

## Determination of speed of sound in the air using Kundt's tube.

The exercise aims to determine the speed of sound in the air with Kundt's pipe's application.

### 1. Theory

Waves could be divided into three main types:

1. **Mechanical waves** – waves propagate as an oscillation of the medium, for example, sea waves, sound waves and seismic waves. They are propagating consistent with classical physics laws and can exist only in water, air or solid-state.

2. **Electromagnetic waves** – waves propagate as continuous changes of electric and magnetic fields. This kind of waves doesn't require a medium, and it can propagate in a vacuum. Light, radio waves, TV waves, microwaves, X-rays are examples of electromagnetic waves. They propagate with the speed of light.

3. **Matter waves** – waves related to all objects in motion. The wavelength of matter waves is given by de Broglie formula  $\lambda = \frac{h}{mv} = \frac{h}{p}$ . It is easier to detect matter waves of particles (e.g. electrons) than heavy objects like a human.

The waves can be divided into two types depending on the oscillation medium's direction with respect to wave propagation direction. If the oscillation direction is **perpendicular to the propagation's direction**, this kind of wave is called the **transverse wave**. If the oscillation **direction is parallel** to the propagation's direction, this kind of wave is called a **longitudinal wave**.

A following formula is used to describe the propagation of the harmonic wave :

$$y(x, t) = A \sin(kx - \omega t),$$

where:

$y(x, t)$  – displacement as a function of position  $x$  and time  $t$ ,

$A$  – amplitude of the wave,

$k$  – wavenumber equal to  $2\pi/\lambda$ , where  $\lambda$  is a wavelength (distance between two minimum or maximum of the wave)

$\omega$  – circular frequency equal to  $2\pi/T$ , where  $T$  means oscillation period

$(kx - \omega t)$  – phase of the wave.

### Sound wave definition

The propagation of the sound wave in the air is an example of wave motion. Air particles are stimulated to vibrations, and they are undergoing periodic densification and thinning, which can be moved far from the sound source. This type of vibrations take place along the same line as distortion propagation, so the **sound wave is a longitudinal mechanical wave**. The sources like human vocal

cords, strings of musical instruments, vibrating speakers membranes produce mechanical airwaves with a frequency range of 20 – 20000 Hz, which can be detected by our ears and analyzed by the brain a sound. The waves with a frequency above 20000 Hz are called ultrasound, and waves with a frequency below 20 Hz are called infrasound.

### The sound wave velocity

For any wave which is propagating in a mechanical medium, the velocity is given by the formula

$$v = \frac{\lambda}{T} = f\lambda$$

where:  $\lambda$  – wavelength,  $T$  – period,  $f$  – frequency [Hz]  $f = \frac{1}{T} = \frac{\omega}{2\pi}$

The sound wave velocity depends on temperature. The sound wave propagation is related to air density in a given temperature by relationship:

$$\rho = \frac{\rho_0}{1 + \alpha T}$$

where:  $\rho_0$  – air density in 0 °C (237,16 K),  $\alpha$  – coefficient of volume expansion of air (1/K),  $T$  – the temperature in Kelvin's scale (K).

Finally, the relationship between sound velocity and temperature is given by the formula:

$$v = v_0 \sqrt{1 + \alpha T}$$

where  $v_0 = 331,8$  m/s in 0 °C.

The above formula works fine for sound waves with low intensity. Generally, the sound velocity is higher for higher temperature (see table).

Temperature [°C]	v [m/s]
-40	306,5
-20	319,3
0	331,8
20	343,8
40	355,3

### Standing waves

The waves undergo a phenomenon of **interference** (overlapping of waves). The result of these phenomena, the wave motion, is complicated. If two waves with the same frequency and amplitude travel in the opposite directions, due to overlapping, we obtain a **standing wave**.

Now suppose, that two waves characterized by the same frequency and amplitude will interfere:

- the first:  $y_1(x, t) = A \sin(kx - \omega t)$

- the second:  $y_2(x, t) = A \sin(kx + \omega t)$

Applying the superposition principle new result wave will be given by equation

$$y(x, t) = y_1(x, t) + y_2(x, t) = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

and after simplification

$$y(x, t) = [2 \sin(kx)] \cos(\omega t)$$

It is called the **standing wave equation**.

The brackets factor can be understood as an amplitude of oscillations in x position, and the factor given by cosine function describes the variation of oscillation in time.

In the case of harmonic wave, the oscillations' amplitude is the same at every point of a space. **In the case of standing wave amplitude depends on the position in space.** The amplitude is equal to zero if the standing wave equation  $kx$  will be given  $\sin(kx) = 0$ . The condition is satisfied when:

$$kx = n\pi \quad (n = 0, 1, 2, \dots)$$

so

$$x = n \frac{\lambda}{2}$$

The points where amplitude = 0 are called **nodes**.

If the amplitude is equal to 1,  $\sin(kx) = 1$ , the condition is satisfied when

$$kx = \left(n + \frac{1}{2}\right) \text{ for } (n = 0, 1, 2, \dots)$$

so

$$x = \left(n + \frac{1}{2}\right) \frac{\lambda}{2}$$

and this kind of points, where amplitude = 1 are called an **arrow**.

