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ELECTRON MOTION IN ELECTRIC AND MAGNETIC FIELDS, EVALUATING THE *e/m* VALUE

1. Fundamentals

1.1. The charge motion in the electric and magnetic fields

A particle of charge q moving in the magnetic field experiences the force called **Lorentz force**:

$$\vec{F} = q(\vec{v} \times \vec{B}), \quad (1)$$

where $\vec{v}$ is the velocity of the charge, $\vec{B}$ - the induction of the magnetic field. According to the properties of the vector product, the force vector is perpendicular to the plane containing vectors $\vec{v}$ and $\vec{B}$, its direction can be found using right hand screw rule. The value of the Lorentz force is:

$$F = qvB\sin\angle(\vec{v},\vec{B}). \quad (2)$$

It can be easily seen that the magnetic force does not act on electric charge in situations when charge is not in motion (v = 0) or when it is moving parallel to induction lines of the magnetic field (sinus of the angle between vectors $\vec{v}$ and $\vec{B}$ is equal to zero). Lorentz force has the biggest value, when the velocity vector direction is perpendicular to magnetic field $\vec{B}$. So the trajectories due to the Lorentz force (line, circle or helix) depend on the angle between vectors of the velocity and the magnetic induction. It must be underlined that Lorentz force is always perpendicular to the motion direction of the charge, what causes (for the steady magnetic field) that **the work of this force is always equal to zero**. For the segment of the trajectory $d\vec{L}$ this work dW is equal $\vec{F}\cdot d\vec{L}$ and is equal to zero, because the vectors $\vec{F}$ and $d\vec{L}$ are always perpendicular. For this reason, the steady magnetic field **cannot change the kinetic energy of the moving charge**, thus its velocity value; magnetic field can change only the direction of the motion.

Let's consider the particular case, when the electron with the velocity $\vec{v}$ enters the uniform magnetic field $\vec{B}$ which is parallel to the axis OZ and is perpendicular to the velocity vector.

According to the definition of the Lorentz force (1) and (2), the force acting on the charge lays on the plane XY and its value is equal to $qvB\sin(\pi/2) = qvB.$ Since the tangential acceleration to the trajectory is equal to zero, the velocity vector has constant value. The motion can take place only in the plane XY and the force is always perpendicular to the motion direction. The acceleration of the charge $\vec{a} = \vec{F}/m$ is constant and perpendicular to the velocity vector.

Only the uniform circular motion has such properties, and the Lorentz force plays a part of the centripetal force:

$$qvB = \frac{mv^2}{r}, \quad (3)$$

where r - is the radius of the circle (the path of the moving particle). This radius can be easily calculated:

$$r = \frac{mv}{qB}. \quad (4)$$

Another value which can be calculated is the time needed to complete one full circle, so called **time period:**

$$T = \frac{2\pi r}{v} = \frac{2\pi m v}{qvB} = \frac{2\pi m}{qB} \quad . \tag{5}$$

This time **does not depend on the velocity of the charge**, but only on the magnetic field and the mass-to-charge ratio of the particle. Above characteristics are used in many different measurement devices like mass spectrometers, bubble chambers, cyclotrons, etc.

A particle of charge *q* in the electric field $\vec{E}$ experiences force:

$$\vec{F} = q\vec{E} \ . \tag{6}$$

The value of this force - on the contrary to the magnetic field – does not depend on the velocity of the moving charge

Let's consider the situation when the electron is moving in the area where **both fields: magnetic and electric are present**, as shown in fig. 1. The directions of the magnetic and the electric fields are parallel, and the angle between the velocity vector of the electron and the direction of the field is equal α. Total force acting on the charge in the presence of both magnetic and electric fields is the sum of the vector forces given by formulae (1) and (6) and can be written as follows:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}). \tag{7}$$

The velocity vector $\vec{v}$ can be resolved into two components: perpendicular $v_{\perp} = v\sin\alpha$ and parallel $v_{||} = v\cos\alpha$ to the magnetic field vector. **According to the principle of the motion superposition, motion in every direction can be treated separately.** In the direction perpendicular to the electric and magnetic fields, the electron experiences only the Lorentz force (3):

$$F = qvB\sin\alpha = qv_{\perp}B. \tag{8}$$

According to the earlier information, the motion in the plane XY (perpendicular to the direction of the electric and magnetic fields) will be circular with the radius given by the formula:

