Triangular Arbitrage in the Foreign Exchange Market

*Yukihiro Aiba, Naomichi Hatano, Hideki Takayasu**
Institute of Industrial Science, Tokyo University, Komaba 4-6-1, Meguro, Tokyo 153-8505, Japan
**Sony CSL, Higashi-Gotanda 3-14-13, Shinagawa, Tokyo 141-0022, Japan*

We show that there are in fact triangular arbitrage opportunities in the spot foreign exchange markets [1], analyzing the time dependence of the yen-dollar rate, the dollar-euro rate and the yen-euro rate. Based on the analysis, we propose a model of foreign exchange rates with an interaction. The model includes effects of triangular arbitrage transactions as an interaction among three rates. The model explains the actual data of the multiple foreign exchange rates well [1]. We show further, on the basis of the model, that the triangular arbitrage makes the auto-correlation function of foreign exchange rates negative in a short time scale [2].

The triangular arbitrage is a financial activity that takes advantage of the three exchange rates among three currencies. When a trader exchanges one Japanese yen to some amount of US dollar, exchanges the amount of US dollar to some amount of euro and exchanges the amount of euro back to Japanese yen instantly at time t, the final amount of Japanese yen is given by the product of the three exchange rates. If the rate product is greater than unity, the trader can make profit through the above transaction. This is the triangular arbitrage transaction. Once there is a triangular arbitrage opportunity, many traders will make the transaction. This makes the rate product converge to a value less than unity, there by eliminating the opportunity. This is the source of the interaction among the rates. Triangular arbitrage opportunities nevertheless appear, because each rate fluctuates strongly [1].

We introduced a stochastic model describing the time evolution of the exchange rates with an interaction. The model gives an explanation to an interesting feature of the fluctuation of foreign exchange rates [2]. The auto-correlation function of the fluctuation of the foreign exchange rates has been known to be negative in a short time scale [3]. Our model suggests that an important ingredient of the negative auto-correlation is the triangular arbitrage [2].

References

[1] Y. Aiba, N. Hatano, H. Takayasu, K. Marumo and T. Shimizu, Triangular arbitrage as an interaction among foreign exchange rates, Physica A 310 (2002) 467-479..

[2] Y. Aiba, N. Hatano, H. Takayasu, K. Marumo and T. Shimizu, Triangular arbitrage and negative auto-correlation of foreign exchange rates, Physica A to be published.

[3] H. Takayasu, M. Takayasu, M.P. Okazaki, K. Marumo, T. Shimizu, in: M.M. Novak (Ed.), Fractal Properties in Economics, Paradigms of Complexity, World Scientific, Singapore, 200, pp. 243-258.