23 - LIGHT REFLECTION ON THE DIELECTRICS' SURFACE

Manual for the virtual experiment at Laboratorium Fizyki I teren południowy

The aim of this exercise is to verify Snell's law and to determine the dielectric refractive index for light with application of various methods.

1. FUNDAMENTALS

1.1 Electromagnetic wave

In diffraction, interference and polarization phenomena, light acts as an electromagnetic wave - i.e. as a series of propagating in space variable electric and magnetic waves. The electromagnetic wave is described by an **electric field vector** ($\vec{E}$), **magnetic field vector** ($\vec{B}$) and **wave vector** $\vec{k}$ that determines the direction of the wave propagation $|\vec{k}| = {}^{2\pi}/_{\lambda}$ where λ is the wavelength.

$\vec{E}$, $\vec{B}$ and $\vec{k}$ vectors are mutually perpendicular, thus an electromagnetic wave is a **transverse wave**. Electric and magnetic fields can be described as sinusoidal functions of position *x* (along the wave propagation direction) and time :

$$\mathbf{E(x,t) = E_0 sin(\omega t - kx + \delta)} \quad (1)$$
$$\mathbf{B(x,t) = B_0 sin(\omega t - kx + \delta)}$$

Where **ω** denotes angular frequency, and **δ** is the initial phase.

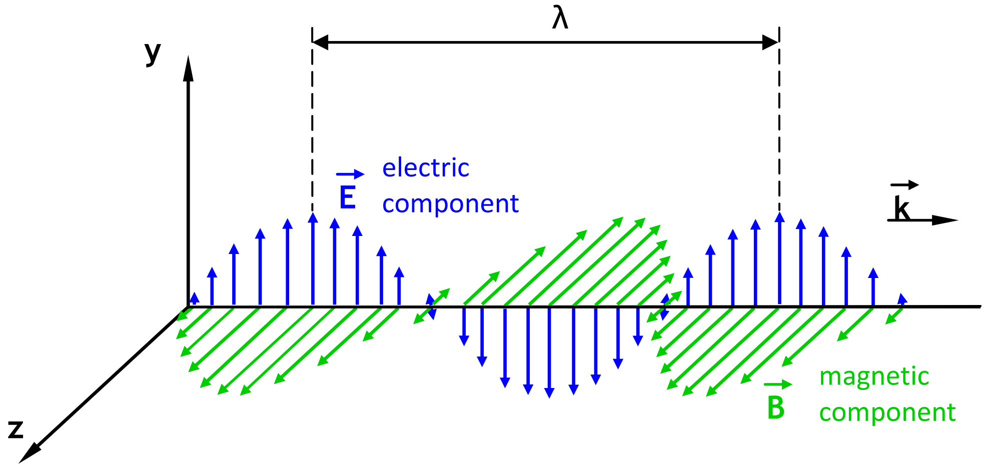


Fig. 1. The image of an electromagnetic wave propagating in x direction.

Since the electric field vector is responsible for all optical phenomena, it is adopted to describe electromagnetic waves with electric field vector $\vec{E}$ and call it "light vector". The electromagnetic waves emitted by an ordinary light sources like the Sun or a bulb are **unpolarized**. It means that the electric field vector is always perpendicular to the propagation direction but it can oscillate in all the possible planes located according to the propagation direction.

The light can be **linearly**, **circularly** and **elliptically** polarized. A linear polarization takes place when the electric vector vibrates in only one plane, that does not change its orientation in space (Fig. 2). Speaking in other words, the vibrations plane of $\vec{E}$ vector remains the same along the entire ray.

If the end of the electric field $\vec{E}$ vector rotates along a circular or ellipsoidal helix, we say that the light is circularly or elliptically polarized.