$$r = \frac{mv_{\perp}}{qB} \tag{9}$$

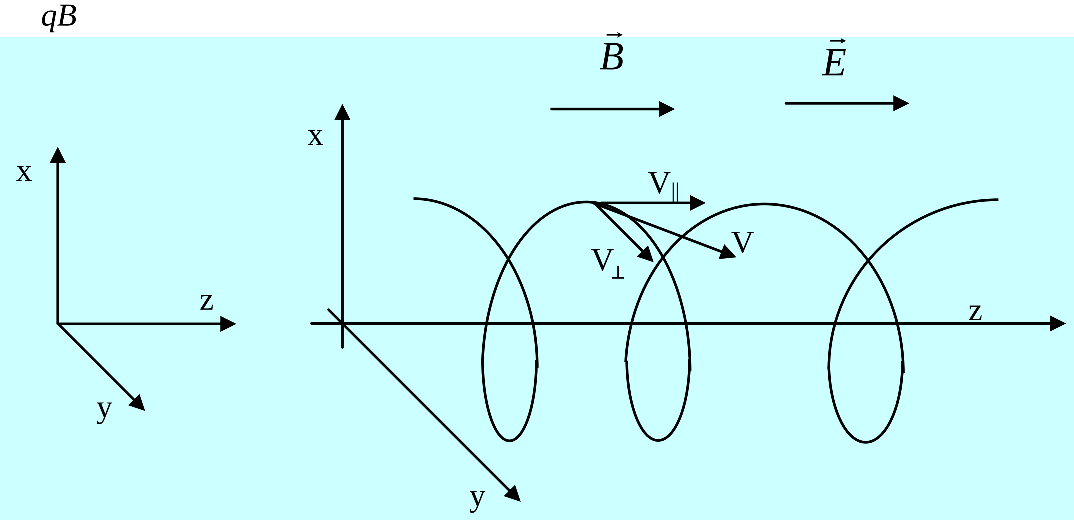
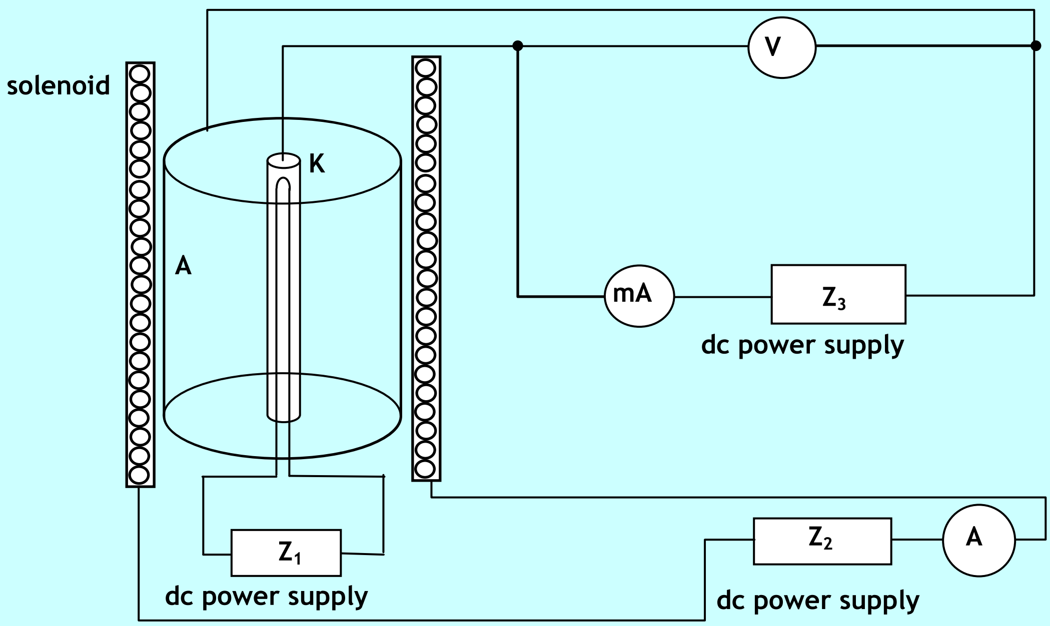


Fig. 1. Electron motion in the electric and magnetic fields.

On the other hand, in the parallel direction (to both fields) the particle moving with the velocity $v_{||}$ experiences force from the electric field only. In the absence of the electric field, the electron would move in this direction with constant velocity $v_{||} = v\cos\alpha$ (component of the Lorentz force in this direction is equal to zero), thus the electron would move on the helix with the constant pitch. The electric field causes the electron movement in this direction with the uniform acceleration. The trajectory is a helix with the variable increasing pitch.

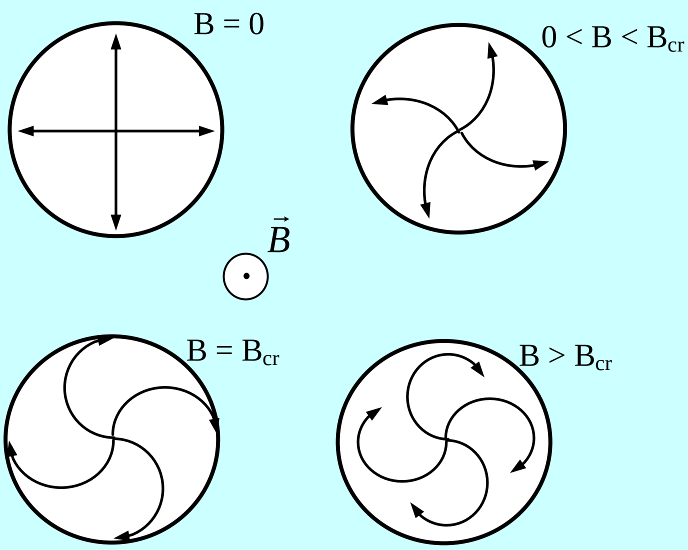
2. Laboratory setup

2.1. Finding the *e/m* value using the magnetron.



Rys.2. Laboratory setup. A - anode, K - cathode, Z_1 - power supply for heating magnetron cathode, Z_2 - power supply for the magnetic field creation, Z_3 - anode power supply.

