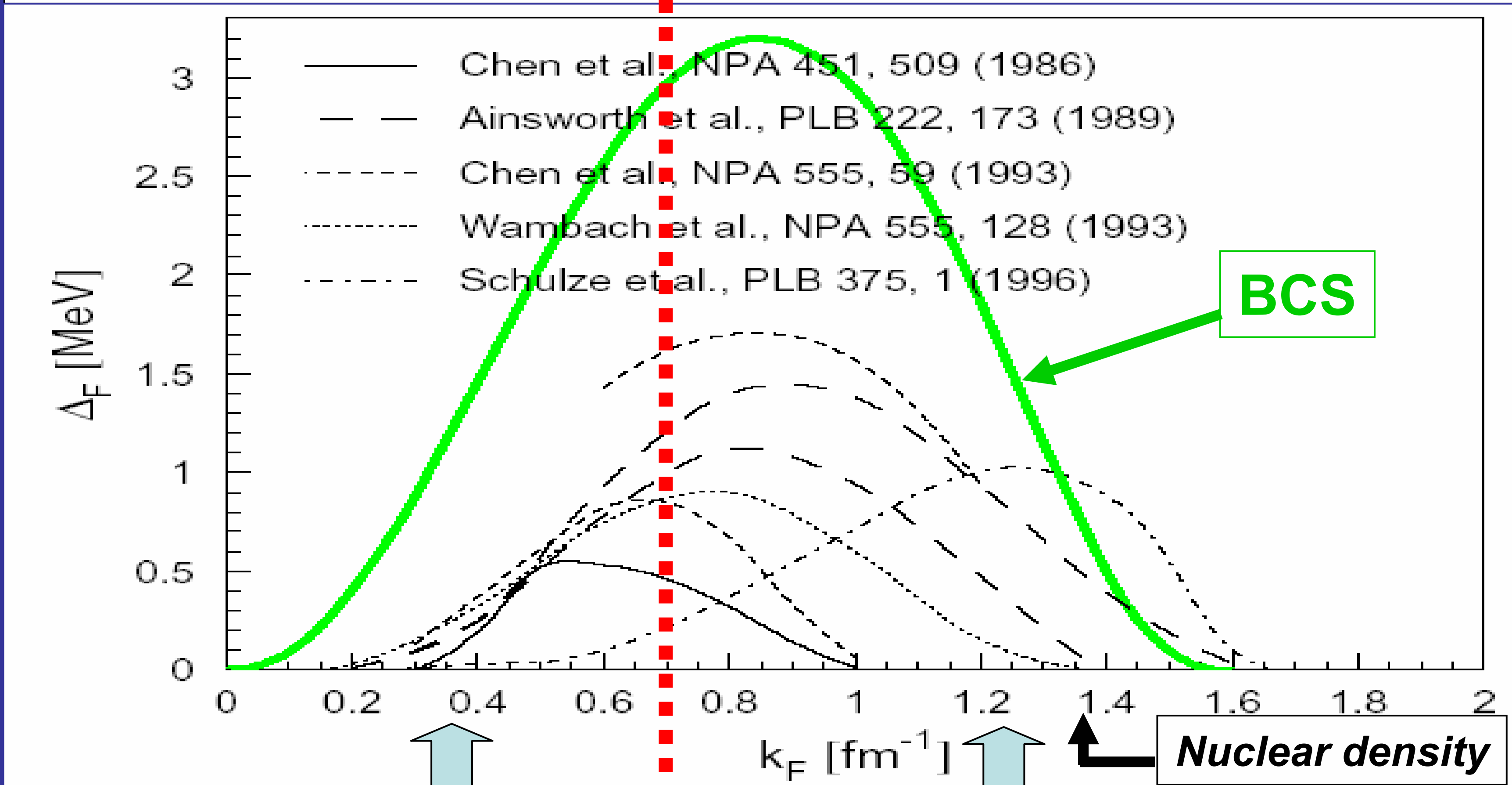
Neutron matter

Neutron-neutron scattering

Scattering length: $a \approx -18.5\ fm$

Effective range: $r_0 \approx 2.8\ fm$

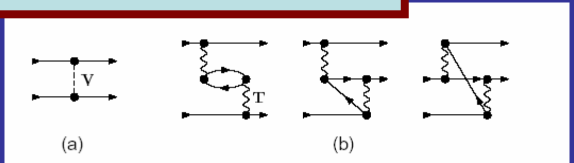
s-wave pairing gap in infinite neutron matter with realistic NN-interactions



Dilute matter: only a, r_0 matter

Details of the n-n potential matter

"Screening effects" are significant!



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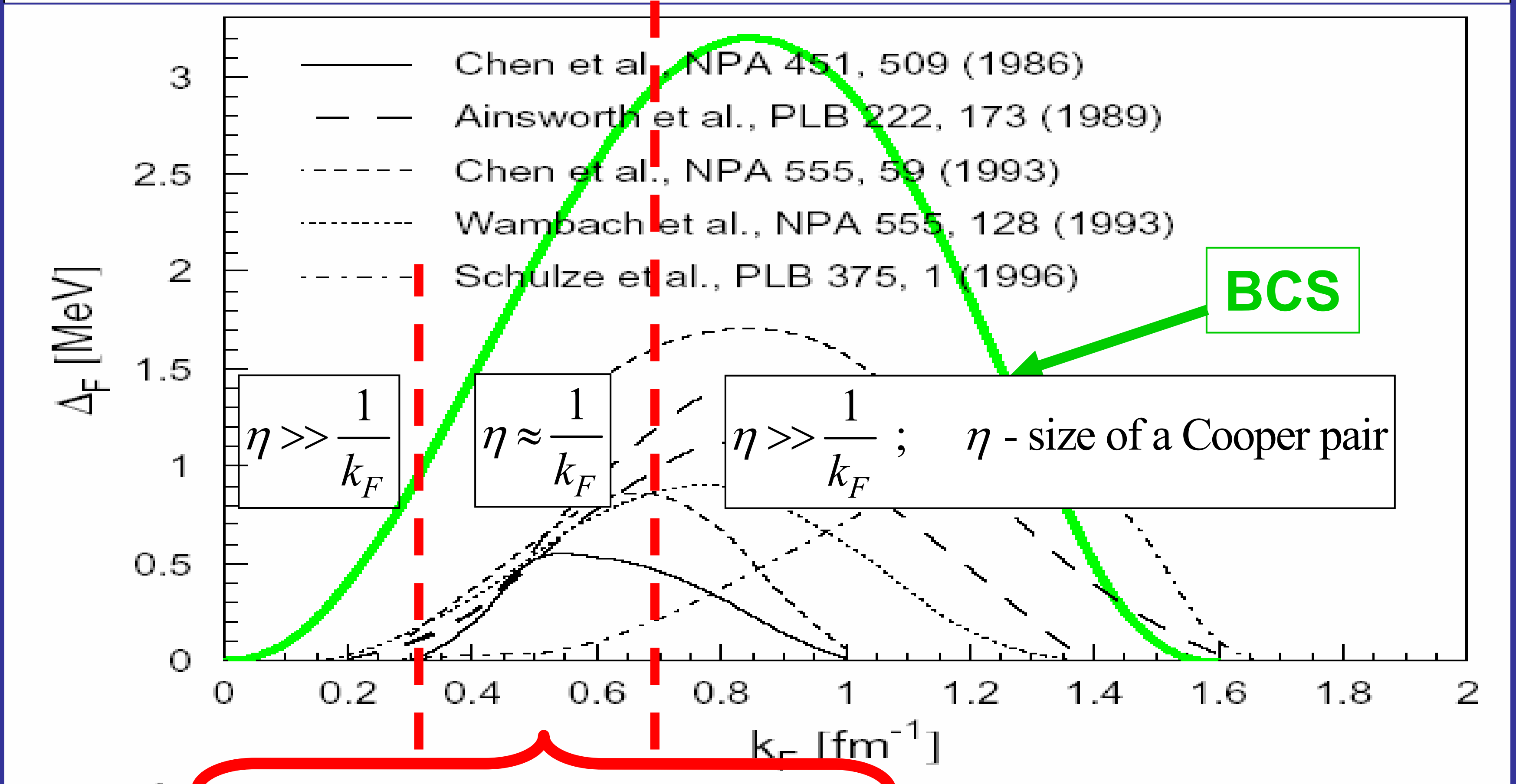