Usually, this type of waves begins when the vibration bumps right off some obstacle. A standing wave can be made in the vibrating string (transverse wave). We can see a standing wave thanks to interfering with two sine-type vibrations with the same amplitude and frequency going into the opposite directions. The distance between two nodes (two arrows) is half of the wavelength  $\lambda/2$ .

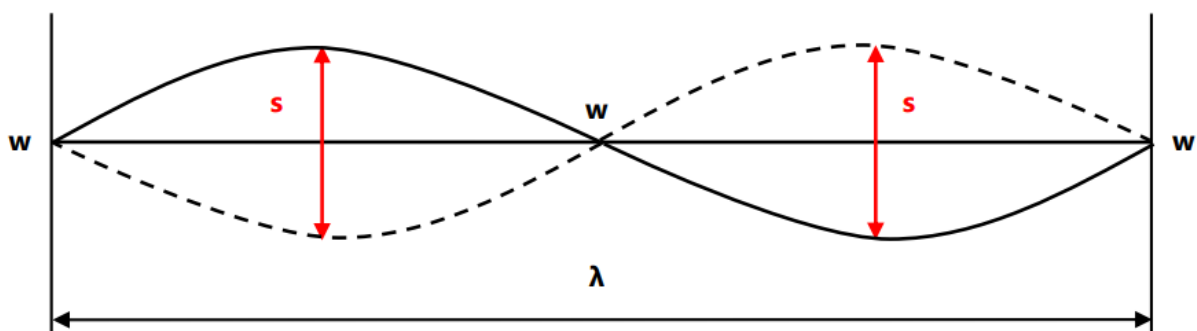
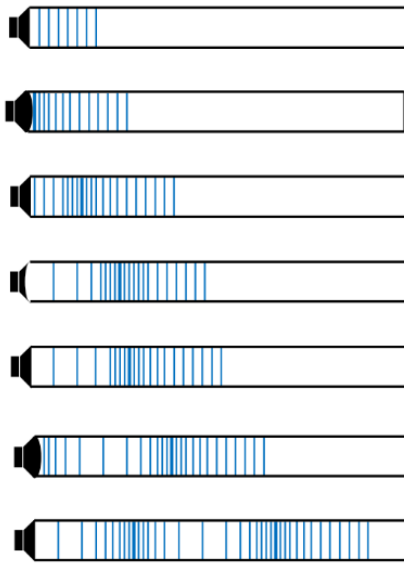


Figure 1 A standing wave on a string fixed at both ends. The nodes (w) and arrows (s) are marked.

In this exercise, there will be examined sound standing waves (or mechanical longitudinal waves). A speaker will generate the wave in a closed tube - Kundt's tube. The speaker membrane is vibrating, and it provides the propagation of high and low-pressure regions (high and low air density) inside the pipe. When the vibration meets the end of the tube, a wave is reflected and returned to the source (the wave changes by  $180^\circ$ ). If the sound wave frequency satisfies a standing wave's

condition, the pipe will be placed with high pressure (arrows) and low pressure (nodes). The nodes and the arrows will not move for standing wave in the pipe. This characteristic frequency is called



**harmonic frequency**, and the first harmonic frequency takes place when the first node is on the end of the tube. The formula gives the condition of the generating standing waves in Kundt's tube:

$$L = (2n + 1) \frac{\lambda}{4} \quad n = 0, 1, 2, \dots$$

where  $L$  is a Kundt's pipe length.

Positions  $x$  of the arrows can be found using the formula

$$x_{n,m}^{(s)} = \frac{2mL}{2n + 1}$$

and the positions of the nodes are

$$x_{n,m}^{(w)} = \frac{(2m + 1)L}{2n + 1}$$

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*Figure 2 Propagation of the sound wave in a pipe using a loudspeaker*

$n$  is the harmonic frequency number, and  $0 \leq m \leq n$  is the next arrow or node from a speaker. For  $m=0$ , the arrow is on the speaker position, and the next node lies at distance  $\lambda/4$ .

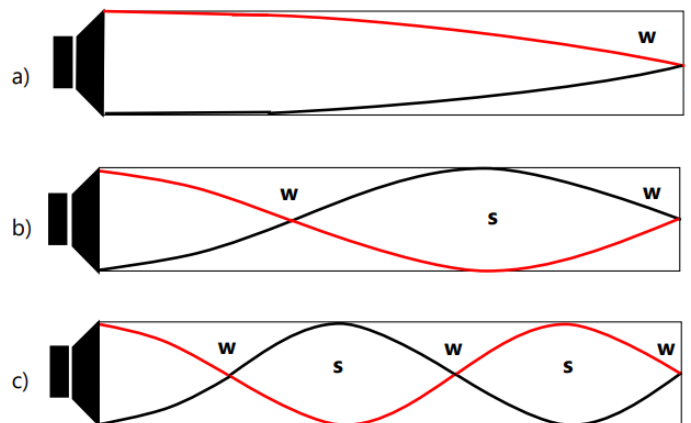
Measurement of distance between the following nodes of the standing wave for known wave frequency gives the possibility of finding the sound speed. The formula provides harmonic frequencies