The electrons are emitted from the heated cathode (as the result of the thermoemission effect). They are moving from the cathode to the anode as a result of the electric field presence. The external magnetic field causes curvature of their trajectories and for the specific critical value of the magnetic field, the curvature is so big that electrons do not reach the anode. The results of the magnetic field influence on the motion of the electron are shown in fig.3.



Rys.3. The magnetic field influence on the motion of the electrons from cathode to anode.

Analyzing dependence between the anode current and the external magnetic field, the charge-to-mass ratio *e/m* can be easily obtained. For this purpose the electron motion in the magnetron has to be analyzed, where the voltage applied between anode and cathode is equal U, and the lamp is in the magnetic field $\vec{B}$. The radius of the cathode is a, and the radius of the anode is b. The magnetic field is created by the solenoid in which the electric current flows. The critical induction B_{cr} can be tied with the electric current I_{cr} flowing through the solenoid using the formula defining magnetic field in the solenoid:

$$B_{kr} = \mu\mu_0 N I_{kr} \ , \tag{10}$$

where μ - permeability of the material, $\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$ - permeability of the free space, N – the number of windings per one unit of the length. As the result of the calculations precisely described in the Appendix, the following formula for the value *e/m* can be obtained (for the free space we assume μ=1):

$$\frac{e}{m} = \frac{8U}{\mu_0^2 N^2 I_{cr}^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2} \ . \tag{11}$$

To get the e/m value the critical current (when the anode current stops flowing) must be obtained. If all electrons would have the same initial velocity (and the final as well), the dependence of the anode current on the magnetic field would look like this one presented on the fig.4a. The critical current would be the current when the anode current becomes zero.

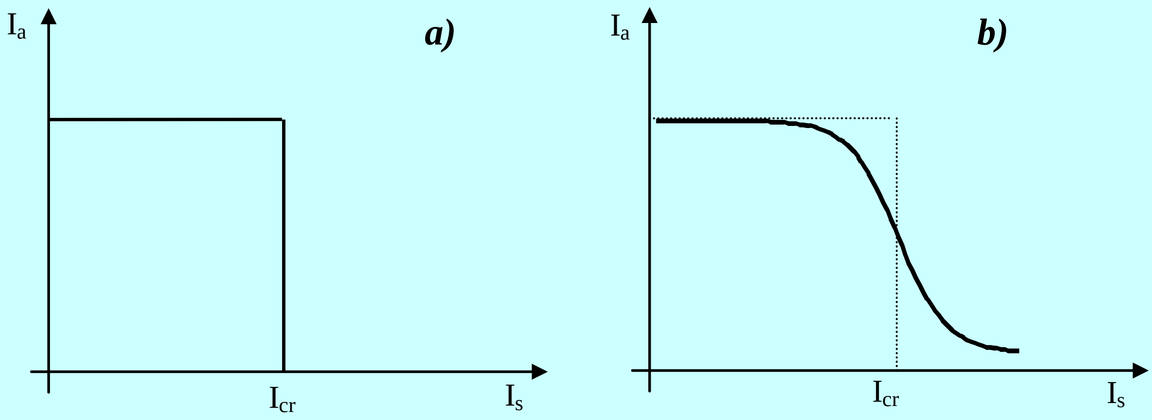


Fig.4. The anode current vs. the current flowing through the solenoid in two cases: a) ideal, b) real.

In real situation it does not happen, because the electrons emitted from the cathode have different velocities (the velocity distribution is the **Maxwell - Boltzmann distribution**), additionally electrons collide each other on the path from the cathode to the anode. Thus the critical magnetic field is different for different electrons, and the dependence of the anode current on the solenoid current looks like the curve presented in the fig.4b. The inflection point of this curve can be treated as the critical current.

2.2. Finding the *e/m* value using the focusing effect of the electron beam (magnetic field parallel to the electron lamp axis)

Fig.5 presents the scheme of the oscilloscope lamp and the coordinate system which will be used to describe the electron motion. The electrons emitted from the cathode are accelerated in the constant electric field (the voltage between anode and cathode is equal U) to the velocity $\vec{v}$ which value can be obtained from the conservation of energy principle:

$$\frac{mv^2}{2} = eU$$

Thus:

$$\mathrm{v} = \sqrt{\frac{2eU}{m}}\,. \qquad (12)$$

The vector $\vec{\mathrm{v}}$ is parallel to axis Z. The electrons move in the constant magnetic field $\vec{B}$ parallel to axis Z. The magnetic field is created by the external solenoid. During their motion between the deflecting plates (electrodes), the electrons experience the force from the weak, alternating electric field $\vec{E}$ parallel to X axis. This field adds to the electron small additional velocity $\vec{\mathrm{v}}_p$ perpendicular to the magnetic field $\vec{B}$.