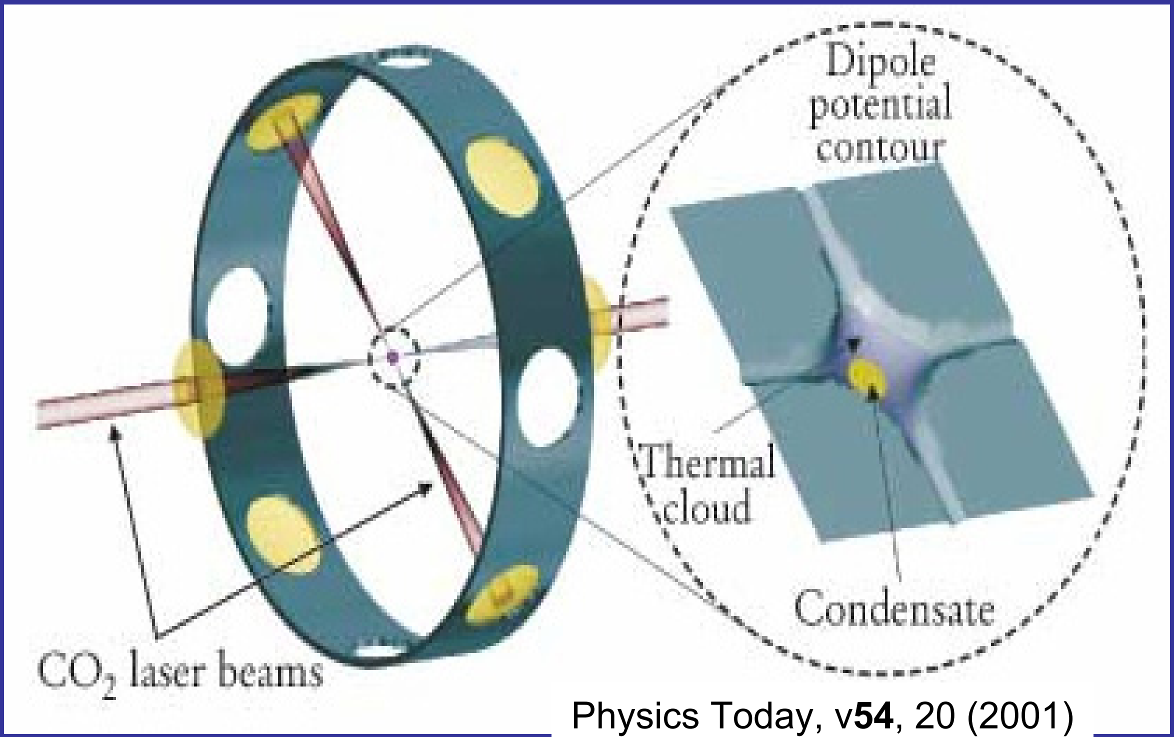
$n \approx 0.001 - 0.01\ fm^{-3}$

$k_F \approx 0.3 - 0.7\ fm^{-1}$

Close to the unitary limit

In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**

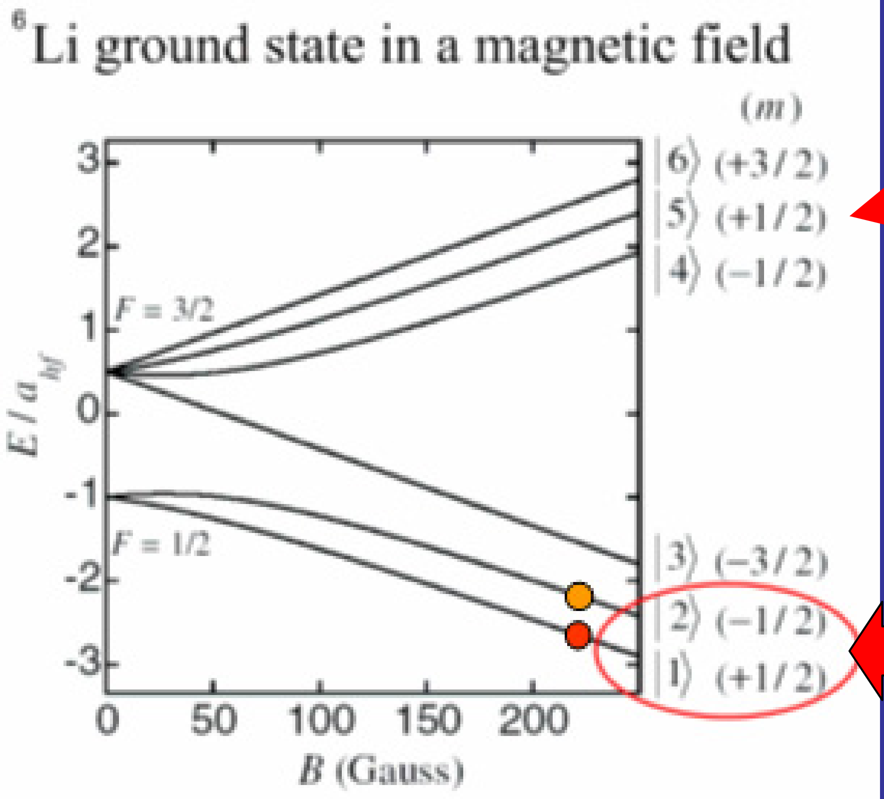


Physics Today, v**54**, 20 (2001)

Who does experiments?

- **Jin's group at Boulder**
- **Grimm's group in Innsbruck**
- **Thomas' group at Duke**
- **Ketterle's group at MIT**
- **Salomon's group in Paris**
- **Hulet's group at Rice**

One fermionic atom in magnetic field



$$|F\, m_F\rangle$$

$$\vec{F} = \vec{I} + \vec{J}\ ;\ \vec{J} = \vec{L} + \vec{S}$$

Nuclear spin   Electronic spin

Two hypefine states are populated in the trap

Collision of two atoms: At low energies (low density of atoms) only L=0 (s-wave) scattering is effective.

- Due to the high diluteness atoms in the same hyperfine state do not interact with one another.
- Atoms in different hyperfine states experience interactions only in s-wave.

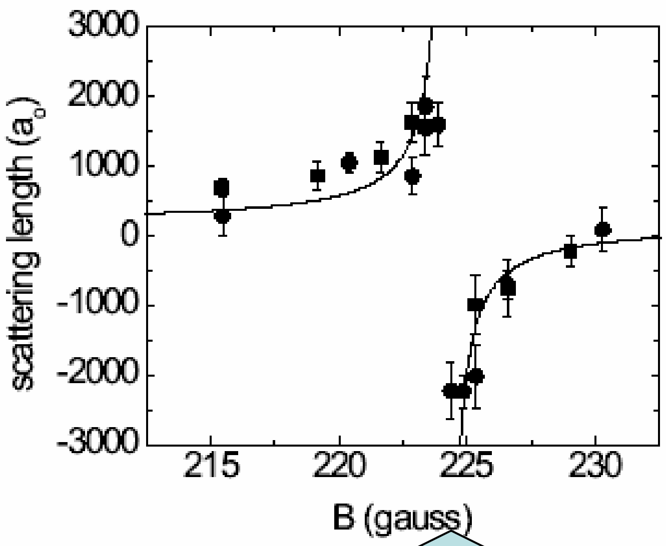
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu} + \sum_{i=1}^{2}\left(V_i^{hf} + V_i^{Z}\right) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{I}\cdot\vec{J},\quad V^{Z} = (\gamma_e J_z - \gamma_n I_z)B$$