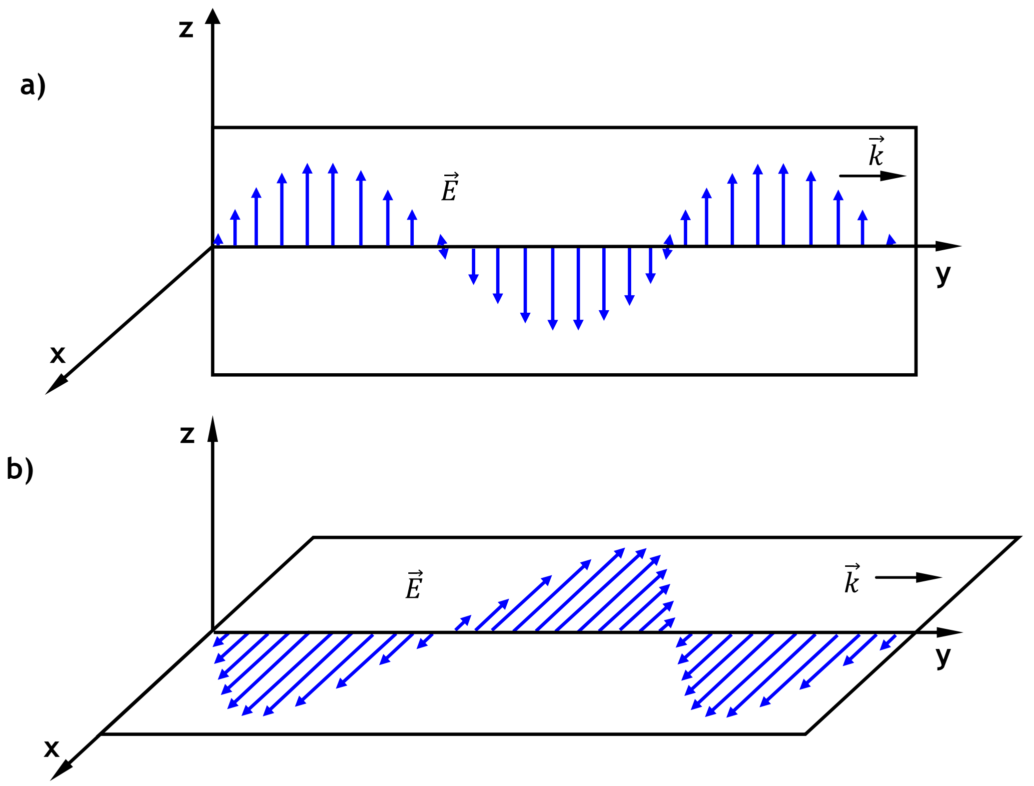


Fig. 2. A wave polarized linearly: $\vec{E}$ vector vibrates in a plane: a) yz , b) xy

The most common method of obtaining linearly polarized light can be achieved by passing it through the polarizer. A polarizer is an optical element that transmits light in a specific direction polarization (Fig. 3). After passing through a polarizer, non-polarized light becomes polarized light linearly with the direction of polarization in line with the polarizer axis. If we set up two polarizers in the path of light with perpendicular axes, the light beam will be completely extinguished.