According to the earlier analysis, when the electrons enter the magnetic field and the angle between the velocity vectors and the magnetic field is not zero, the motion trajectories are helices laying on the side area of the cylinder. The axis of this cylinder is parallel to the magnetic field, and the radius according to the formula (4) is $r = \frac{m|\vec{\mathrm{v}}|_p}{eB}$ ($\vec{\mathrm{v}}_\mathrm{p}$ - is the velocity component in the direction OY), and the time period (time needed to make a full circle 2π in the plane perpendicular to the cylinder axis) according to the formula (5) is:

$$T = \frac{2\pi m}{eB}\,.$$

This time does not depend on the velocity of the electrons. The image on the oscilloscope lamp screen shows the cross section of the electron trajectories with the plane XY perpendicular to the lamp axis.

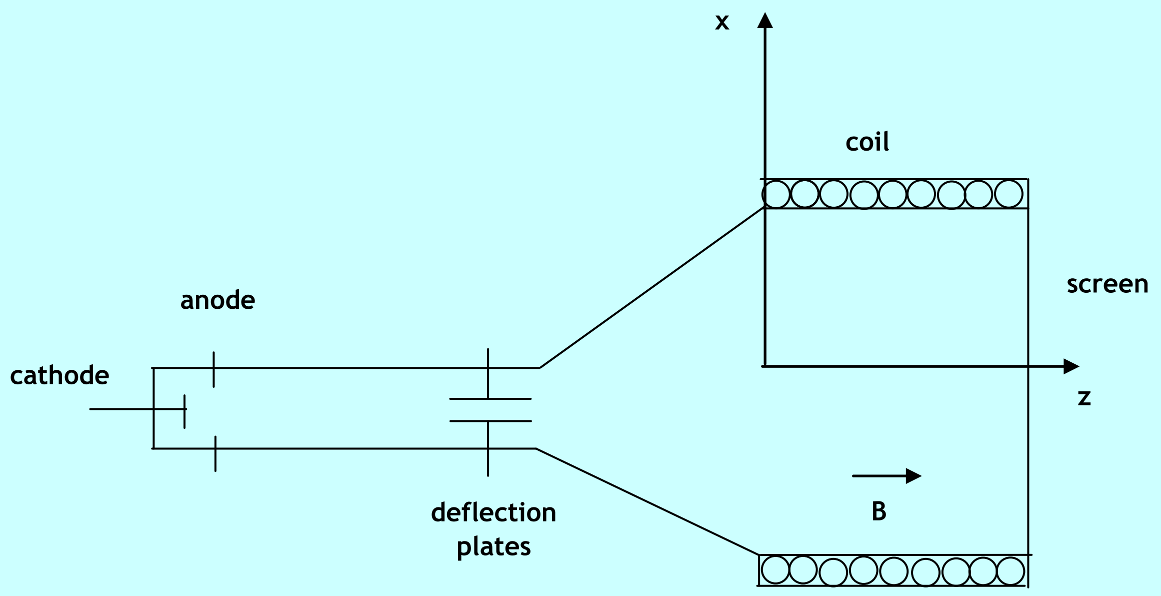


***Fig.5**. The scheme of the electron lamp and the coordinate system.*

Fig.6 shows the image on the screen, when **to the deflection plates the alternating voltage is applied** ($E = E_0 \sin \omega t$), **in the presence of the constant magnetic field**. In such situation, the electrons have different velocities $\vec{\mathrm{v}}_p$ and their trajectories lay on the side areas of the different cylinders.

Since the time period is the same for different electrons (equation (5)), the image on the screen corresponds to the points where the electrons hit the screen after going round the same angle in the different helix trajectories. This angle is equal to $2\pi t/T$, where t denotes the time of flight from the deflection plates to the screen, and T denotes the time needed to make full angle 2π in the plane perpendicular to the lamp axis.

When this angle is the multiple of 2π, which corresponds to the distance nTv, the electrons meet on the axis of the lamp. Changing the voltage U between anode and cathode and the magnetic field B inside the coil, we can alter v and T in such way that the path from the deflection plates to the screen will be the integer multiple of vT. At this time on the screen instead of the segment the shining point will be observed. This phenomenon is called **the focusing of the electron beam**.

