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30

HALL EFFECT IN SEMICONDUCTORS

1. Background

1.1. Electron motion in electric and magnetic fields

A particle of charge *q* moving in the magnetic field experiences the force called **Lorentz force**:

$$\vec{F} = q(\vec{v} \times \vec{B}), \qquad (1)$$

where $\vec{v}$ is the velocity of the charge, $\vec{B}$ - the induction of the magnetic field. Taking into account properties of the vector product, the force vector is perpendicular to the plane containing vectors $\vec{v}$ and $\vec{B}$, its direction can be found using right hand screw rule. The value of the Lorentz force is:

$$F = qvB \sin\angle(\vec{v}, \vec{B}). \qquad (2)$$

It can be easily seen that the magnetic force does not act on electric charge in situations when the charge is not in motion (v = 0) or when it is moving parallel to induction lines of the magnetic field (sinus of the angle between vectors $\vec{v}$ and $\vec{B}$ is equal to zero). Lorentz force has the biggest value, when the velocity vector direction is perpendicular to magnetic field $\vec{B}$. So the trajectories due to the Lorentz force (line, circle or helix) depend on the angle between vectors of the velocity and the magnetic induction. It must be underlined that Lorentz force is always perpendicular to the motion direction of the charge, what causes (for the steady magnetic field) that **the work of this force is always equal to zero**. For the segment of the trajectory $d\vec{L}$ this work dW is equal $\vec{F} \cdot d\vec{L}$ and is equal to zero, because the vectors $\vec{F}$ and $d\vec{L}$ are always perpendicular. For this reason, the steady magnetic field **cannot change the kinetic energy of the moving charge**, thus its velocity value; magnetic field can change only the direction of the motion.

A particle of charge *q* in the electric field $\vec{E}$ experiences force:

$$\vec{F} = q\vec{E}. \qquad (3)$$

The value of this force - on the contrary to the magnetic field – does not depend on the velocity of the moving charge.

Total force acting on the charge in the presence of both magnetic and electric fields is the sum of the vector forces given by formulae (1) and (3) and can be written as follows:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

1.2. Microscopic characterization of the electric current

All materials can be conventionally divided into three groups depending on their electrical conduction, in other words the ability to transport electric charges. These groups are: **conductors, semiconductors and isolators**. In conductors (mainly metals) the charge carriers are free electrons, their number is approximately equal to the number of atoms; conductors conduct electric current very well. In semiconductors the number of the charge carries is several orders lower comparing to the metals. As charge carriers can act both electrons (negative charge) and holes (positive charge); conductivity of semiconductors is much smaller than metals and additionally can be altered in the wide range using doping (the intentional introduction of impurities into semiconductor). Isolators conduct electric current insignificantly, because of lack of the free charge carriers. More specific description of the electric current conduction mechanism can be found **in Appendix A** in this manual.

So the flow of the electric current in metals and semiconductors is related to the existence of the free charge carriers – electrons in metals and electrons and holes in semiconductors. **The density of the current depends on the charge carriers' concentration *n* and their velocity called the drift velocity v_d:**

$$\vec{j} = ne\vec{v}_d \,, \tag{4}$$

where *e* is the electron charge.

In the absence of the external fields, free carriers are moving chaotically in random directions with big velocities (about 10^5 – 10^6 m/s). When the electric field *E* is applied, the free carriers gain the velocity component parallel to the electric field direction and start to move - „drift" – accordingly to this direction with the velocity dependant on the electric field magnitude. This mean, steady velocity is called **drift velocity v_d**:

$$\vec{v}_d = \mu\vec{E} \,. \tag{5}$$

The drift velocity depends linear on the electric field. The proportionality coefficient between electric field and drift velocity is called **mobility μ.** Mobility is the one of the fundamental microscopic parameters characterizing given material. If we put equation (5) into (4), the relation called **microscopic Ohm's Law** will be obtained:

$$\vec{j} = ne\mu\vec{E} = \sigma\vec{E} \,, \tag{6}$$

where **σ = neμ is the electrical conductivity of the material**. The formulae given above describe the situation when all the charge carriers have the same sign. If in the semiconductor two types of the charge carriers exist (electrons and holes), the equation defining conductivity must take into consideration both types of the charge carriers.

