Modelling financial market by the multiplicative sequence of trades

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The inherent origin of the $1/f$ noise of the signals consisting of a sequence of pulses may be the Brownian fluctuations of the average interevent time between subsequent pulses of the pulse sequence [1, 2]. In this contribution we generalize the model of interevent time to reproduce a variety of self-affine time series exhibiting the power spectral density $S(f)$ scaling as a power of the frequency f. The specific interest of our analysis is related with the financial markets, where the long-range correlations of the price fluctuations and trading activity largely depend on the number of transactions [3, 4].

We consider a signal $I(t)$ as a sequence of the δ-type random correlated pulses $I(t) = \sum_k \delta(t - t_k)$ and define the number or trades N_j in the time interval τ_d as a signal integral, $N_j = \int_{t_j}^{t_j+\tau_d} I(t)\, dt$. Such point process is completely described by the set of times of the events $\{t_k\}$ or equivalently by the set of interevent intervals $\{\tau_k = t_{k+1} - t_k\}$. We base our study on the model for the stochastic multiplicative interevent time τ_k written as Langevine stochastic differential equation in k-space, $d\tau_k/dk = \gamma\tau_k + \tau_k\sigma\xi\,(k)$, where $\langle \xi\,(k)\,\xi\,(k')\rangle = \delta(k - k')$. Here σ denotes the standard deviation of the white noise and $\gamma \ll 1$ is a damping constant. Such model system exhibits power-law spectral density $S(f) \sim 1/f^\beta$ for various values of β, including $\beta = 1/2$, 1 and 3/2. We derive explicit expressions for the power spectra in the low frequency limit and for the distribution density of the interevent time.

Furthermore, we analyze the relation between the power-law autocorrelations and the origin of the power-law probability distribution of the trading activity N_j [5]. The counting statistics of the events is analyzed numerically and analytically. We analyze the spectral density and counting statistics of the number of transactions, as well. The model reproduces the spectral properties of trading activity and explains the mechanism of power law distribution in the real markets. The study provides the evidence that the statistical properties of the trading activity in the financial markets are involved in the statistics of the time interval between trades. A multiplicative stochastic point process serves as a consistent model generating this statistics.

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