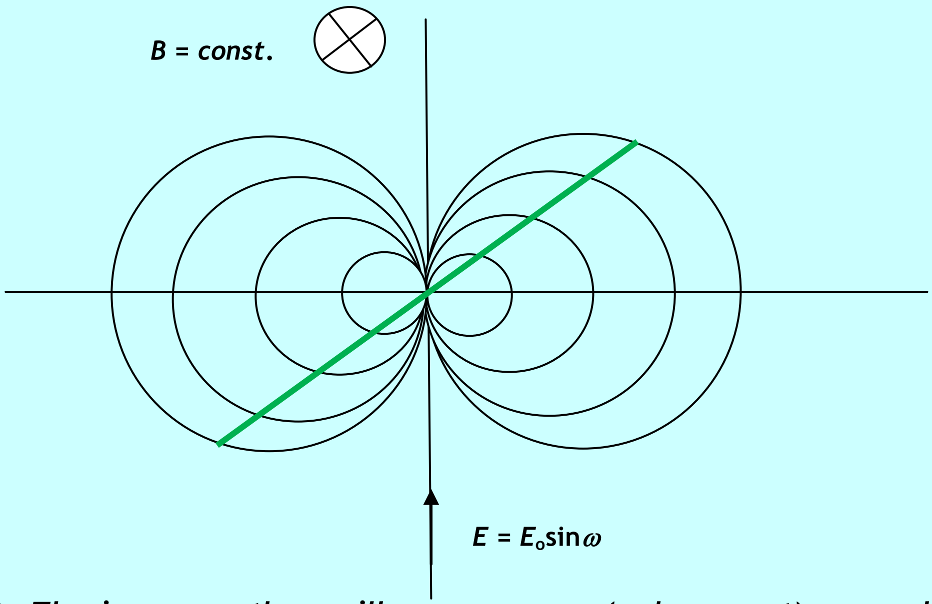


Fig.6. The image on the oscilloscope screen (red segment) seen when the alternating voltage is applied to deflecting plates in the presence of the constant magnetic field. The tiny circle lines illustrate the projection of the electrons trajectories on the screen plane (they are not visible on the screen!!!).

When the focusing happens, the condition of the focusing is fulfilled nTv = d (n – focusing multiple), thus from the equation (5) results:

$$n\mathrm{v}\frac{2\pi m}{eB} = d\,. \tag{13}$$

The velocity obtained from the formula (12) should be inserted to the above equation and after some simple calculations we obtain:

$$\frac{e}{m} = \frac{8\pi^2 U n^2}{d^2 B^2}\,, \tag{14}$$

where n = 1,2,3,..., and d – is the length of the magnetic field area.

2.3. Finding the *e/m* value with the electron beam deflection method (magnetic field perpendicular to the electron lamp axis)

Let's consider the electron motion in the oscilloscope lamp as it is shown in fig. 7*a*. The magnetic field vector is parallel to the Y axis. It is obvious, that in the situation when there is no voltage on the deflection plates nor magnetic field inside the lamp, on the lamp screen the bright point will be observed. Applying constant magnetic field (in the Y direction) will cause the point shift (x) in the vertical direction.

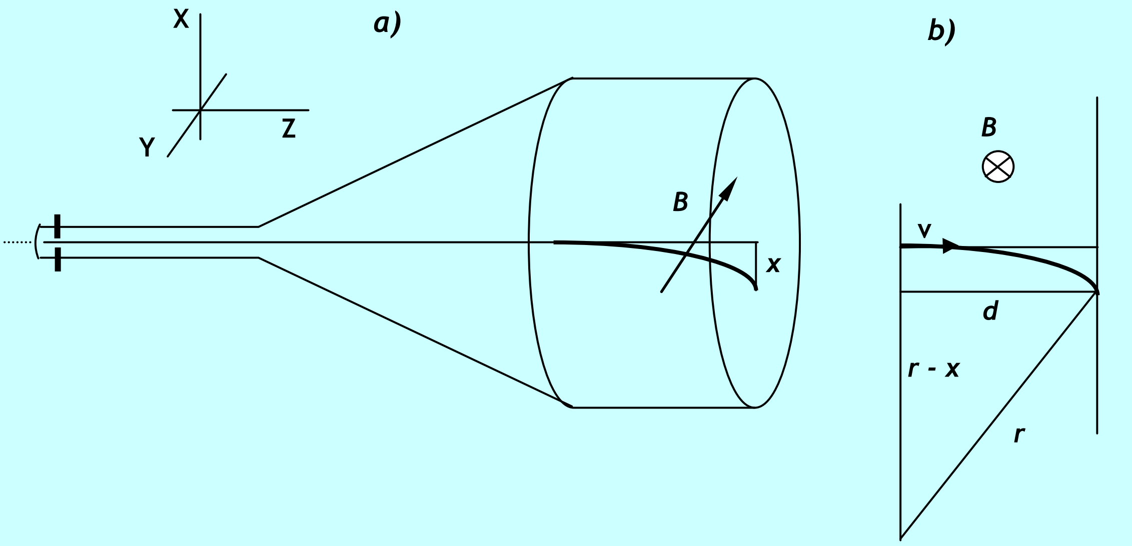


Fig.7. The electron trajectory when the magnetic field id perpendicular the direction of the electron velocity: a) setup scheme, b) geometrical description.