The electrical conductivity (specific conductance) is the reciprocal of the **resistivity (specific resistance) ρ**, which defines **resistance R** of the given sample:

$$R = \frac{1}{\sigma}\frac{l}{S} = \rho\frac{l}{S} \,, \tag{7}$$

where l is the length of the sample and S is its cross section.

1.3. Hall effect

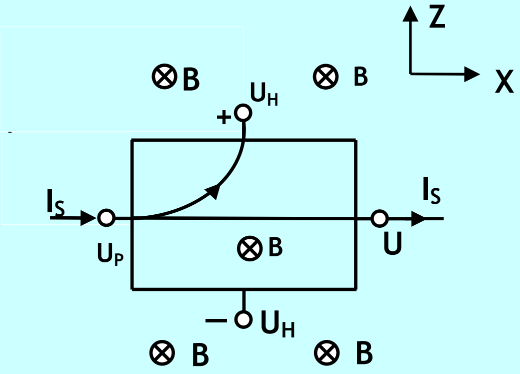


Fig. 1. Creation of the Hall voltage. (I_s -current flowing through the sample, called control current, U_H - Hall voltage measurement terminals, U_p - terminals to measure voltage in the direction of the flowing control current, B - external magnetic field)

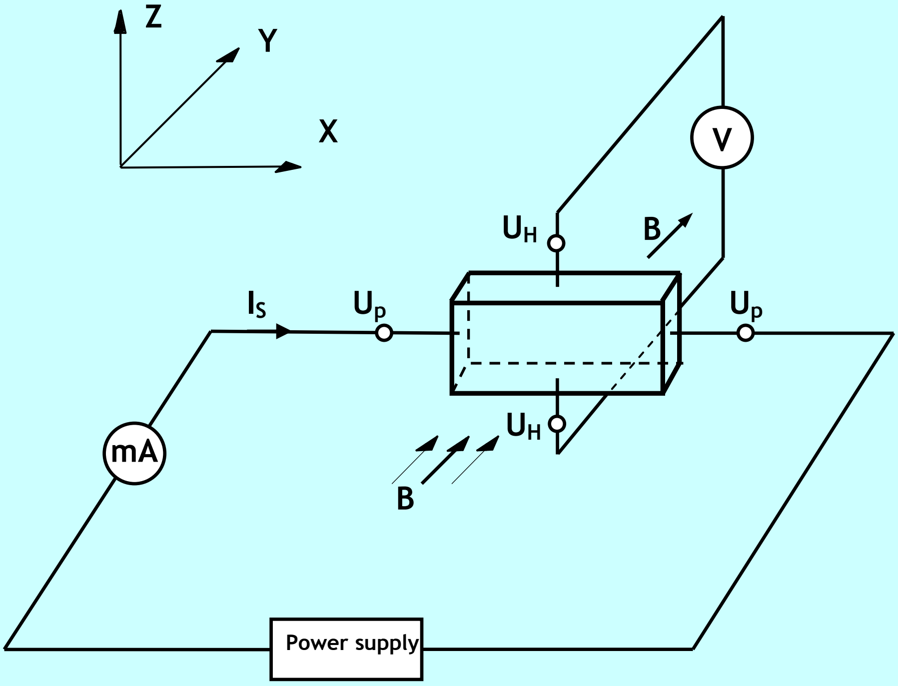


Fig. 2. Setup for the Hall effect measurements.

Hall effect will be described in the sample with the one type of charge carriers only (see fig. 1 and 2). Dimensions of the sample are as follows: **l (length** in the direction X of the control current), **w (width** in the direction Z) and **t (thickness** in the direction Y). After the switching on the power supply, the electric field in the sample is created and the charge carriers start to move with the drift velocity v_d parallel to the direction of this field (X). When the magnetic field B perpendicular to this direction (in the direction Y) is applied, their trajectories are curved as the result of the Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$. The moving charges accumulate on the one face of the sample (in the direction Z), and on the opposite face the amount of charges will be smaller. As the result, in the direction Z the new internal electric field E_H will be created. The accumulation of the charges will be finished when both forces (from the magnetic and newly created electric fields) applied to the charge carriers will balance each other, and the voltage U_H across faces in the Z direction will stabilize. **The production of this**

new voltage is called Hall effect. Both forces have the same axis and opposite directions, so the balance condition can be written as follows:

$$\begin{aligned} q\mathrm{v}_d B &= qE \\ \mathrm{v}_d B &= E \end{aligned} \tag{8}$$