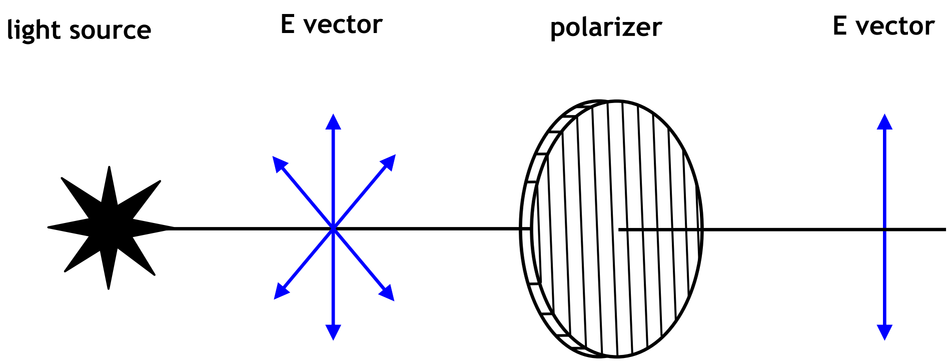
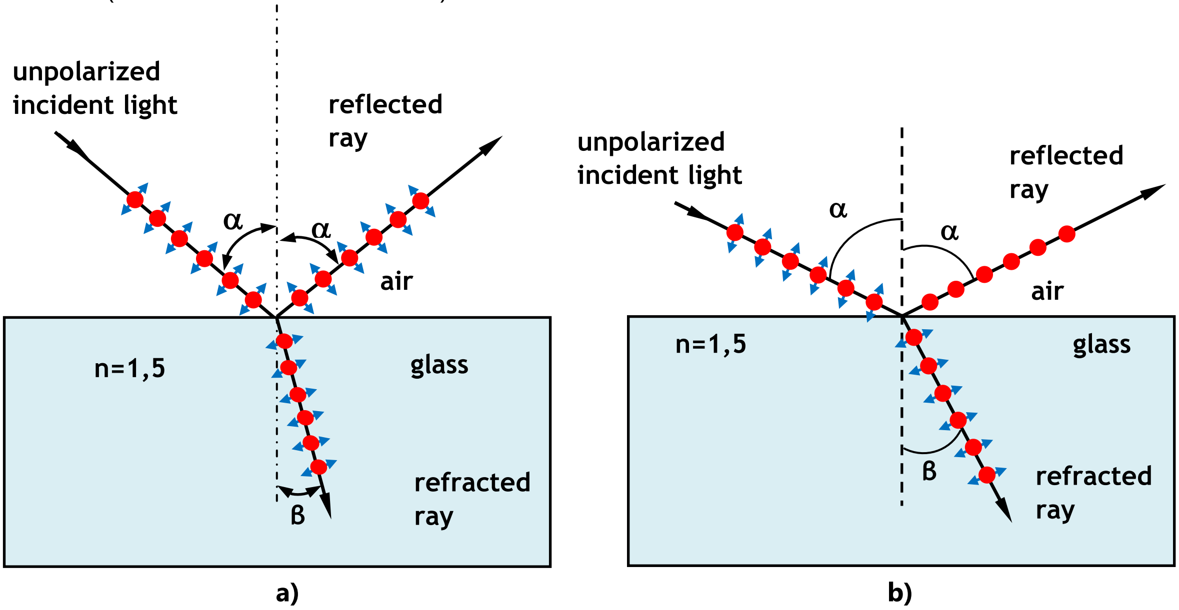


Fig. 3. Polarization of an unpolarized light beam with application of a polarizing filter.

1.2 Dielectrics and the law of refraction

Dielectrics are bodies in which, due to the lack of free electric charges, the electric field can be generated and maintained without any losses (ideal dielectric). Because there exist insignificant amounts of free charges in dielectrics, one conventionally considers as dielectrics (insulators) those materials, whose resistivity at room temperature is greater than ρ = 100 Ωm. Dielectric placed in the electric field gets **polarized**, which means that the positive and negative electric charges are separated, forming electric dipoles. This is also how the electric field of an electromagnetic wave acts: it induces the **formation of electric dipoles**. Because the electric field of an electromagnetic wave is variable in time, the resulting dipoles vibrate with a frequency equal to the frequency of incident light, generating their own electromagnetic wave. This secondary electromagnetic wave interferes with the primary wave causing creation of the output wave which: (**1) has a velocity lower than the primary wave and (2) a different propagation direction**. Both phenomena can be easily observed when light (electromagnetic wave) passes from one medium to another of different dielectric properties. There is a change of direction of wave propagation at the border of both media, i.e. a **refraction** phenomenon occurs. At the same time, part of the wave does not pass into the other medium and undergoes the phenomenon of **reflection** (see fig.3a.)
Let us consider a situation where an electromagnetic wave passes from vacuum (where the speed of wave propagation is is equal to the speed of light c) to the dielectric medium. Such a wave begins due to the phenomena described above propagate at a speed v, **lower than the speed of light**. The speed v is dependent on the properties of the medium, and the ratio of the speed of light in a vacuum to the speed of light in a medium is called the **absolute refractive index** of the medium denoted by the letter n (note that n is always greater than one). In case of laboratory analysis of the wave transfer from air to a dielectric, the obtained measurement result of the refractive index is the same as the transition from a vacuum as the speed of light in air is almost equal the speed of light in a vacuum (air refractive index is 1.0003).