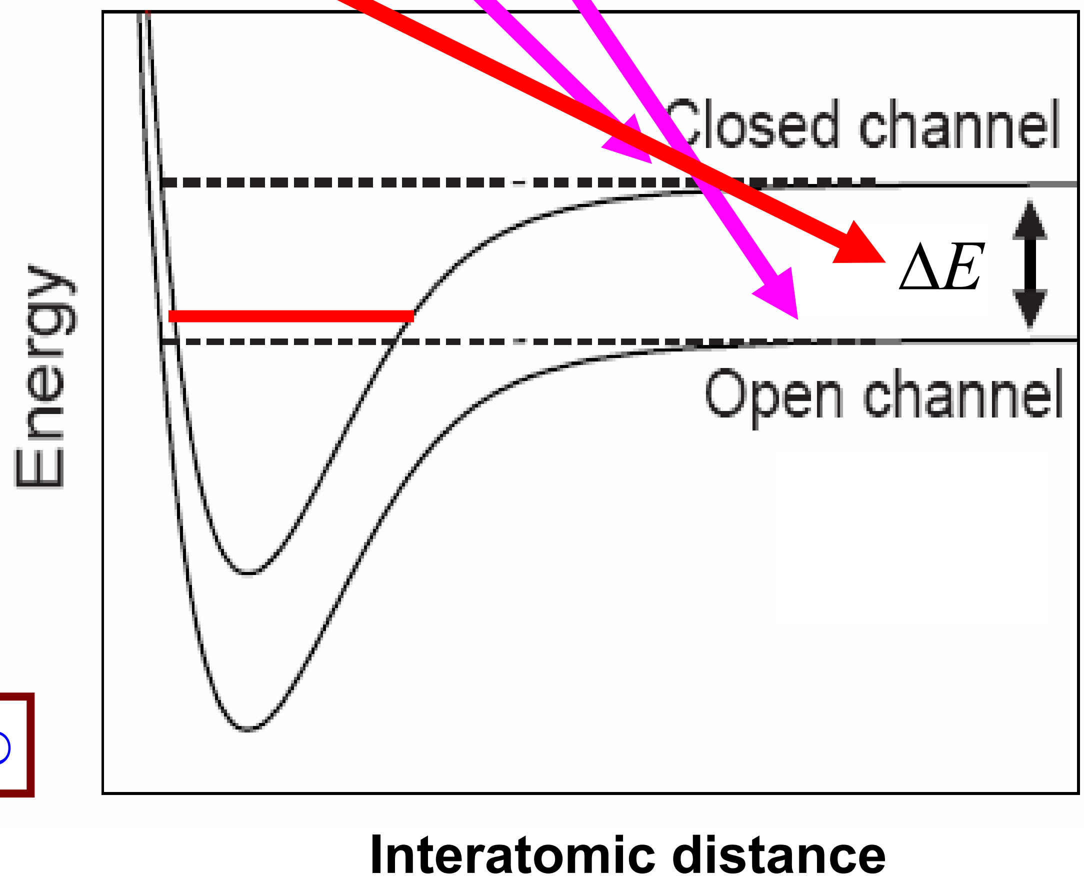
Tiesinga, Verhaar, Stoof, Phys. Rev. A47, 4114 (1993)

Channel coupling



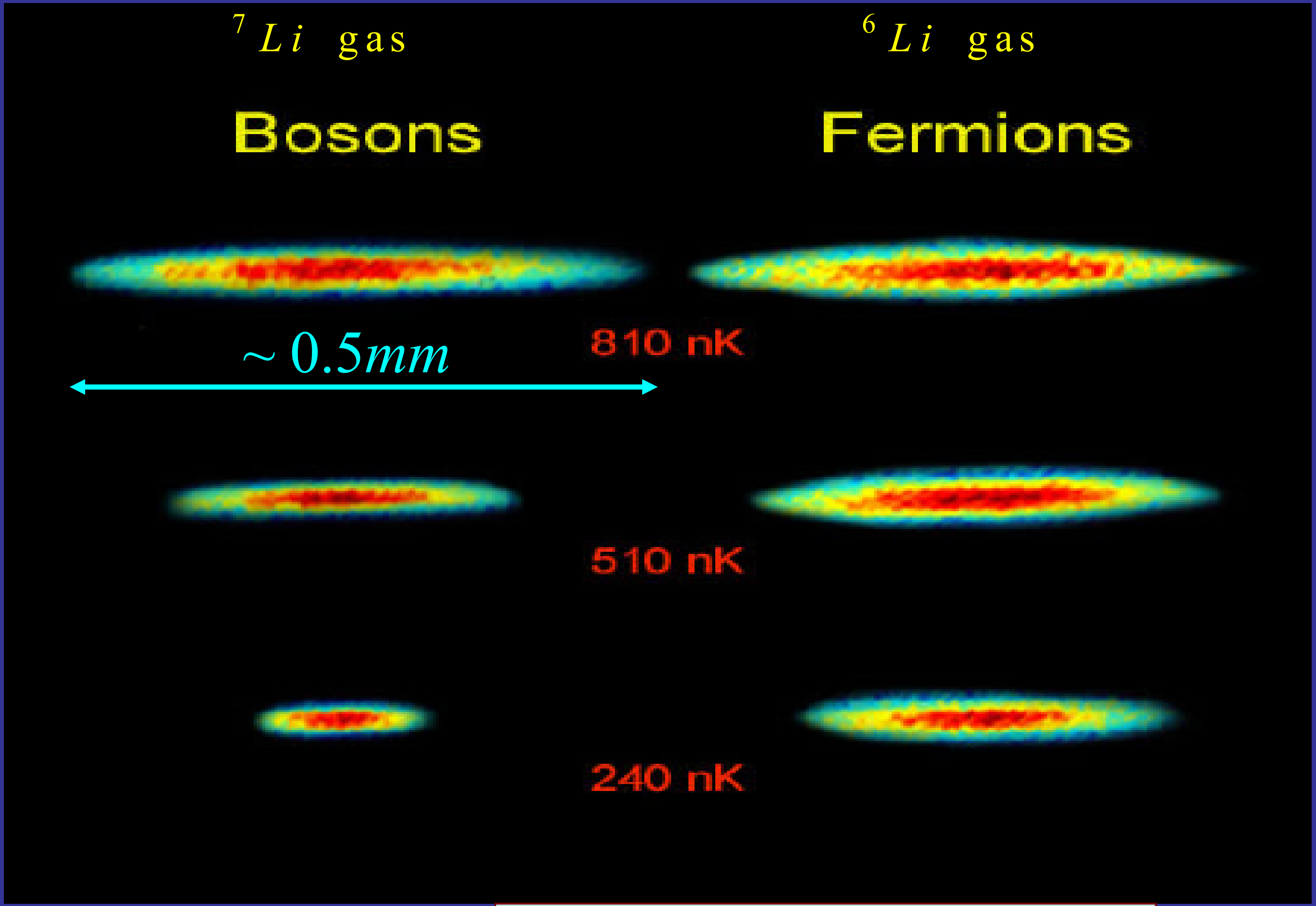
resonance: $a \to \pm\infty$

Regal and Jin, PRL 90, 230404 (2003)



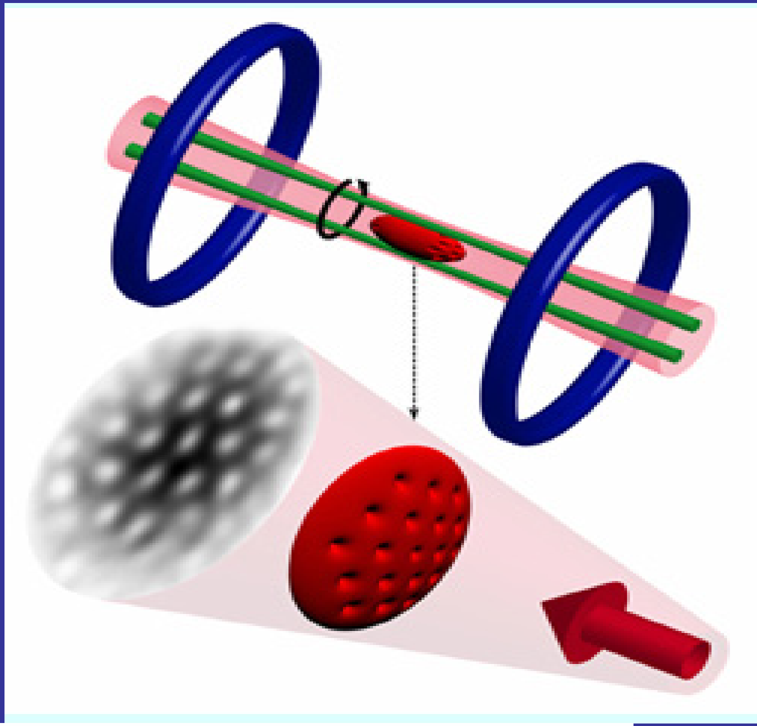
Size of the atomic cloud as a function of temperature around the critical temperature:

Pauli exclusion principle at work!