In the fig.7b the geometrical dependencies allowing finding the *e/m* value are shown. It can be noticed, that according to the Pythagorean Theorem:

$$d^2 + (r - x)^2 = r^2,$$

and after the reduction:

$$d^2 + x^2 = 2xr \ .$$

Taking into account that the radius of the circle of the electrons trajectories is (5) $r = \frac{m\mathrm{v}}{eB}$, after the simple calculations one can obtain:

$$d^2 + x^2 = 2x\frac{m\mathrm{v}}{eB} \ ,$$

thus:

$$\frac{e}{m} = \frac{2x\mathrm{v}}{B(d^2 + x^2)} \ . \qquad (15)$$

Above formula cannot be used to calculate directly the e/m value, **because the velocity of the electron depends on this ratio**. The way the e/m value can be calculated is described in the result preparation section. The value of the magnetic field B can be calculated using the formulae written on the information plates.

2.4. Finding the e/m value using the Helmholtz coils.

In this part of the experiment, a special lamp filled with low-pressure neon (about 4x10-4 Pa) is used. This gas plays a key role in the experiment as the electrons colliding with gas particles cause their ionization. The ion recombination leads to luminescence phenomenon and **the electrons' trajectory can be observed**. On the other hand, due to electrostatic interaction between ions and electrons, the electron beam is being collimated. The electrons are injected to the lamp bulb from an electron gun that accelerates them to the energy of E=eU. The lamp is located between two coaxial coils made of copper wire that are called **the Helmholtz coils**. The current flowing through the coils induces magnetic field inside the lamp of the B induction vector perpendicular to the lamp axis. **The Helmholtz coils are relatively large (big radius) to assure a homogenous field (approximately) inside the lamp**. The lamp can be rotated around its own axis, which enables to change the direction of electron motion with respect to the direction of the magnetic field. At a certain position of the lamp, the electrons will move on the circle with the radius defined by the equation (5). Inside the lamp there is also a metal ladder coated with a fluorescent paint which enables to determine the diameter of the electron trajectory.

According to the equation (5): $r = \frac{m\mathrm{v}}{eB}$, the radius of the circle can be changed by altering the velocity of the electrons or the magnetic field *B*. The value of the velocity v can be changed by changing the accelerating voltage U according to the equation $\frac{m\mathrm{v}^2}{2} = eU$. The velocity v obtained from this equation can be inserted into equation (5), and after simple calculations the e/m value will be given by:

$$\frac{e}{m} = \frac{2U}{r^2 B^2} \quad . \tag{16}$$

The induction of the magnetic field B can be altered changing the current flowing through the Helmholtz coils. Taking assumption that the current flowing through both coils is the same, B can be calculated using the following experimental equation: $B = 0{,}715\mu_0 \frac{nI}{R}$, where: n - the number of windings = 154; R - radius of the Helmholtz coils = 200 mm = 0,2 m; μ_0 - permeability of the free space = $1{,}256 \cdot 10^{-6} \frac{Tm}{A}$.

The above values should be put in the equation defining the magnetic field, so the following dependence could be obtained:

$B = 0{,}632\ 10^{-3} I$ [the induction will be in T, since the current will be expressed in A]

In this way we can express the *e/m* value as the function of the physical parameters measured in the laboratory setup:

$$\frac{e}{m} = 4{,}17 \cdot 10^6 \frac{U}{I^2 r^2} \tag{17}$$

To obtain the *e/m* value in SI units, the voltage U must be expressed in volts, the current I in amperes, and the radius r in meters.

3. Measurements

3.1. Finding the e/m value with the application of a magnetron

1. Connect the measurement setup according to the diagram on the information plate. Detailed parameters of all devices, the measurements ranges and technical data can be found on the information plates.
2. Once all the electrical connections in the setup are checked and confirmed by the supervisor, turn on all the devices, starting from the power supply of the glowing circuit (a few minutes are needed to stabilize the anode current)
3. Perform the anode current measurement with relation to the coil current, at the constant anode voltage. Please pay a particular attention to the stability of the anode voltage during one measurement series. If at the moment of change of the coil current the anode voltage is changed, before each measurement, one has to correct it to the value chosen for this series.

3.2. Finding the *e/m* value using the focusing effect of the electron beam (magnetic field parallel to the electron lamp axis)

1. Connect the measurement setup according to the diagram on the information plate.
2. Once all the electrical connections in the setup are checked and confirmed by the supervisor, turn on the power supply of the oscilloscope lamp, and wait until the bright dot is seen on the screen.
3. Switch on the time base. Power the coil with the direct current and change the value of the current to obtain the effect of the electron beam focusing on the oscilloscope screen.

3.3. Finding the e/m value with the electron beam deflection method (magnetic field is perpendicular to the oscilloscope lamp axis)

1. Connect the measurement setup according to the diagram on the information plate.
2. Once all the electrical connections in the setup are checked and confirmed by the supervisor, turn on the power supply of the oscilloscope lamp, and wait until the bright dot is seen on the screen.
3. Adjust the focus to see a sharp dot in the central point of the oscilloscope.
4. Turn on the power supply of the coil and increase the current flowing through the coil, causing the dot shift by 1, 2 … sections in vertical direction. Write down the current values. The measurements have to be done for both directions of the magnetic field vector tip. This can be obtained by changing the current polarity. (One segment on the screen is 6 mm).

