Volatility Multi-scale Stochastic Dynamics and Scaling Sources

*Enrico Capobianco**

Abstract

A recent and important advance in financial mathematics has been the idea of exploiting high frequency market information for estimating the volatility process underlying the observed return dynamics. This problem has a well defined stochastic characterization by adopting a semimartingale probabilistic approach, and by looking at two related measures: realised and integrated volatility. The former is an empirical measure used to approximate the latter, since it is its integral counterpart. This approximation is found by averaging a certain number of high frequency returns observed intra-daily so to obtain an estimate of the daily volatility. Semimartingales are key because according to the quadratic variation principle, the convergence of realised to integrated volatility can be verified both in time and frequency coordinates. From the experimental standpoint, the cumulative squared high frequency returns can be seen as consistent estimators for the integrated volatility. We show through some simulations that equally valid convergence properties hold when scale coordinates are considered and wavelet-based estimators are adopted. Then, we show with an empirical study that multi-scaling behavior characterizes the volatility dynamics underlying a stock index return series.

Keywords: *Semimartingales; Quadratic Variation; Realised and Integrated Volatility; Inverse Problems; Wavelet Estimators; Multi-scaling*

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*The author is currently a visiting scientist at the International Centre for Theoretical Physics, Condensed Matter group. Most of this work has been done while at CWI, Amsterdam.