Rys.3. Reflection and refraction of light on the border of two media: a) incident at any angle, b) the refracted and reflected rays are perpendicular (The E vector component perpendicular to the sheet surface is marked with the red dots - σ polarization, the parallel one – with the blue arrow- π polarization)

The directions of propagation of the refracted and reflected rays are precisely defined - see Figure 3a. All three the rays lie in one plane. **All angles (incidence, reflection, refraction) are relative to normal to the material surface at the point of wave incidence. The reflection angle is always equal to the angle of incidence**, while the relationship between the angles of incidence and refraction is determined by **Snell's law**:

$$\frac{sin\alpha}{sin\beta} = \frac{n_2}{n_1},$$

where n2 and n1 denote, respectively, the absolute refractive indices of the medium 2 and 1. If we assume that medium 1 is air with a refractive index equal to 1 (as for vacuum), then Snell's law can be written as:

$$\frac{sin\alpha}{sin\beta} = n\,,$$

where n is the absolute refractive index of the medium in which the light wave is refracted.
The laws of reflection and refraction are the basic laws of geometric optics that were formulated 400 years ago and can be easily derived from the Fermat principle (a light ray travels the path between two points on the shortest possible optical path, i.e. the travel time is the minimum).
This form of the law of reflection and refraction determines only the directions of propagation of reflected and refracted rays in relation to the direction of propagation of the incident ray and the plane separating the media. Snell's law does not define how much of the incident light intensity is reflected, and how of it much goes to the other medium. This is defined by the **reflection coefficient R**, which is the ratio of the intensity of the reflected light to the intensity RI_0 of the light incident on the boundary of the media. Since the sum of the intensities of the reflected and refracted wave must be equal to the intensity of the incident wave, the intensity of the wave passing to the second medium is *(1-R)*I_0. The value of the reflection coefficient R depends on the angle of incidence and the value of the refractive indexes of both media, as well as on the polarization of the incident wave. We can distinguish two borderline cases of linear polarization, as shown in Figure 3a. The polarization in which the **electric field vector $\vec{E}$ vibrates in the plane of incidence** (the plane defined by the direction of incidence and the direction perpendicular to the medium boundary) is called the **π polarization**. The polarization in which the **electric field vector $\vec{E}$ vibrates perpendicularly to the incidence plane** is called **polarization σ**. For σ polarization, the incident wave vector $\vec{E}$ is tangent to the surface separating both media, and for polarization π is directed to this surface in terms of incidence.

