Probability density functions of returns and volatility forecasting for financial markets

E. Alessio[1)], V. Frappietro[1)], M. I. Krivoruchenko[1,2,3)], and L. J. Streckert[1)]

[1)]Metronome-Ricerca sui Mercati Finanziari, C. so Vittorio Emanuele 84
10121 Torino, Italy
[2)]Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25
117259 Moscow, Russia
[3)]Institute für Theoretische Physik, Tübingen Universität, Auf der Morgenstelle 14
D-72076 Tübingen, Germany

ABSTRACT

Lévy stable truncated distributions [1] are known to provide for financial time series (i) an approximate scaling invariance of probability density functions (PDFs) of returns with a slow convergence to the Gaussian behavior and (ii) existence of "fat tails" in the PDFs. We show that the k-dimensional Student PDFs have useful features for modeling joint PDFs of returns in a financial time series. These distributions have marginal PDFs which satisfy the scaling property exactly and have fat tails from the start. The k-dimensional Student PDFs can be modified further to describe quantitatively two other well established "stylized facts" which are (iii) long ranged volatility-volatility correlations that are also known as volatility clustering [2] and (iv) return-volatility correlations that are also known as leverage effect [3,4]. We then fix free parameters of the modified k-dimensional Student PDFs over a short term period by fitting 3 to 8 years of trade-by-trade quotes from the Eurex Bund, Bobl, DAX, and EuroSTOXX futures contracts, and over the long term by fitting 100+ years of the DOW Jones 30 Industrial Average (DJIA) and 50+ years of the Standard & Poors 500 (S&P 500) daily prices. The joint PDFs of returns are used then for the probabilistic forecasting of volatility. Two estimators are considered to quantify volatility. These are short-term dynamic and long-term static PDFs of volatility which are then constructed. The volatility distributions are compared with the historical ones of the Eurex Bund, Bobl, DAX, and EuroSTOXX futures contracts and with the historical volatility distributions for the DJIA and S&P 500 indices. Cycles in the financial time series are identified using the Fisher statistical criterion and considered along with trends as causal components of the time evolution. We construct, in particular, for the DJIA index, short-term dynamic PDFs of volatility for one to six month time horizons, starting from the 13.11.03.

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