The Hall voltage U_H can be calculated:

$$U_H = E_H w = \mathrm{v}_d B w = \frac{J_s}{ne} B w = \frac{I_s}{newt} B w = \frac{1}{ne} \cdot \frac{I_s B}{t} \tag{9}$$

The value 1/ne is called the **Hall constant R_H**:

$$R_H = \frac{1}{ne} \cdot \tag{10}$$

The above formula is valid only with **two provisions**:

1. Taking assumption that all the charge carriers move with the same velocity. This is true only in the metals. In the laboratory setup the Hall effect is measured in the p-type semiconductor, in which mainly the holes act as the charge carriers. These holes moving with different velocities are scattered by the lattice vibrations. Thus the Hall constant is defined as $R_H = r/ne$, where r is constant depending on the scattering mechanism. When the charge carriers scatter on the lattice vibrations, $r = 3\pi/8$.

2. Above formulae are valid only for weak magnetic fields. „Weak field” means the field which causes small changes of the trajectory between collisions. The magnetic field can be named “weak”, when the following condition is fulfilled:
   $\mu B << 1$.

If the value of the Hall constant is known, the concentration of the charge carriers can be evaluated using the equation (10). From the equation (6) the following expression can be derived:

$$\sigma = ne\mu = \frac{\mu}{R_H} \tag{11}$$

This relation can be used to obtain the mobility of charge carriers since the conductivity of the sample is known. The conductivity of the sample can be obtained using equation (7), measuring the resistivity of the sample in the flowing current direction and knowing the dimensions of the sample.

1.4. Application of the Hall effect

The Hall effect has found the wide application in many industries in the form of the Hall effect sensors. These sensors generate voltage in the presence of the magnetic field - in the laboratory setup, the teslameter is equipped with Hall probe with the semiconductor sensor. The Hall effect sensors are widely used in the automation devices (limit switches, displacement transducers, electric motors control), in automotive industry (ABS and ESP devices, injection devices, ignition systems), high current measurements without circuit disruption (proximity sensors which measure magnetic field generated by the flowing electric current) and many other applications.

2. Measurements

2.1. The Hall voltage measurements

1. Connect the laboratory setup accordingly to the suggestions presented on the information plates near the setup. After the correct connection you should obtain the setup similar to the one presented in fig. 2. The source of the magnetic field is the electromagnet. The electric current flows through its windings (the electromagnet is not shown in fig. 2, in order not to complicate the scheme). The sample is located in the narrow gap between poles in the homogenous magnetic field. The specific data of the used devices, the measurement ranges and technical data can be found on the information plates.
2. Connect the millivoltmeter to (4) terminals to measure the Hall voltage.
3. Once all the connections have been checked by the supervisor, turn all the devices on, starting from the power supply of the measurement module.
4. Make sure that the teslameter shows magnetic field induction equal to zero (2000 mT range). Otherwise adjust it with the knob (2). Make sure the teslameter shows 0 mT.
5. Use the knob (2) to set the control current flow (I_c) on 30 mA. Measure the Hall voltage (U_H) against the magnetic field induction (B) (changed by the adjustment of **the coil current**). The measurement range is from -320 to +320 mT with the increment of 40 mT. To change the direction of the field, change the polarity of terminals. The number and conditions of measurement series (for various current values) will be defined by the tutor.
6. Set the induction on 250 mT. Measure the Hall voltage against **the control current** in the range from -30 to +30 mA with the step 5 mA. The control current can be changed with the knob (2). The number and conditions of measurement series (for various induction values) will be defined by the tutor.

2.2. The resistivity of the sample measurements

1. Disconnect the wires powering the coil to make sure that the magnetic field is off. Connect the millivoltmeter to (3) terminals to measure the voltage in accordance with the control current flow.
2. Measure the sample voltage against the current flowing through the sample in the range from 5 to 30 mA with the step 5 mA. The control current can be changed with the knob (2).

3. Results

3.1. The Hall constant evaluation

1. Basing on the measurements done in points 2.1.5 and 2.1.6, draw two graphs ($U_H \cdot t$) vs. ($B \cdot I_c$): (a) the first one when the magnetic field is constant and (b) the second one when the control current is constant. The plotted measurement points must have the uncertainty bars.
2. Using the least squares method (in Origin software), calculate the Hall constant and its uncertainty, taking into consideration the uncertainty types A and B. Does the χ^2 test confirm the theoretical dependence?