$$f_n = \frac{2n + 1}{4L} v$$

Furthermore, the distance between the following resonance frequencies is constant, and it depends on the ratio wave speed and pipe length:

$$\Delta f = f_{n+1} - f_n = \frac{v}{2L}$$

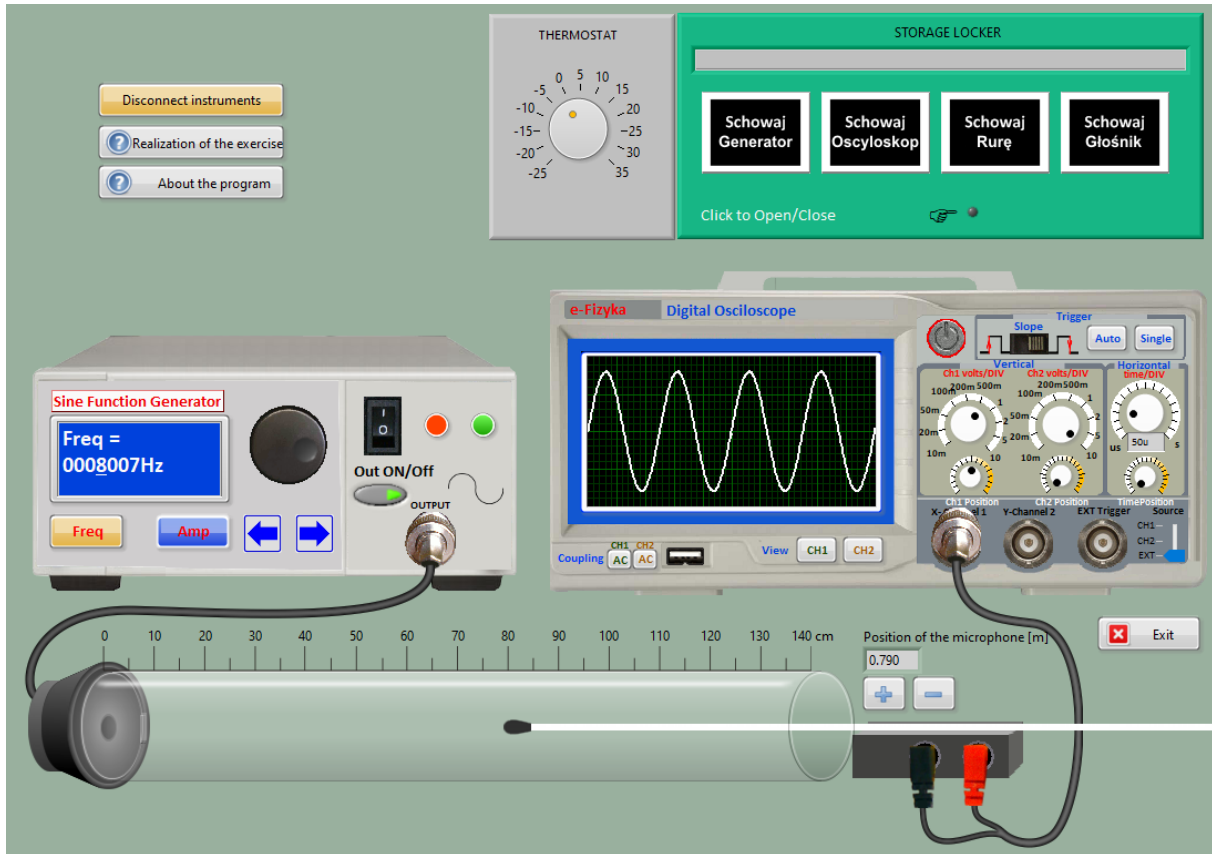
If we know the distance between nodes for standing wave in Kundt's tube, we can determine the sound speed for a given air temperature.



*Figure 3 Standing waves in Kund's tube for 1st, 2nd and 3rd harmonic frequency.*

## 2. Experimental Setup

In this experiment, we will use Virtual Laboratory Exercise called "Kundt's tube". Simulation is written in LabView 2020.



The experimental setup will be described during classes.

## 3. Measurements

**Accuracy of frequency measurement:**  $c_1=0.5\%$ ,  $c_2=0.02\%$ , ranges: 1, 10, 100 kHz.

**Accuracy of the microphone position:**  $\Delta x=0.003$  m

### Method A

1. Set an air temperature on the thermostat (choose any).
2. Measure the distance between nodes of the standing wave for 10 chosen frequencies.
3. Based on the dependency of frequency and wavelength, make a plot. Data points should lie on the line. Put on the chart uncertainties and make a linear fitting.
4. From the line's slope, determine the speed of the sound for chosen air temperature inside the pipe.
5. Determine the A type uncertainty of the speed of sound. For one chosen point, calculate the B type uncertainty and find combined uncertainty if it is necessary.
6. Report the final result with the uncertainties and compare your result with the known sound speed for chosen temperature.