The Multi-Fractal Model of Asset Returns:

Its Estimation via GMM and Its Use for Volatility Forecasting

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Abstract: Multi-fractal processes have been proposed as a new formalism for modeling the time series of returns in finance. The major attraction of these processes is their ability to generate various degrees of long memory in different powers of returns - a feature that has been found to characterize virtually all financial prices. Furthermore, elementary variants of multi-fractal models are very parsimonious formalizations as they are essentially one-parameter families of stochastic processes. The aim of this paper is to provide the characteristics of a *causal* multi-fractal model (replacing the earlier combinatorial approaches discussed in the literature), to estimate the parameters of this model and to use these estimates in forecasting financial volatility. We use the auto-covariances of *log increments* of the multi-fractal process in order to estimate its parameters consistently via GMM (Generalized Method of Moment). Simulations show that this approach leads to essentially unbiased estimates, which also have much smaller root mean squared errors than those obtained from the traditional 'scaling' approach. Our empirical estimates are used in out-of-sample forecasting of volatility for a number of important financial assets. Comparing the multi-fractal forecasts with those derived from GARCH and FIGARCH models yields results in favor of the new model: multi-fractal forecasts dominate all other forecasts in one out of four cases considered, while in the remaining cases they are head to head with one or more of their competitors.

Keywords: multi-fractality, financial volatility, forecasting

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