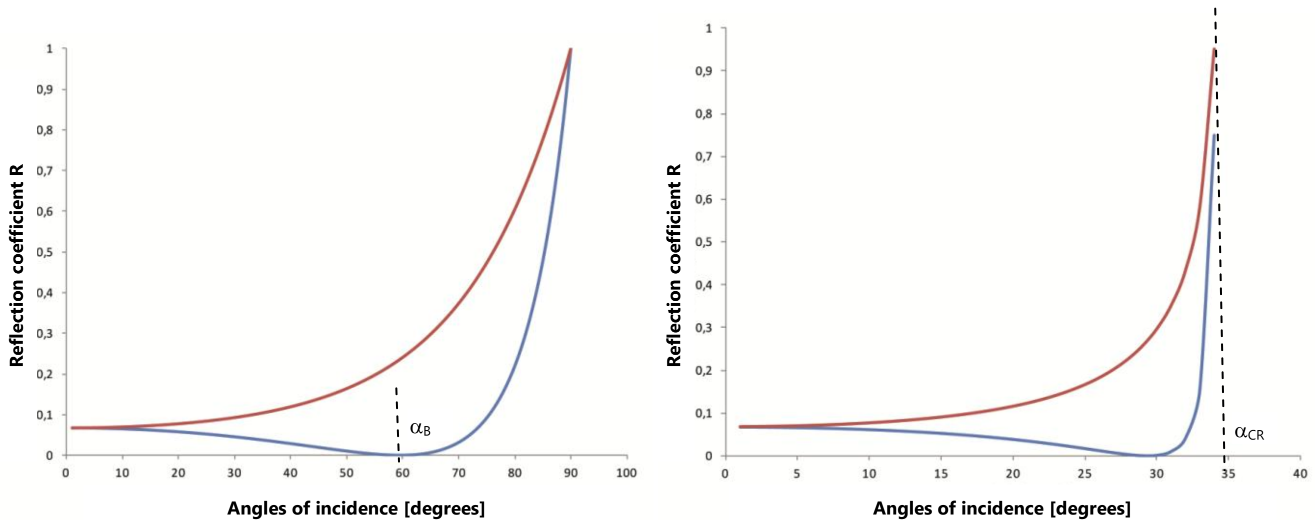


Fig.4. The dependence of the reflection coefficient R for polarized light - red color σ polarization, blue color π polarization - for the passage of light: a) from air to glass b) from glass to air

The reflectance values for both types of polarization define Fresnel formulas:

$$R_\pi = \frac{tg^2(\alpha-\beta)}{tg^2(\alpha+\beta)} \;\; and$$

$$R_\sigma = \frac{sin^2(\alpha-\beta)}{sin^2(\alpha+\beta)}\,,$$

where α and β are the angles of incidence and refraction, respectively, related by Snell's law. The dependence of the reflection coefficients on the angle of incidence for different polarizations is shown in Fig. 4. Fig. 4a shows a graph of the dependence for the case where light falls from the air onto a material with a refractive index equal to n2 = 1.7,

1.3 Air-dielectric transition - Brewster angle

If the light incides perpendicularly to the interface of the media, i.e. the angle of incidence is zero degrees, the reflectance coefficients for both polarizations are equal (why?). It also means that the reflectivity of the light falling on the border of the media is independent of the side from which the light falls. This coefficient is the same for light passing from air to dielectric and from dielectric to air.

For angles of incidence greater than zero, the reflection coefficients for different polarizations start to differ from each other - see Fig. 4. As the angle of incidence increases, the reflectance values for both polarizations change differently. For the polarization σ, the reflection coefficient increases to the value of 1 for the angle of incidence of 90°, which means that the light moves parallel to the media boundary and there is no refracted wave. However, for the polarization π, the reflection coefficient first decreases, for the angle of incidence α_B it reaches zero (the intensity of the reflected light is equal to zero), and then it increases.

The angle of incidence α_B for which there is no reflected wave with a polarization π (polarization for which the electric field strength vector lies in the plane of incidence) is called the **Brewster angle. For the Brewster angle, the reflected and refracted beams form the 90^0 angle** with each other. For the π polarization, the vector of the electric field strength of the refracted wave has the direction in which the reflected wave should appear. As you know, the electric field induced by the electric field does not send a secondary wave towards natural vibrations. Therefore, in the direction in which the π polarized reflected wave should appear, no secondary waves are sent and therefore no reflected wave occurs.

The angle of refraction corresponding to the Brewster angle α_B is equal to $\beta=90°-\alpha_B$, which after substituting Snell's law to obtain:

$$n_1 sin\alpha_B = n_2 cos\alpha_B \ ,$$

$$tg\alpha_B = \frac{n_2}{n_1} \ .$$

If unpolarized light incides on the boundary of two media at the Brewster angle, then only the component of the electromagnetic wave whose polarization is consistent with the σ polarization (linear polarization for which the electric field intensity vector is perpendicular to the plane of incidence) will be reflected. The reflected wave will then be a linearly polarized wave - see Fig. 3b. This phenomenon is used to build light polarizers.