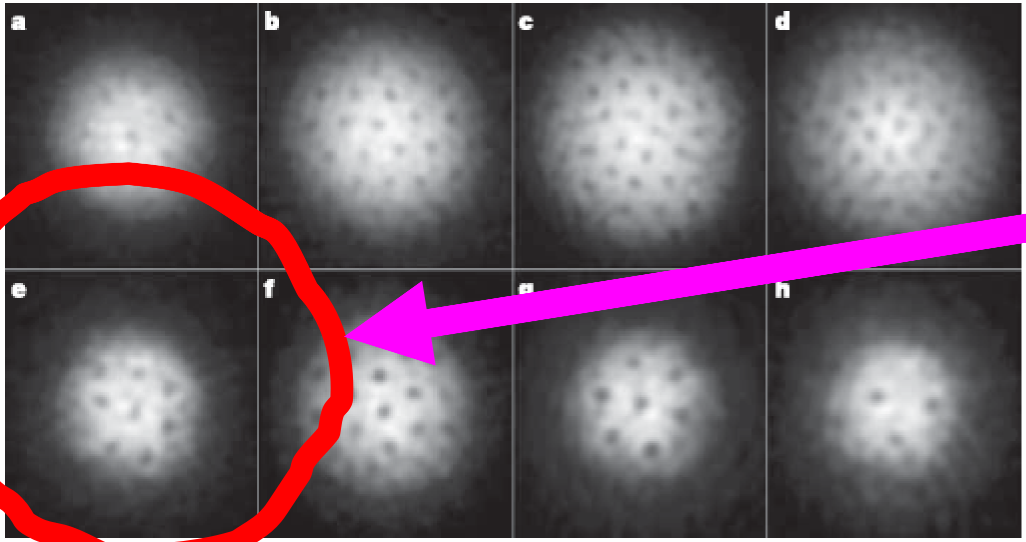
J.R. Anglin, W. Ketterle, Nature v416, 211 (2002)

Evidence for fermionic superfluidity: vortices!



system of fermionic ^6Li atoms

Feshbach resonance: B=834G



BEC side: a>0

BCS side: a<0

UNITARY REGIME

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

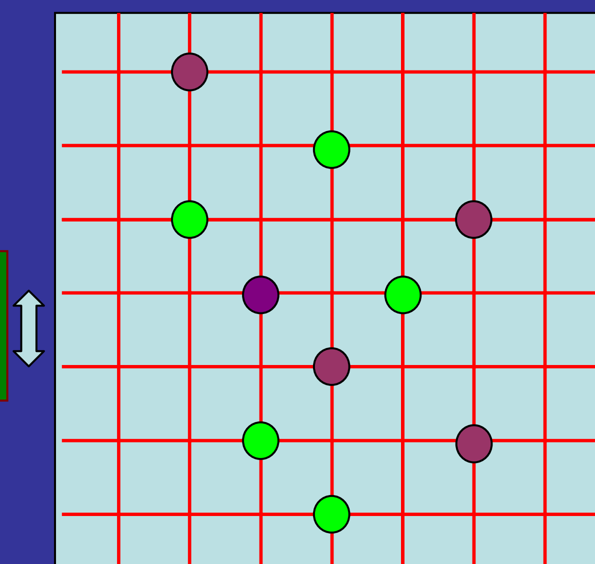
$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \;\; \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

Theoretical approach: Fermions on 3D lattice

Coordinate space

L –limit for the spatial correlations in the system

$$k_{cut} = \frac{\pi}{\Delta x} \; ; \;\; \Delta x$$



$$Volume = L^3$$

$$lattice \; spacing = \Delta x$$

- Spin up fermion:

- Spin down fermion:

External conditions:

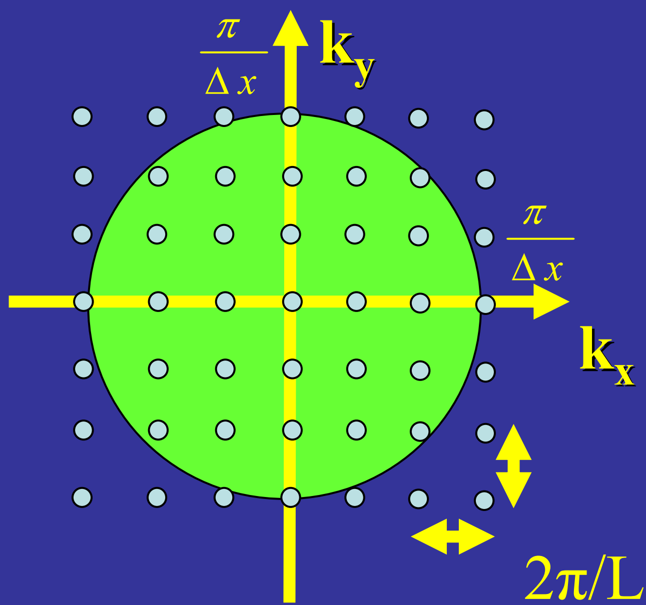
T - temperature

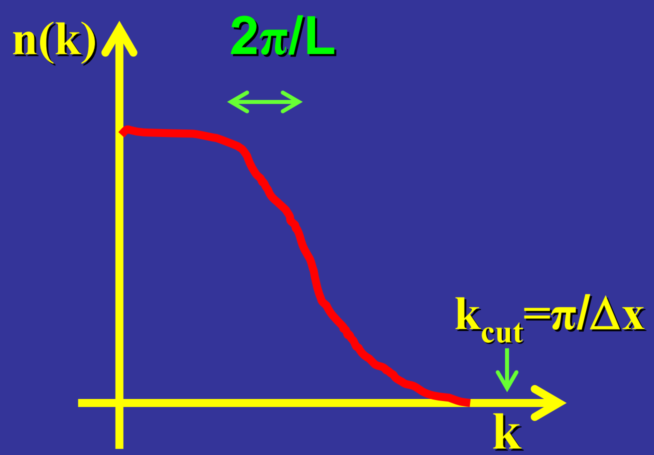
μ - chemical potential

Periodic boundary conditions imposed

Theoretical approach: Fermions on 3D lattice

Momentum space





UV momentum cutoff $\Lambda_{UV} = \frac{\pi}{\Delta x}$

IR momentum cutoff $\Lambda_{IR} = \frac{2\pi}{L}$

$$\frac{\hbar^2 \Lambda_{IR}^2}{2m} << \varepsilon_F, \ \Delta << \frac{\hbar^2 \Lambda_{UV}^2}{2m}$$