3.4. Finding the e/m value with the application of Helmholtz coils

1. Once the instrument has been heated, set the appropriate acceleration voltage (the full brightness of the electron beam is obtained in about 3 minutes after the instrument is on).
2. Turn on the power supply of the current flowing through the Helmholtz coils and observer the electron trajectory in gas. **(Caution: the maximum current flowing through the coil is 5 A.)**
3. Rotate the lamp to such a position that the direction of electrons is exactly perpendicular to the direction of magnetic field lines. If the lamp is correctly set, the electrons move on the circle trajectory.
4. By controlling the coil current, set such a diameter, that the trajectory goes through the horizontal steps of the ladder. The ladder is coated with a paint that lights when the electrons are falling on it so in case the appropriate diameter is set, the respective ladder stem emits the light. When the trajectory reaches a ladder step, only half of the circle can be seen. The circle radiuses are 2, 3, 4 and 5 cm (when the trajectory reaches respective steps).
5. For a chosen accelerating voltage U measure the respective coil current I values for which respective ladder steps light.
6. Perform two next measurement series for various U values.

CAUTION:
If the measurement is going to be interrupted for a few minutes, please turn both control knobs of the power supply to the zero position. This will increase the lamp lifetime.

4. Results

4.1. Finding the e/m value with the application of a magnetron

1. Plot a graph of the anode current versus the solenoid current. All the measurement points must have the uncertainty segments.
2. For every measurement series find graphically the critical current value and its uncertainty (the way of the uncertainty evaluation has to be discussed with the supervisor!!!).
3. Calculate *e*/*m* value using the equation (11). All constants needed for the calculation can be found on the information table. Calculate the combined uncertainty and the expanded uncertainty of the *e*/*m* value. Check your result against the true value.

4.2. Finding the *e/m* value using the focusing effect of the electron beam (magnetic field parallel to the electron lamp axis)

Using the equation (14) calculate *e*/*m* value taking into account information presented on the information table. Please remember, that not only the measured current value has its uncertainty, but also other values defining the *e*/*m* ratio have their uncertainties.

4.3. Finding the e/m value with the electron beam deflection method (magnetic field is perpendicular to the oscilloscope lamp axis)

1. Note all the values defining geometry of the oscilloscope lamp and solenoid, which can be found on the instrument chassis.
2. Using the formulae (12) and (15) and the formula defining *B*(*I*) that can be found on the instrument chassis, present the *e*/*m* value as the function of the magnetic field induction *B*. Do not calculate the velocity value! Assuming value $t = \frac{x}{d^2 + x^2}$ as the independent variable, convert the obtained formula to the linear dependence in this way that the value *e*/*m* would be the relation factor (the slope) between the magnetic field function and the oscilloscope dot shift. Calculate the *e*/*m* value with application of the least squares method using the Origin software.

4.4. Finding the e/m value with the application of Helmholtz coils

Using the formula (17) calculate *e*/*m* value for every measurement. Obtained results should be treated as **the one measurement series,** calculate the mean value and both types of the uncertainty.

Compare all methods. Which one is the most precise, gives the best results and why?

5. Questions (see www for full list of the questions)

1. Describe the electron motion in the uniform magnetic field. Calculate the pitch of the helix when the electron enters the magnetic field and the angle between the velocity and magnetic fields vectors is equal α.
2. What is the magnetron?
3. Why does not the value of the anode current in a magnetron decline to zero, although the magnetic field is much greater than the critical field? Describe the method of finding the e/m value with the application of a magnetron.
4. Draw and describe the image on the oscilloscope lamp screen obtained in the case when the alternating voltage to the deflection plates is applied and the magnetic field is constant and a) parallel to the electron lamp axis, b) perpendicular to the oscilloscope lamp axis.
5. How can the Helmholtz coils be used to find the e/m value?

6. References

1. Halliday D., Resnick R., Walker J., Fundamentals of Physics, Six Edition, Chapters 23-29.
2. The Feynman Lectures on Physics, http://www.feynmanlectures.caltech.edu/II_29.html

Appendix. e/m value for a magnetron

Analyzing dependence between the anode current and external magnetic field, it is easy to evaluate the *e/m* value. For this purpose, the electron motion in a magnetron from cathode to anode (as presented in the Fig. D1) will be analyzed. Between anode and cathode the voltage U is applied and the lamp is in the magnetic field $\vec{B}$.

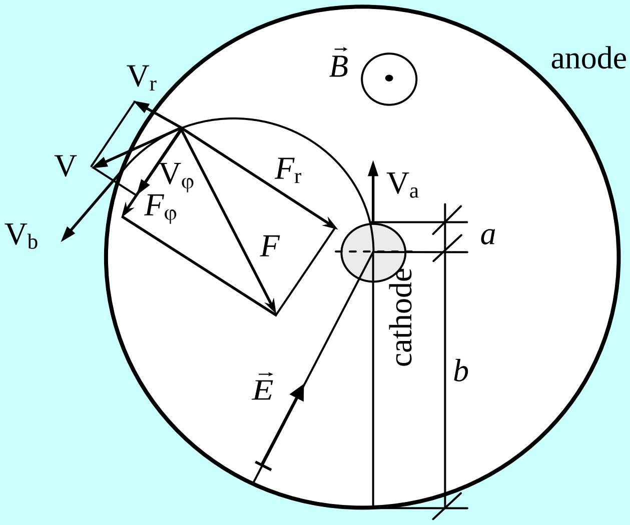


Fig.D1. Forces acting on the moving electron in the magnetron.