3.2. The resistance and the microscopic parameters of the sample evaluation

1. Basing on the measurements done in the point 2.2.2, using the least squares method, find the sample resistance and its uncertainty, taking into consideration the uncertainty types A and B. Does the χ^2 test confirm the Ohm's law?
2. Find the remaining microscopic parameters of the semiconductor sample, according to the description given in this manual. Assume that the charge carriers are scattered on the lattice vibrations.
3. Calculate the uncertainties of all obtained parameters.
4. Check the "weak field" condition in the laboratory setup.

4. Questions (the complete list - see laboratory web site)

1. Describe the electron motion in the uniform magnetic field. Calculate the pitch of the helix trajectory, when the electron enters the magnetic field with the angle α to magnetic field vector.
2. Microscopic Ohm's law.
3. What is the Hall effect?
4. How the mobility and the concentration of the charge carriers can be evaluated?

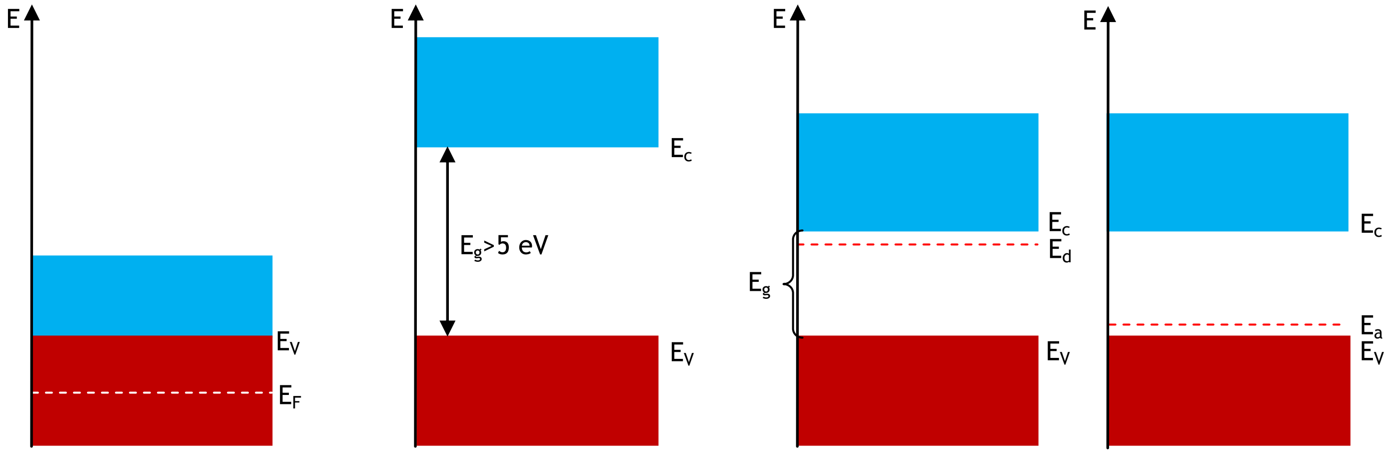
5. References

1. Halliday D., Resnick R., Walker J., Fundamentals of Physics, Wiley, 2001, chapter 29-4.

Appendix A

The electrical conductivity of the metals and semiconductors

The **band model** is the key to understand the mechanism of the electrical conductivity of the metals and semiconductors. How can we imagine the origin of the energetic bands in any material? Each atom has precisely defined the number of the electrons which can occupy the specific energetic levels only. It means that the electrons in the atom can have only certain energetic states (the simplest case is the energetic structure of the hydrogen atom described in the manual for the **exercise 29**). According to the **Pauli exclusion principle**, two or more fermions (electrons) cannot occupy the same quantum state. It means that only two electrons can have the same energy, but thus the electrons must have opposite half-integer spins. When the two similar atoms are coming near they start to interact. Before this interaction, the energetic levels of the atoms were identical, so to fulfill the Pauli exclusion principle after the close proximity of the atoms, the slight change of the energy levels must occur. It is often said that each level of the atom must be split into two new levels. The number of the available energetic levels for the electron will double. Adding consecutive atoms creates the crystalline lattice of the material, in which every level of the single atom is split into the sublevels which number is equal to the number of the atoms forming the lattice. The number of atoms in materials is extremely huge - in our sample there are approximately 10^{23} atoms, so the number of these new levels is huge and the differences in their energy are close to zero. These extremely close levels can be considered as a continuous energy bands separated by the energy gaps - the bands of the energy which the electrons cannot have. Depending on the atoms forming the sample, the energetic bands can be occupied by the electrons in different ways. In the fig. A1 the electronic band structures for the **metals, isolators and semiconductors** are presented.