REAL SPACE ⟺ (FFT) ⟺ MOMENTUM SPACE

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \;\; \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{m k_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

$$\frac{1}{g} = \frac{m}{2\pi\hbar^2 \Delta x}$$ - UNITARY LIMIT

Grand-canonical ensemble:

$$E(T) = \langle \hat{H} \rangle = \frac{1}{Z(T)} Tr \left\{ \hat{H} \rho(\hat{H}, \hat{N}, T) \right\} = \frac{1}{Z} \sum_n E_n e^{-\frac{1}{kT}(E_n - \mu N_n)}$$

$$Z(T) = Tr \left\{ \rho(\hat{H}, \hat{N}, T) \right\} = \sum_n e^{-\frac{1}{kT}(E_n - \mu N_n)}; \;\; \rho(\hat{H}, \hat{N}, T) = e^{-\frac{1}{kT}\left(\hat{H} - \mu \hat{N}\right)}$$

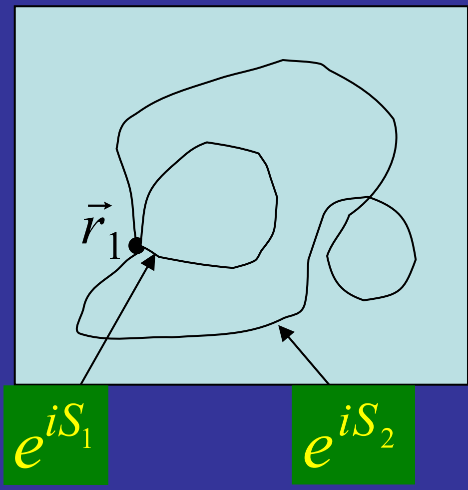
Eigenenergies of the Hamiltonian are unknown!

Path integral approach:

Single-particle quantum mechanics:

$$\left\langle \vec{r}_1 \left| e^{-i\hat{H}(t-t_0)} \right| \vec{r}_1 \right\rangle = \int D[r(t)] e^{i\int_{t0}^{t} L\left(\vec{r}(t),\dot{\vec{r}}(t)\right)dt}$$

$$L\left(r(t),\dot{r}(t)\right) = \frac{m\dot{\vec{r}}(t)}{2} - V(\vec{r}); \quad e^{i\int_{t0}^{t} L\left(\vec{r}(t),\dot{\vec{r}}(t)\right)dt} = e^{iS[\vec{r}(t)]}$$



Quantum statistical mechanics:

$$Z(\beta) = Tr\left\{\exp(-\beta(\hat{H} - \mu\hat{N})\right\} = \sum_{\substack{n-many \\ body\ states}} \left\langle n \right| \exp(-\beta(\hat{H} - \mu\hat{N}) \left| n \right\rangle$$

$$\beta = 1/kT \ ; \ \text{imaginary time: } \tau = it$$

$$Z(\beta) = \int D\left[\sigma(\vec{r},\tau)\right] \ e^{\ln\{\det[1+\hat{U}(\{\sigma\})]\}}$$

$$S[\sigma(\vec{r},\tau)] = -\ln\{\det[1+\hat{U}(\{\sigma\})]\} \text{ - action}$$

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\};\ \hat{h}(\{\sigma\}) - \text{one-body operator}$$

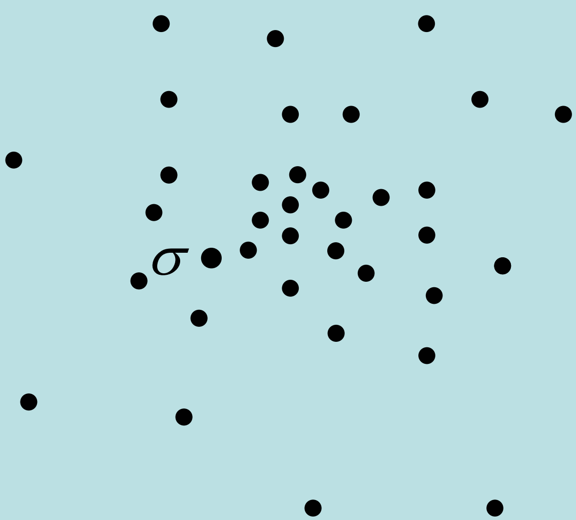
$$U(\{\sigma\})_{kl} = \left\langle \psi_k \middle| \hat{U}(\{\sigma\}) \middle| \psi_l \right\rangle; \quad \left|\psi_l\right\rangle \text{- single-particle wave function}$$

$$E(T) = \left\langle \hat{H} \right\rangle = \int \frac{D[\sigma(\vec{r},\tau)]e^{-S[\sigma]}}{Z(T)} E[U(\{\sigma\})]$$

$E[U(\{\sigma\})]$- energy associated with a given sigma field

Quantum Monte-Carlo:

Sigma space sampling



σ

$$P(\sigma) \propto e^{-S[\sigma]}$$

$$\bar{E}(T) = \frac{1}{N_\sigma} \sum_{k=1}^{N_\sigma} E\left(U(\{\sigma_k\})\right)$$

$\bar{E}(T)$ - stochastic variable

$$\left\langle \bar{E}(T) \right\rangle = E(T)$$

$$\sqrt{\left\langle \bar{E}(T)^2 \right\rangle - \left\langle \bar{E}(T) \right\rangle^2} \propto 1/\sqrt{N_\sigma}$$

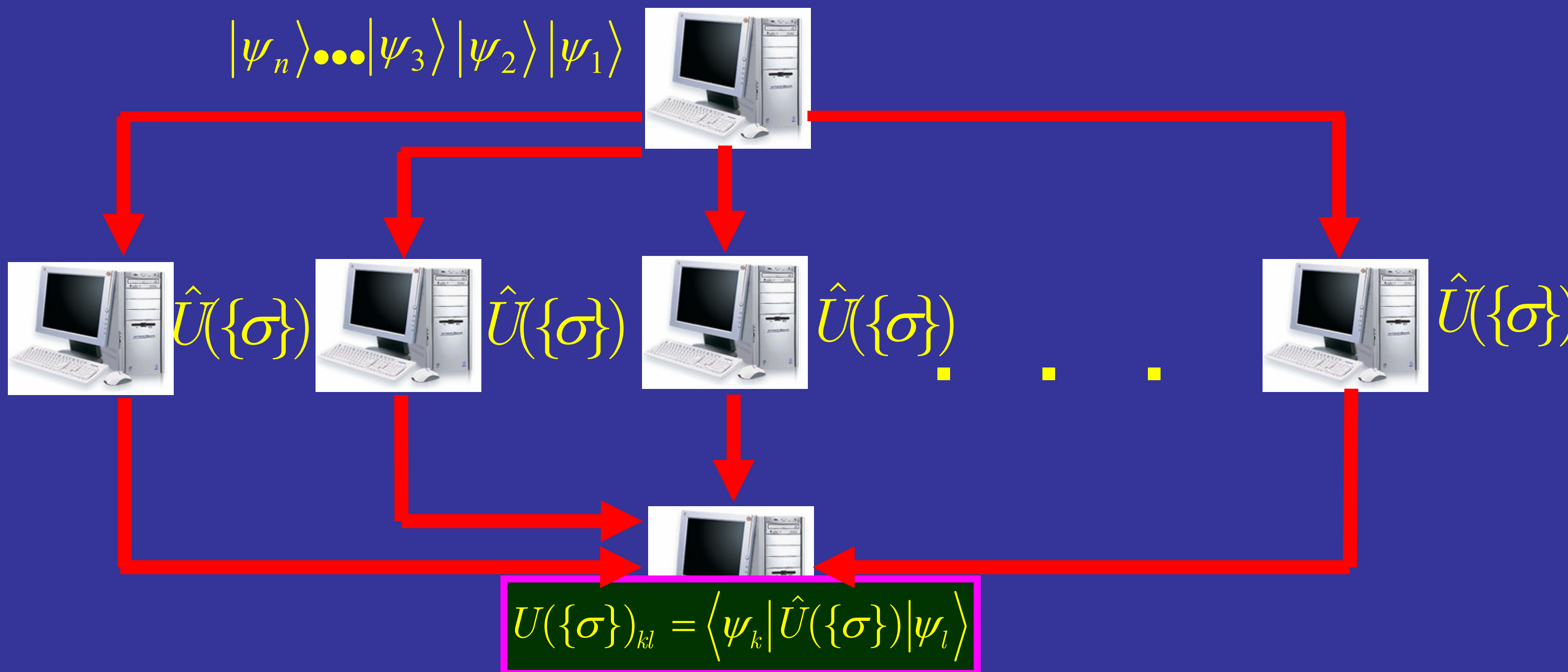
N_σ - number of uncorrelated samples

Quantum Monte-Carlo: parallel computing

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\};\ \hat{h}(\{\sigma\}) - \text{one-body operator}$$

$$U(\{\sigma\})_{kl} = \langle \psi_k | \hat{U}(\{\sigma\}) | \psi_l \rangle; \quad |\psi_l\rangle \text{- single-particle wave function}$$

For each sigma *n* single particle states have to be evolved.