The Lorentz force, similarly to the velocity, can be decomposed into two components: in the direction to the centre of the lamp (alongside the radius) $\vec{F}_r$ and in the direction perpendicular do the radius $\vec{F}_\varphi$. Overall force acting on electron has two components $\vec{F}_\varphi$ and $\vec{F}_r + e\vec{E}$ (force $e\vec{E}$ is the effect of the electric field between anode and cathode). Forces $\vec{F}_r$ *and* $e\vec{E}$ act alongside the radius r and cannot change the angular momentum of the electron respect to the lamp axis ($\vec{J} = \vec{p} \times \vec{r} = m\vec{\mathrm{v}}_r \times \vec{r} = 0$, because vectors $\vec{\mathrm{v}}$ and $\vec{r}$ are parallel). The angular momentum $\vec{J}$ of the electron respect to the magnetron axis (equal to the modulus $|\vec{J}| = |\vec{p} \times \vec{r}| = |m\vec{\mathrm{v}}_\varphi \times \vec{r}| = m\mathrm{v}_\varphi r$) can be change only by the component $\vec{F}_\varphi$ of the Lorentz force - its direction and value are defined by the formula (2). The direction of the force $\vec{F}_\varphi$ will be perpendicular to the magnetic vector $\vec{B}$ and to the component $\vec{\mathrm{v}}_r$ of the electron velocity. Using the formula (1) one can write:

$$\vec{F}_\varphi = e(\vec{\mathrm{v}}_r \times \vec{B}) \qquad \text{(D1)}$$

It must be underlined that the value of the Lorentz force is changing, because the component v_r of the velocity is changing due to the presence of the electric filed. Using the Newton's second law of motion for the rotational motion:

$$\vec{M} = \frac{d\vec{J}}{dt} \qquad \text{(D2a)}$$

where $\vec{M} = \vec{r} \times \vec{F}_\varphi$ is the momentum of the force, which according to formula (D1) is equal:

$$|\vec{M}| = |\vec{r} \times [e(\vec{\mathrm{v}}_r \times \vec{B})]| \;, \qquad \text{(D2b)}$$

and $\vec{J}$ - is the angular momentum equal:

$$\left|\vec{J}\right| = \left|m\vec{v}_\varphi \times \vec{r}\right| = m v_\varphi r \, . \qquad \text{(D2c)}$$

Inserting formulae (D2b) and (D2c) to (D2a) and taking into account that in the formula (D2b) $v_r = dr/dt$ we obtain:

$$Ber\frac{dr}{dt} = \frac{d}{dt}(mv_\varphi r) \, . \qquad \text{(D3)}$$

Multiplying the last equation by dt and integrating both sides along the radius from cathode to anode, i.e. from *a* to *b*:

$$eB_{kr}\int_a^b r\,dr = \int_{amv_a}^{bmv_b} d(mv_\varphi r), \qquad \text{(D4)}$$

we obtain:

$$eB_{kr}\frac{b^2 - a^2}{2} = bmv_b - amv_a \, . \qquad \text{(D5)}$$

Assuming that v_a equals to zero does not create significant error, because majority of electrons emitted from the cathode has velocity direction precisely alongside the radius and has not the component v_φ. The value of the v_b can be calculated in very easy manner. It can be assumed that the electrons reaching the anode have only the component v_φ (for $B = B_{kr}$ the direction of the electrons velocity is tangent to the anode surface - see fig.4). The kinetic energy of the electrons is equal to the work of the electric field force:

$$\frac{mv_b^2}{2} = eU \qquad \text{(D6)}$$

Hence:

$$v_b = \sqrt{\frac{2eU}{m}} \, . \qquad \text{(D7)}$$

Inserting this result to the equation (D5) we obtain:

$$\frac{1}{2}eB_{kr}(b^2 - a^2) = bm\sqrt{\frac{2eU}{m}} \, . \qquad \text{(D8)}$$

The magnetic field B_{kr} can be expressed as the function of the current I_{kr} flowing through the solenoid generating magnetic field:

$$B_{kr} = \mu\mu_0 N I_{kr} \; , \qquad \text{(D9)}$$

where μ - permeability of the material, $\mu_0 = 4\pi \cdot 10^{-7}\frac{Vs}{Am}$ - permeability of the free space, N – the number of coils per one unit of the length. After a few simple transformation of the formula (D8) and taking into account (D9), we obtain the formula defining *e/m* value (for the free space we take μ=1):

$$\frac{e}{m} = \frac{8U}{\mu_0^2 N^2 I_{kr}^2 b^2\left(1 - \frac{a^2}{b^2}\right)^2} \, . \qquad \text{(D10)}$$