1.4 Dielectric-air transition – critical angle

The reflection of light at the boundary of two different media takes occurs always, except for the case of the Brewster phenomenon described in the previous chapter. The transmission of light to the second medium and the refractive effect observed in the process can only take place in a certain range of incidence angles. This limitation occurs when light is incident from the side of a medium with a higher refractive index, for example for incidence from the side of a glass with a coefficient n1 to the border with air, the refractive index of which is n2 = 1. The value of the refractive index then reaches the maximum value equal to 1 for the angle of incidence α_{CR} (Fig. 4b). For angles greater than α_{CR}, the light is completely reflected. The critical angle of incidence is the angle for which the refracted wave moves along the boundary separating the two media. It means that $\beta=90°$, thus after plugging in into Snell's law gives:

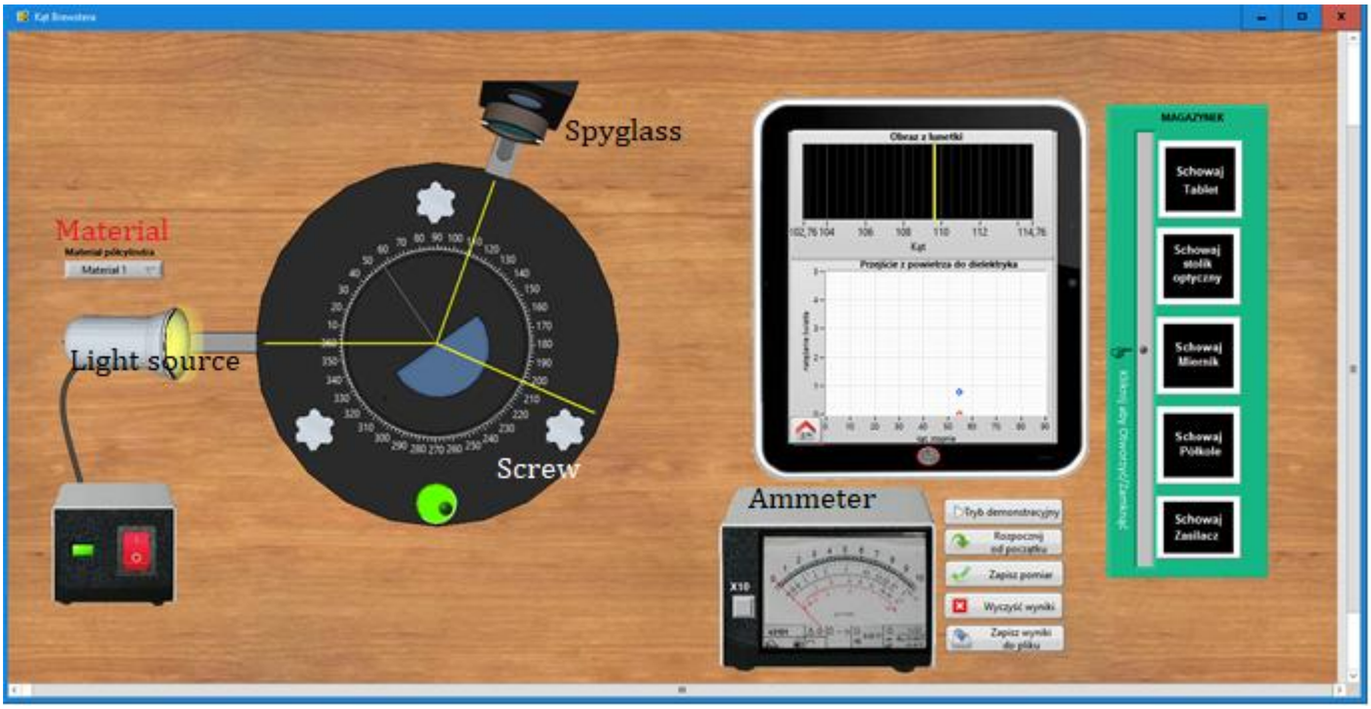
$$sin\alpha_{CR} = \frac{n_2}{n_1} \ .$$

For angles of incidence greater than the critical angle, the value $n_1/n_2\, sin\alpha_{CR}$ is greater than 1 and the refraction law is no longer valid. The wave remains in a medium with a higher refractive index. This phenomenon is called **total internal reflection (TIR)**.