More details of the calculations:

- Lattice sizes used from 8^3 x 257 (high Ts) to 8^3 x 1732 (low Ts), <N>=50, and 6^3 x 257 (high Ts) to 6^3 x 1361 (low Ts), <N>=30.

- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.

- Update field configurations using the Metropolis importance sampling algorithm.

- Change randomly at a fraction of all space and time sites the signs the auxiliary fields $\sigma(r,\tau)$ so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .

- Thermalize for 50,000 – 100,000 MC steps or/and use as a start-up field configuration a $\sigma(x,\tau)$-field configuration from a different T

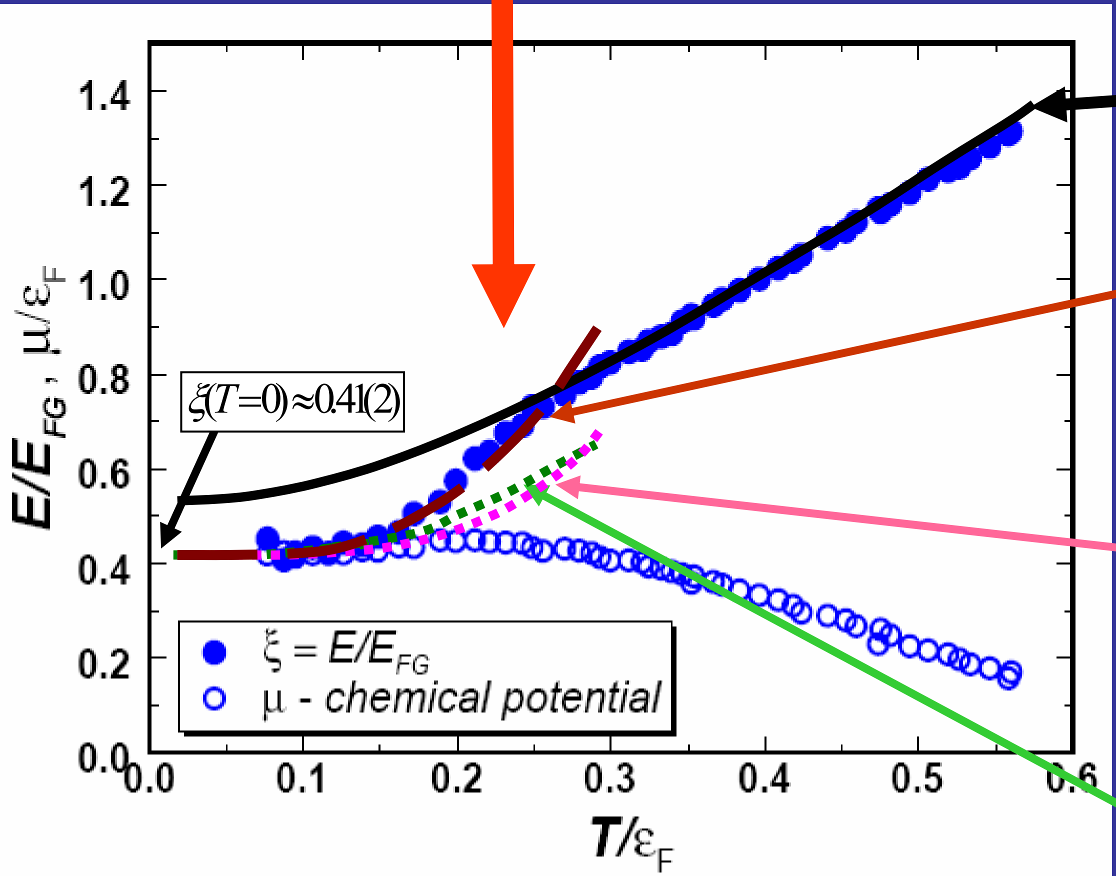
- At low temperatures use Singular Value Decomposition of the evolution operator $U(\{\sigma\})$ to stabilize the numerics.

- Use 200,000-2,000,000 $\sigma(x,\tau)$- field configurations for calculations

- MC correlation "time" ≈ 150 – 200 time steps at $T \approx T_c$

$a = \pm\infty$

Superfluid to Normal Fermi Liquid Transition



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution
(dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

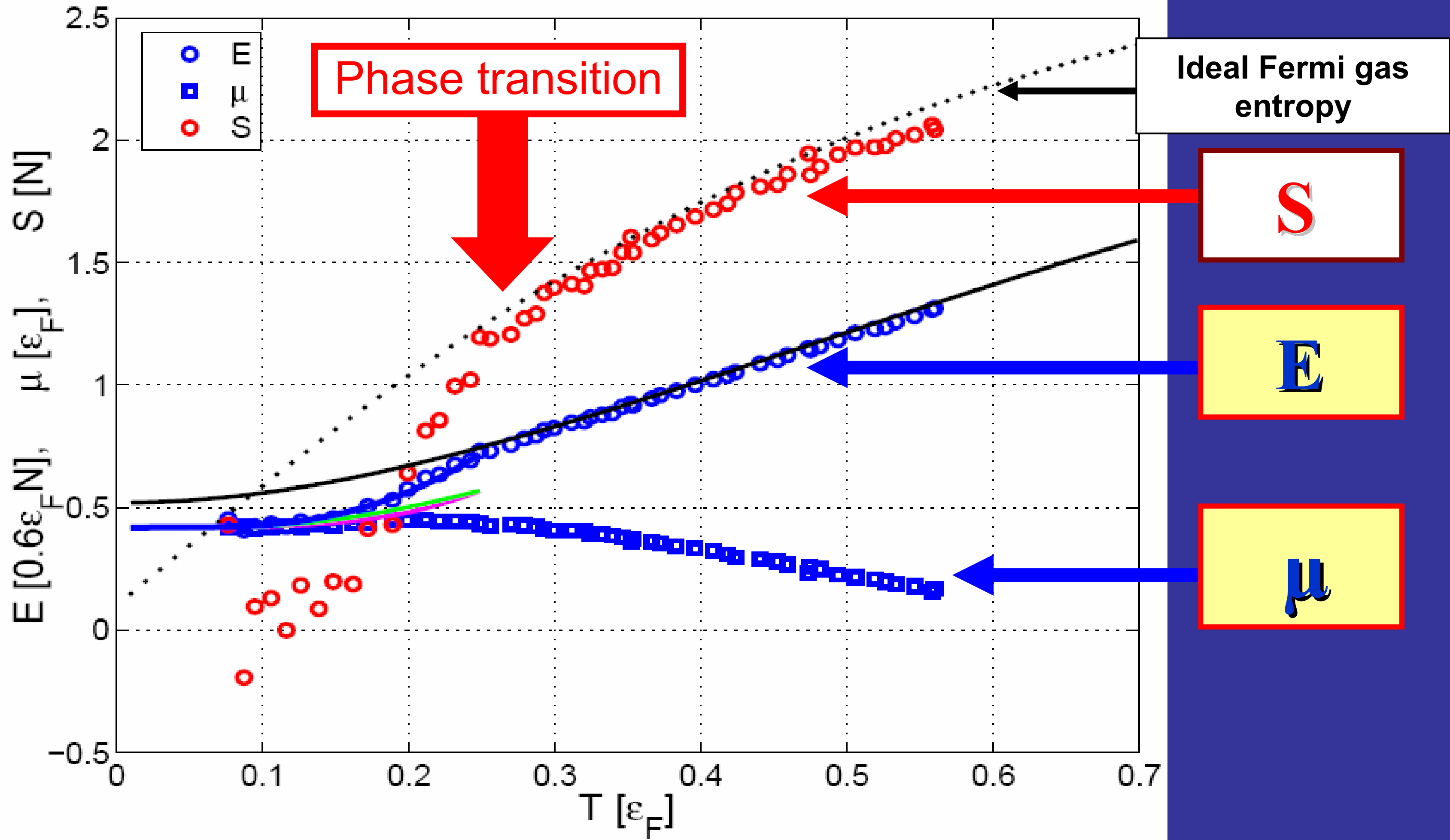
Quasi-particle contribution only
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

