Possible origin of long-term correlations in financial time series

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Abstract

In the last decade an increased effort of statistical physicists has been undertaken to understand the long-term power-law correlations present in the financial time series [1–7]. In this context, we extend the recently considered toy model of Weierstrass walks with varying velocity of the walker [8] by introducing a more realistic possibility that the walk can be occasionally intermitted by its momentary localization; the localizations themselves are again described by the Weierstrass process. The direct empirical motivation for developing this combined model is, for example, the dynamics of financial high-frequency time series or hydrological and even meteorological ones where variations of the index are randomly intermitted by flat intervals of different length exhibiting no changes in the activity of the system. This combined Weierstrass walks was developed in the framework of the non-separable generalized continuous-time random walk (GCTRW) formalism developed very recently [9]. This approach makes it possible to study by stochastic simulations the whole spatial-temporal range while analytically we cannot study the intermediate one. Our approach is possible since the Weierstrass walks is a geometric superposition of regular random walks each of which can be simply treated by stochastic simulations. This non-Markovian two-state (walking-localization) model makes possible to cover by the unified treatment a broad band of known up to now types of non-biased diffusion from the sub- over the normal, super- up to the hyperdiffusion. We observed that anomalous diffusion is characterized here by three fractional exponents: the temporal characterizing the localized state and complementary temporal and spatial characterizing the walking one. Moreover, by considering successive dynamic exponents we constructed a series of different diffusion phase diagrams on the plane defined by the temporal and spatial exponents (characterizing the walking state). To adapt the model to the description of empirical data, which are collected with a discrete time-step (i.e., the discrete time series), we used in the basic continuous-time series produced by the model a discretization procedure. We observed that such a procedure generates, in general, long-range non-linear autocorrelations even in the Gaussian regime, which appear to be similar to those observed, e.g., in the financial time series [10–13], although single steps of the walker within continuous time are, by definition, uncorrelated. This suggests a supprising origin of long-range non-linear autocorrelations alternative to the one proposed very recently (cf. [14] and refs. therein) although both aproaches involve related variants of the well-known CTRW formalism applied yet in many different branches of knowledge [15–17].

Key words: Weierstrass or Lévy walks with varying velocity, Weierstrass and combined Weierstrass walks, Stochastic hierarchical spatial-temporal coupling, Anomalous diffusion, Long-term non-linear autocorrelations, Non-Gaussian stochastic process, Fractional spatial and temporal dynamic exponents, Power-law, Scaling relations
PACS: 89.90.+n, 05.40.+j, 02.50.-r, 02.50.Ey, 02.50.Wp

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