The phenomenon of total internal reflection has found wide application in optics and photonics. It enables the transmission of light (and encoded information) through optical fibers over very long distances. It is makes it possible to built elements that play a role of mirrors - for example, in a prism created by cutting a corner of a glass cube, the light returns exactly to the direction from which it was reflected after several internal reflections. This is how sets of prisms located on the surface of the Moon work, reflecting laser pulses from the Earth, and enabling super-accurate measurement of the Earth-Moon distance (up to millimeters).

2. Measurement setup

The simulation of the light reflection and refraction experipents is performed with ""Brewster's angle" software.



The virtual set consists of:
1. Optical table with the light source, the spyglass and the level display.
2. The semicircle made of dielectric.
3. The light source power supply.
4. The ammeter connected to thl ight detector.
5. Tablet.

Open a locker and take out all the devices. Turn on the light source, the ammeter and tablet. Choose the type of material of semicircle.

3. Measurements

3.1 The Snellius Law and the Brewster angle

1. The first step is leveling of the table. One does that using 3 screws on the table. After the correct adjustment of the screws, the air bubble in the leveler should be located at the centre of green field.
2. On the tablet, run the software "Transition from air to a dielectric"
3. Choose the material of semicircle (it should be chosen by your tutor).
4. Set the semicircle angle at c.a. 10 deg.
5. Turn the spyglass to the position appropriate for measurements of the reflected ray. Measure the light intensity for the vertical position of the polarizer and click "Save measurement point".
6. Change the orientation of the polarizer (click on it). Measure light intensity for the horizontal position of the polarizer and save measurement point.
7. Change the semicircle angle in 5 deg steps up to 85 deg and for each semicircle position make measurements described in 5 and 6.
8. At the end, click "Save all data". The column structure in the file is as follows: intensity of the light in the first polarizer position, intensity of the light in the second polarizer position, angle of the reflected and refracted ray.

3.2 Determining the critical angle

1. The first step is leveling of the table. One does that using 3 screws on the table. After the correct adjustment of the screws, the air bubble in the leveler should be located at the centre of green field.
2. On the tablet, run the software "Transition from dielectric to air"
3. Set the angle of incidence of the light at c.a. 10 deg.
4. Turn the spyglass to the position appropriate for measurements of the reflected ray. Measure the intensity of the light for the vertical polarizer position. Click "save measurements point".
5. Change the polarizer position into a horizontal position and save the measurement.
6. Repeat the steps 4 and 5.
6. Change the angle of incidce with 5 deg steps as long as ray will to glide on the medium surface and repeat points 4 and 5 for next semicircle orientations.
7. Save the angle value, when the ray starts to glide on the surface.

4. Results analysis

4.1. The Snellius Law and the Brewster angle

1. Estimate uncertainty of the angle measurements (in radian!).
2. Calculate given angles values for reflected and refracted rays into absolute values and convert it into radian.
3. Plot a common graph reflectivity for both polarization setting.
4. For the π polarization, estimate Brewster angle.
5. Calculate reflectivity for dielectric and its uncertainty.
6. Calculate uncertainty of the sine angle of reflection and refraction.
7. Taking into account a Snellius law, determine reflectivity of the dielectric using least square method.
8. Calculate A and B type uncertainty for reflectivity.
9. Does chi-square test confirm the Snellius law?

4.2. Determining the critical angle

1. Estimate angle measurements uncertainty.
2. On the common graph plot reflectivity for both polarizations.
3. Based on the graph for both polarization function, estimate critical angle.
4. Calculate the dielectric reflectivity and its uncertainty.

What can you say about results for both method? What material was the semicircle made of?