A. Bulgac, J.E. Drut, P. Magierski,PRL96,090404(2006)



$$E = \frac{3}{5}\varepsilon_F(n) N \, \xi\left(\frac{T}{\varepsilon_F(n)}\right)$$

$$n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F(n) = \frac{\hbar^2 k_F^2}{2m}$$

$$S(T) = S(0) + \int_0^T \frac{\partial E}{\partial T}\frac{dT}{T}$$

$$S(T) = \frac{3}{5} N \int_0^{T/e_F} dy \, \frac{\xi'(y)}{y}$$

$$\rho_2(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = \left\langle \hat{\psi}^\dagger_\uparrow(\vec{r}_1)\hat{\psi}^\dagger_\downarrow(\vec{r}_2)\hat{\psi}_\downarrow(\vec{r}_4)\hat{\psi}_\uparrow(\vec{r}_3)\right\rangle$$

$$\rho_2^P(\vec{r}) = \frac{2}{N}\int d^3r_1 d^3r_2 \rho_2(\vec{r}_1+\vec{r},\vec{r}_2+\vec{r},\vec{r}_1,\vec{r}_2)$$

$$\lim_{r\to\infty}\rho_2^P(\vec{r}) = \alpha \text{ - condensate fraction}$$

More Results...

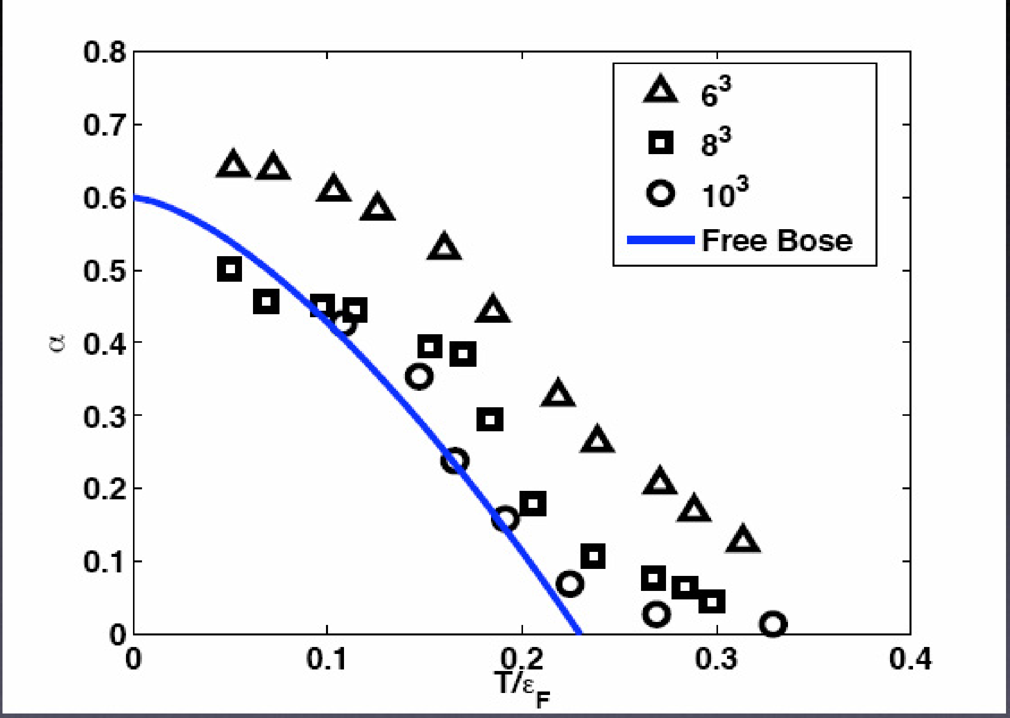
Condensate fraction α:
Order parameter for
Off Diagonal
Long Range Order
(C.N. Yang)

Free Bose gas-like:

$$\alpha(T) = \alpha(0)\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]$$

Free : $\alpha(0) = 1$
Unitary: $\alpha(0) \cong 0.6$

$T_c = 0.23(2)$



From a talk of J.E. Drut

Low temperature behaviour of a Fermi gas in the unitary regime

$$F(T) = \frac{3}{5}\varepsilon_F N \varphi\left(\frac{T}{\varepsilon_F}\right) = E - TS \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \text{ for } T < T_C$$

$$\mu(T) = \frac{dF(T)}{dN} = \varepsilon_F\left[\varphi\left(\frac{T}{\varepsilon_F}\right) - \frac{2}{5}\frac{T}{\varepsilon_F}\varphi'\left(\frac{T}{\varepsilon_F}\right)\right] \approx \varepsilon_F \xi_s$$

$$\varphi\left(\frac{T}{\varepsilon_F}\right) = \varphi_0 + \varphi_1\left(\frac{T}{\varepsilon_F}\right)^{5/2}$$

$$E(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^n\right]$$

Lattice results disfavor either n≥3 or n≤2 and suggest n=2.5(0.25)

This is the same behavior as for a gas of <u>noninteracting</u> (!) bosons below the condensation temperature.

Energy of noninteracting Bose gas

(up to arbitrary constant):

$$E(T) \approx E(0) + \frac{m_B^{3/2}\Gamma\left(\frac{3}{2}\right)\varsigma\left(\frac{3}{2}\right)}{2^{1/2}\pi^2\hbar^3} T^{5/2}V, \quad \text{if} \quad T \gg m_B c^2$$

and fitting to lattice results $\Rightarrow \quad m_B \approx 3m$

- **Why this value for the bosonic mass?**
- **Why these bosons behave like noninteracting particles?**

