Time evolution of the probability distribution of returns in the Heston model of stochastic volatility compared with the high-frequency stock-market data

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An analytical formula for the probability distribution $P_t(x)$ of the log-returns x after the time lag t was derived in Ref. [1] for the Heston model of stochastic volatility [2]. In Refs. [1] and [3], this formula was compared with the daily data for the major stock-market indexes, and a good agreement was found for the time lags t from a day to a year. Now we compare the formula with the high-frequency, tick-by-tick data for share prices of selected companies, such as Johnson & Johnson (JNJ) and Intel (INTC). With a given set of three parameters, we find a reasonable agreement between the data and the theory for the time lags from 5 minutes to about 3 months. The formula derived in Ref. [1] evolves continuously from an exponential, tent-shape distribution for short t to a Gaussian distribution for long t. This shape evolution correctly captures the main features of data behavior.

Ref. [1] gives a closed-form and relatively simple formula for the characteristic function (the Fourier transform) $\tilde{P}_t(k)$ of $P_t(x)$. For this reason, we have developed a method for direct calculation of the characteristic function $\tilde{P}_t(k)$ from the data set. Straightforward comparison between the empirical and theoretical characteristic functions greatly simplifies determination of the model parameters and improves the accuracy of the fit. This general method may useful for other quantitative studies of stock market as well.

References

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