metal *isolator* *type n semiconductor* *type p semiconductor*

Fig. A1. Electronic band structure for different materials - the valence bands are presented in red and the conduction bands in blue.

The valence band is the highest band containing electrons in the ground states in the temperature 0 K. **The conduction band** is the lowest band with the vacant electronic states which can be filled with the electrons from the valence band. The flow of the electrons from valence to conduction band may occur as a result of the energy supply in the form of the heat, photon, electric or magnetic field etc. The electrons in the conduction band act as charge carriers and form the electric current. Between the valence and conduction bands there is a gap called **band gap**. Another important parameter used in the description of electrical conductivity is **the Fermi level**. It defines the highest energy level occupied at the temperature 0 K. More information about the Fermi level and Fermi-Dirac distribution can be found in the manual for **exercise 21**.

In the **metals** the conduction band overlaps the valence band and the Fermi level is located under the top of the valence band close to the center of this band. Thus in such material there is a huge number of the non occupied states with nearly the same energies, to which the electrons can jump without adding any energy. Thanks to this fact the metals have very high conductivity.

In the **semiconductors** the situation is quite different. In the semiconductors the Fermi level is situated between the top of the valence band E_v and the bottom of the conduction band E_c. So the Fermi level lies in the energetic gap; it means that at the temperature 0 K there are no electrons in the conduction band. The resistance of the semiconductor is infinite. When the temperature rises, the number of the electrons in the conduction band rises as well and the semiconductor starts to conduct the current. Its conductivity is significantly smaller comparing to the metal, because of the small number of the electrons – charge carriers. The energetic gap cannot be too big (must be smaller than 2-3 eV), in order to enable the electron jumping to the conduction band. When the electron jumps to the conduction band, the unoccupied state in the valence band is created which is called the **hole**. The external electric field moves the electrons in the specific direction and the holes move in the opposite. The holes act as the positive charge carriers **(and their charge is equal to the electron charge)**. The electrons in the conduction band and the holes in the valence band can be the current charge carriers in the semiconductors.

In the figure A1 two additional energetic levels E_d and E_a are presented. These levels are connected with **doping**, so as the result of insertion the atoms with the number of valence electrons different than number of valence electrons of the atoms creating this material to the crystalline lattice. **In the n-type semiconductors**, the dopant atoms have more valence electrons than the material atoms. As the n-type doping we can describe adding the phosphorus atoms to the silicon crystal. The phosphorus atom called **donor** has one more electron than the silicon atom and creates additional energetic level (called the **donor level)**, in the space E_d from the bottom of the conduction band. The energy distance between the donor level and the bottom of the conduction band is much smaller than the energetic gap E_g, so the energy required moving the electron from the donor level to the conduction band is small. It causes the significant rise of the conductivity of the doped semiconductor comparing with the undoped (intrinsic) semiconductor. The conductivity growth depends on the number of atoms of the dopant.

Similar situation exists when the dopant atoms have less valence electrons than the material atoms. In such situation the **p-type semiconductor** is obtained. As the p-type doping we can describe adding the aluminum atoms to the silicon crystal. The aluminum atom called **acceptor** has one less electron than the silicon atom and creates additional energetic level (called the **acceptor level)**, in the space E_a from the top of the valence band. The energy distance between the acceptor level and the top of the valence band is much smaller than the energetic gap E_g, so the energy required moving the electron from valence band to the acceptor level is small. It causes the significant rise of the conductivity of the doped semiconductor comparing with the undoped (intrinsic) semiconductor. Similar to the n-type semiconductors, the conductivity growth depends on the number of atoms of the dopant.

When the energetic gap is very big, for example 5.5 eV for the diamond, the possibility of moving the electron to the conduction band is low. The materials with big gaps have nearly no charge carriers in the conduction band, so they do not conduct the electric current and are called **isolators**.