Conclusions

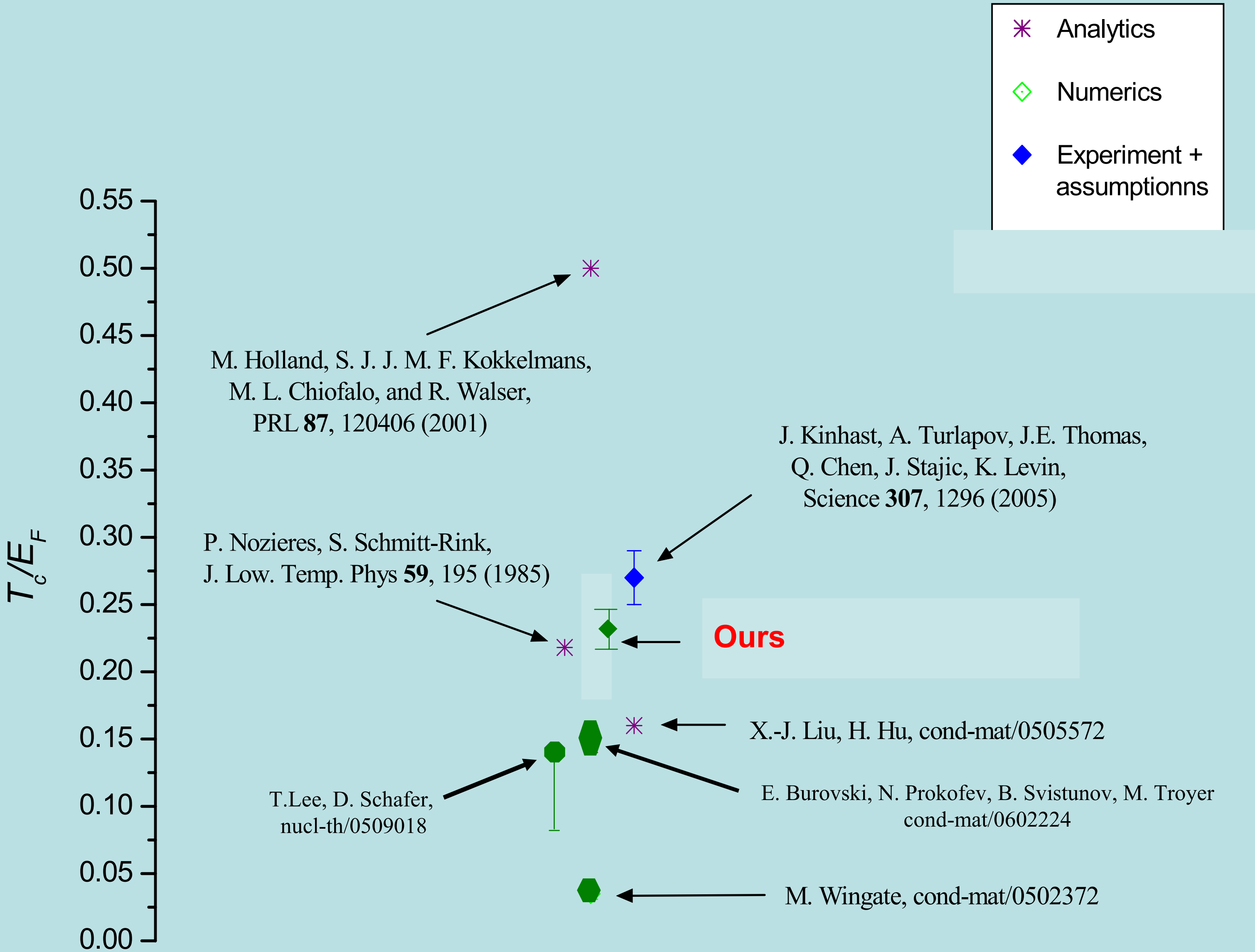
- ✓ Fully non-perturbative calculations for a spin ½ many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at $T_c = 0.23\ (2)\ \varepsilon_F$
  (Exp: $T_c = 0.27(2)\ \varepsilon_F$, J. Kinast *et al.* Science, 307, 1296 (2005): Based on theoretical assumptions).

- ✓ Chemical potential is constant up to the critical temperature – note similarity with Bose systems!

- ✓ Below the transition temperature, both phonons and fermionic quasiparticles contribute almost equaly to the specific heat. In more than one way the system is at crossover between a Bose and Fermi systems.

There are reasons to believe that below the critical temperature this system is a new type of fermionic superfluid, with unusual properties.

Quest for unitary point critical temperature



Boris Svistunov's talk (updated), Seattle 2005

Open questions and future prospects:

- We have a theoretical tool that enable us to study various aspects of strongly interacting Fermi system.

Dilute Fermi gases off the unitary regime: T_C ($k_F a$)

Phase diagram for polarized and diluted Fermi system.

Generalization to finite effective range. Equation of state for dilute neutron matter (neutron star crust: specific heat, neutrino scattering)

Extension to denser nuclear matter

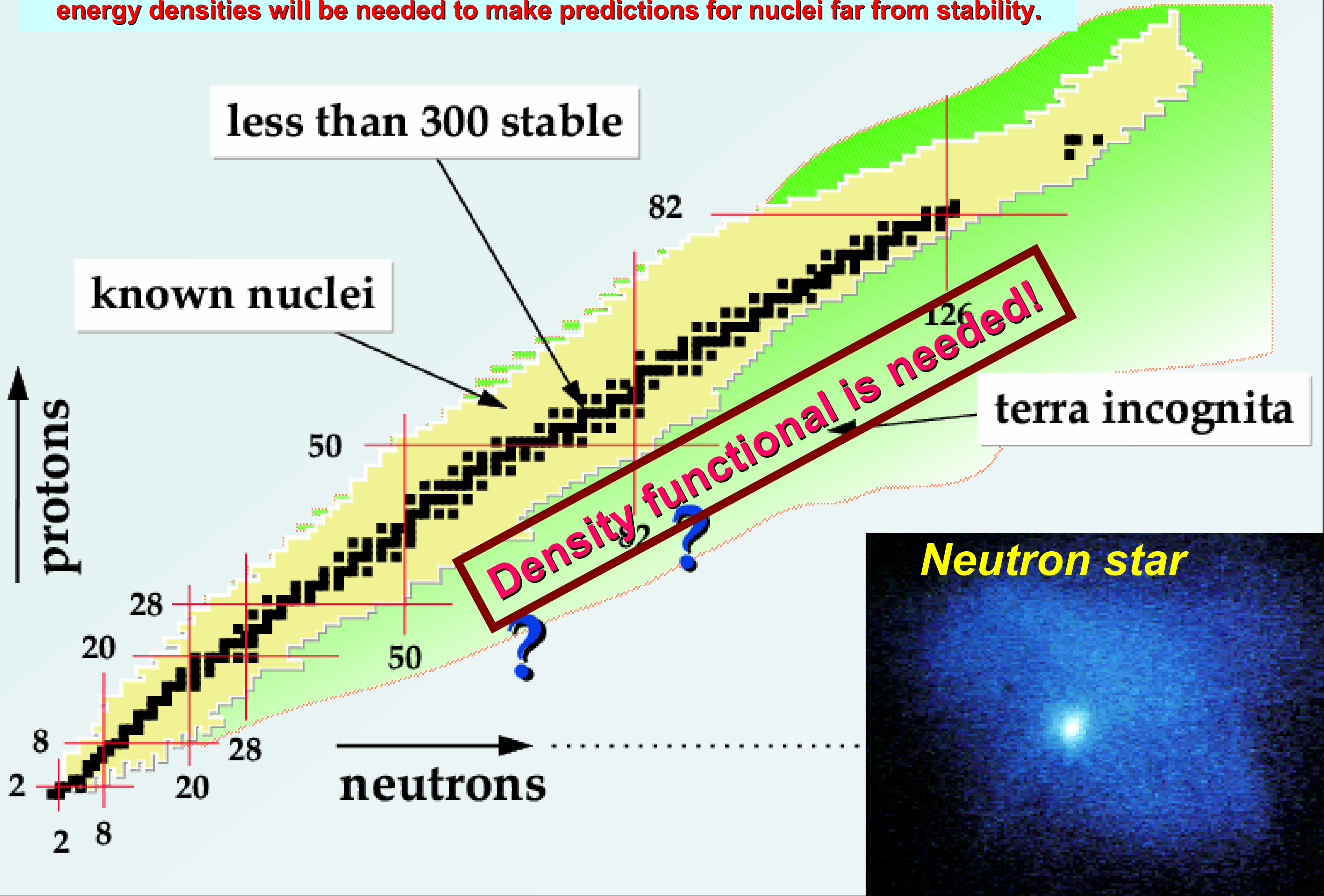
What we can learn from physics of cold Fermi atoms?

Pairing dependence on the density. Pairing field in an inhomogeneous nuclear matter.

We should try to get away from the heavily phenomenological approach which dominated nuclear pairing studies most of last 40 years and put more effort in an *ab initio* and many-body theory of pairing and be able to make reliable predictions, especially for neutron stars and nuclei far from stability. The studies of dilute atomic gases with tunable interactions could serve as an extraordinary testing ground of theories.

Limits of the extreme N/Z ratios

Reliable nuclear density functional depending on both normal and superfluid energy densities will be needed to make predictions for nuclei far from stability.



One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart[A)]. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

Robert B. Laughlin, Nobel Lecture, December 8, 1998