# Statistical laws and dynamics of an open market in very short time scales

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We analyze high-resolution foreign exchange data consisting of about 20 million data points of USD-JPY for 13 years. We confirm the following statistical laws:

1. The cumulative distribution of exchange rate fluctuations is approximated by a power law distribution with exponent -3.
2. Autocorrelation function for the rate change at 1 tick $[dP(t) = P(t+1\text{tick}) - P(t)]$ shows a negative value –0.5 when the time lag is 1 tick and the correlation value is zero in the case of other time lags.
3. There are higher order correlations. We investigate conditional probability after n-successive signs, where the up-down signal of "+" and "-" denotes the exchange rate goes up and down. We confirm that the conditional probability depends on n.

Next, we derive the following nonlinear probabilistic equation from trader's basic actions,

$$P(t+1) = P'(t) + b_1(t) \cdot \Delta P'(t) + b_2(t) \cdot (P(t) - P'(t)) + f(t), \qquad (1)$$

where, $P$ is market price and $P'$ is a moving average of $P$, $b_1$ is a market trend and $b_2$ is fluctuation a trend, and $f$ is a white noise. We explain the mechanism that produces above statistical laws from this equation.

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| 1 | : | The present price fluctuation is dependent on past price fluctuations. |
| 2&3 | : | There are violent noises and gradual trends in price change dynamics. |

In workshop, we report these results and also recreate the statistical laws of volatility and the price diffusion in very short time scales.